

FUZZY CONNECTEDNESS IN FUZZY PENTA TOPOLOGICAL SPACE

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Abstract- *The purpose of this paper is to study fuzzy connected space and fuzzy separation in fuzzy penta topological space (Fp-topological space) and study some of their properties.*

Keywords- Penta topological spaces, p-connected space and p-separation, Fp-topological spaces, Fp-connected space and Fp-separation.

Mathematics subject classification: 54A40

I.INTRODUCTION

J.C. Kelly [4] introduced the concept of bitopological space. W. J. Pervin [9] was define connectedness in a bitopological space. J. Swart [12] studied connectedness in bitopological spaces. Tri topological space is a generalization of bitopological space. The tri topological space was first initiated by Martin Kovar [5]. S. Palaniammal [8] studied tri topological space and he also introduced fuzzy tri topological space. D.V. Mukundan [7] introduced quad topological space. We [13] [14] introduced tri connectedness in tri topological space and quad connectedness in quad topological space. Penta topological space was introduced by Muhammad Shahkar Khan and Gulzar Ali Khan[6]. We [15] discussed open sets in Penta topological space.

In 1965, Zadeh L.A. [16] introduced the concept of fuzzy sets. In 1968 Chang C.L. [1] introduced the concept of fuzzy topological spaces. K.S. Sethupathy Raja

and S. Lakshmivarahan [11] introduced connectedness in fuzzy topological space. Kandil A. [2] [3] introduced fuzzy bitopological spaces. We [10] introduced fuzzy connectedness in fuzzy tri topological space. In this paper, we introduce fuzzy connectedness and fuzzy separated sets in fuzzy penta topological space.

II. PRELIMINARIES

Definition 2.1[13]: Let (X, T_1, T_2, T_3) be a tri topological space, a subset A of X is said to be tri disconnected if and only if it is the union of two non-empty tri separated sets. That is, if and only if there exist two non-empty separated sets C and D such that $C \cap tri\,cl(D) = \phi$, $tri\,cl(C) \cap D = \phi$ and $A = C \cup D$, A is said to be tri connected if and only if it is not tri disconnected.

Definition 2.2[11]: A fuzzy topology X is said to be disconnected if $X = A \cup B$, where A and B are non-empty open fuzzy sets in X such that $A \cap B = \phi$. A fuzzy topological X is said to be connected if X cannot be represented as the union of two non-empty, disjoint open sets on X .

Definition 2.3[6]: Let (X, τ) be a p-topological space. A subset A of X is called penta-open (p-open) if $A \in \tau_i$; $i \in \{1, 2, 3, 4, 5\}$ and its complement is said to be penta-closed (p-closed).

III. CONNECTEDNESS IN Fp-TOPOLOGICAL SPACE

Definition 3.1: Let X be a nonempty set $\tau_1, \tau_2, \tau_3, \tau_4$ and τ_5 are fuzzy topologies on X . Then a fuzzy subset χ_λ of space X is said to be fuzzy p-open if

$\chi_\lambda \prec \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4 \cup \tau_5$ and its complement is said to be fuzzy p-closed and set X with five fuzzy topologies called fuzzy p-topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$.

Definition 3.2: Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a fuzzy penta topological space and let $\chi_\lambda \prec X$. The intersection of all fuzzy p-closed sets containing χ_λ is called the fuzzy p-closure of χ_λ & denoted by $p-cl(\chi_\lambda)$. We will denote the fuzzy p-interior (resp. fuzzy p-closure) of any fuzzy subset, say of χ_λ by fuzzy $p-int(\chi_\lambda)$ ($p-cl(\chi_\lambda)$), where $p-int(\chi_\lambda)$ is the union of all fuzzy p-open sets contained in χ_λ , and $p-cl(\chi_\lambda)$ is the intersection of all fuzzy p-closed sets containing χ_λ .

Definition 3.3: Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a fuzzy penta topological space. X is said to be Fp-connected if X cannot be written as the union of two disjoint non-empty Fp-open sets.

Example 3.4: Let $X = \{1, 2, 3, 4\}$ be a nonempty fuzzy set. Consider five fuzzy topologies $\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1\}}, \chi_{\{1,2\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1\}}, \chi_{\{1,3\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{4\}}\}$, $\tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{2\}}\}$ $\tau_5 = \{1_X, 0_X\}$ Fp-open sets are $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{1\}}, \chi_{\{4\}}, \chi_{\{2\}}, \chi_{\{1,2\}}, \chi_{\{1,3\}}\}$, X cannot be written as the union of two non-empty disjoint Fp-open sets. Hence X is Fp-connected.

Theorem 3.5: Fp-topological space X is called Fp-connected if and only if X cannot be written as the union of two non-empty disjoint Fp-closed sets.

Proof: Suppose X is Fp-connected. If $X = \chi_{\lambda_1} \vee \chi_{\lambda_2}$ where χ_{λ_1} and χ_{λ_2} are two non-empty disjoint Fp-closed sets. $\chi_{\lambda_1} = \tilde{1}_X - \chi_{\lambda_2}$ And $\chi_{\lambda_2} = \tilde{1}_X - \chi_{\lambda_1}$. Since χ_{λ_1} and χ_{λ_2} are Fp-closed sets and $\chi_{\lambda_2}, \chi_{\lambda_1}$ are Fp-open sets. $X = \chi_{\lambda_1} \vee \chi_{\lambda_2}$ Where χ_{λ_1} and χ_{λ_2} are non-empty disjoint Fp-open sets.

Claim: X is Fp-connected.

If not, let $X = \chi_{\lambda_1} \vee \chi_{\lambda_2}$ where χ_{λ_1} and χ_{λ_2} are two nonempty disjoint Fp-open sets.

$\chi_{\lambda_2} = \tilde{1}_X - \chi_{\lambda_1}$ And $\chi_{\lambda_1} = \tilde{1}_X - \chi_{\lambda_2}$, χ_{λ_1} and χ_{λ_2} are Fp-closed sets.

$X = \chi_{\lambda_1} \vee \chi_{\lambda_2}$ Where χ_{λ_1} and χ_{λ_2} are non-empty disjoint Fp-closed sets.

Hence X is Fp-connected.

Theorem 3.6: An Fp-topological space X is Fp-connected if and only if there does not exist a non-empty fuzzy set which is both Fp-open and Fp-closed.

Proof: Suppose X is Fp-connected. If there exists a non-empty fuzzy set χ_δ which is both Fp-open and Fp-closed. Then $\tilde{1}_X - \chi_\delta$ is a non-empty fuzzy subset of X which is both Fp-open and Fp-closed. Hence $\chi_\delta \vee \tilde{1}_X - \chi_\delta = X$ where χ_δ and $\tilde{1}_X - \chi_\delta$ are non-empty disjoint Fp-open sets. Hence there does not exist a non-empty fuzzy set which is both Fp-open and Fp-closed.

Conversely, if there does not exist a fuzzy non-empty set which is both Fp-open and Fp-closed.

Claim: X is Fp-connected. If not, let $\chi_{\delta_1} \vee \chi_{\delta_2} = \tilde{1}_X$ where χ_{δ_1} and χ_{δ_2} are disjoint non-empty Fp-open sets. Since $\chi_{\delta_2} = \tilde{1}_X - \chi_{\delta_1}$, $\tilde{1}_X - \chi_{\delta_1}$ is Fp-open set.

χ_{δ_1} is Fp-closed set. χ_{δ_1} is a non-empty fuzzy set which is both Fp-open and Fp-closed. Hence X is Fp-connected.

Theorem 3.7: X is Fp-connected if and only if X cannot be written as the union of two non-empty fuzzy sets χ_{δ_1} and χ_{δ_2} where

1. $\chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$

2. $\chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} = \tilde{0}_X$

Proof: Suppose X is Fp-connected.

If $\chi_{\delta_1} \vee \chi_{\delta_2} = \tilde{1}_X$ where χ_{δ_1} and χ_{δ_2} are non-empty fuzzy sets such that

$$\chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X \text{ And } \chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} = \tilde{0}_X.$$

Since $\chi_{\delta_1} \wedge \chi_{\delta_2} \prec \chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$, $\chi_{\delta_1} \wedge \chi_{\delta_2} = \tilde{0}_X$

Hence χ_{δ_1} and χ_{δ_2} are disjoint non-empty fuzzy sets.

Let $\chi_{\{x\}} \leq Fp-cl\chi_{\delta_1} \Rightarrow \chi_{\{x\}} \succ \chi_{\delta_2} \Rightarrow \chi_{\{x\}} \leq \chi_{\delta_1}$ [Since $\chi_{\delta_1} \vee \chi_{\delta_2} = \tilde{1}_X$]

$Fp-cl\chi_{\delta_1} \leq \chi_{\delta_1}$ Always $\chi_{\delta_1} \leq Fp-cl\chi_{\delta_1}$

Hence $\chi_{\delta_1} = Fp-cl\chi_{\delta_1}$

Let $\chi_{\{x\}} \leq Fp-cl\chi_{\delta_2} \Rightarrow \chi_{\{x\}} \succ \chi_{\delta_1} \Rightarrow \chi_{\{x\}} \leq \chi_{\delta_2}$,

$Fp-cl\chi_{\delta_2} \leq \chi_{\delta_2} \Rightarrow Fp-cl\chi_{\delta_2} = \chi_{\delta_2}$, since $\chi_{\delta_2} \leq Fp-cl\chi_{\delta_2}$

χ_{δ_1} And χ_{δ_2} are Fp-closed sets.

Hence $\chi_{\delta_1} \vee \chi_{\delta_2} = \tilde{1}_X$ where χ_{δ_1} and χ_{δ_2} are disjoint non-empty Fp-closed sets. Hence X is Fp-connected.

Hence X cannot be written as the union of two nonempty fuzzy sets A and B where

$\chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$ and $\chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} = \tilde{0}_X$

Conversely, X cannot be written as the union of two nonempty sets χ_{δ_1} and χ_{δ_2}

where $\chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$ and $\chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} = \tilde{0}_X$

Claim: X is Fp-connected. If not, $\chi_{\delta_1} \vee \chi_{\delta_2} = \tilde{1}_X$ where χ_{δ_1} and χ_{δ_2} disjoint non-empty Fp-closed sets.

$\Rightarrow \chi_{\delta_1} = Fp-cl\chi_{\delta_1}$ And $\chi_{\delta_2} = Fp-cl\chi_{\delta_2}$.

And $\Rightarrow \chi_{\delta_1} = \tilde{1}_X - \chi_{\delta_2}$ and $\chi_{\delta_2} = \tilde{1}_X - \chi_{\delta_1}$ Hence

$\Rightarrow \chi_{\delta_1} \wedge (\tilde{1}_X - \chi_{\delta_1}) = \tilde{0}_X$, $\chi_{\delta_1} \wedge \chi_{\delta_2} = \tilde{0}_X$

$\Rightarrow \chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$. Similarly $\chi_{\delta_2} \wedge (\tilde{1}_X - \chi_{\delta_2}) = \tilde{0}_X \Rightarrow \chi_{\delta_2} \wedge \chi_{\delta_1} = \tilde{0}_X$.

Hence $\chi_{\delta_1} \vee \chi_{\delta_2} = \tilde{1}_X$ where χ_{δ_1} and χ_{δ_2} are non-empty fuzzy sets such that

$\chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$ and $\chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} = \tilde{0}_X$. Hence X is Fp-connected.

IV. Fp-SEPARATED SETS IN Fp-TOPOLOGICAL SPACE

Definition 4.1 Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a fuzzy penta topological space. Two non-empty fuzzy subsets χ_{δ_1} and χ_{δ_2} of X are called Fp-separated if $\chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$ and $\chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} = \tilde{0}_X$.

Theorem 4.2: If χ_{δ_1} and χ_{δ_2} are Fp-separated then χ_{δ_1} and χ_{δ_2} are disjoint.

Proof: $\chi_{\delta_1} \wedge \chi_{\delta_2} \prec \chi_{\delta_1} \wedge Fp-cl\chi_{\delta_2} = \tilde{0}_X$ Since χ_{δ_1} and χ_{δ_2} are Fp-separated.

$\Rightarrow \chi_{\delta_2} \wedge \chi_{\delta_1} = \tilde{0}_X$. χ_{δ_1} And χ_{δ_2} are disjoint sets.

Result 4.3: Converse is not true.

Example 4.4: Let $X = \{a, b, c\}$ be a non-empty fuzzy set, consider five topologies

$$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}, \tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}\}, \tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,c\}}\}, \tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\},$$

$$\tau_5 = \{\tilde{1}_X, \tilde{0}_X\}$$

Fp-open sets are $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}\}$

Fp-closed sets are $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{b,c\}}, \chi_{\{c\}}, \chi_{\{b\}}\}$

Let $\chi_{\delta_1} = \chi_{\{a,b\}}$ and $\chi_{\delta_2} = \chi_{\{c\}}$, χ_{δ_1} and χ_{δ_2} are disjoint sets. $Fp-cl\chi_{\delta_1} = Fp-cl\chi_{\{a,b\}} = \tilde{1}_X$

$$\chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} = \chi_{\{c\}} \wedge Fp-cl\chi_{\{a,b\}} = \chi_{\{c\}} \wedge \tilde{1}_X = \chi_{\{c\}} [\neq \tilde{0}_X]$$

Since $\chi_{\delta_2} \wedge Fp-cl\chi_{\delta_1} \neq \tilde{0}_X$, χ_{δ_1} and χ_{δ_2} are not Fp-separated.

Definition 4.5: Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be an Fp-topological space. Let $\chi_{\delta} \leq \tilde{1}_X$. χ_{δ} is called Fp-dense if $Fp-cl\chi_{\delta} = \tilde{1}_X$.

Example 4.6: Let $X = \{a, b, c\}$ be a non-empty fuzzy set, consider five topologies

$$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}, \tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}\}, \tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,c\}}\}, \tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\}, \tau_5 = \{\tilde{1}_X, \tilde{0}_X\}$$

Fp-open sets are $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}\}$

Fp-closed sets are $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{b,c\}}, \chi_{\{c\}}, \chi_{\{b\}}\}$

Let $\chi_{\delta} = \chi_{\{a,b\}}$, $Fp-cl\chi_{\delta} = Fp-cl\chi_{\{a,b\}} = \tilde{1}_X$. Hence χ_{δ} is Fp-dense.

V. Fp-HYPER CONNECTED Fp-TOPOLOGICAL SPACE

Definition 5.1: An Fp-topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ is said to be Fp-hyper connected if every non-empty Fp-open set is Fp-dense in X.

Example 5.2: Let $X = \{a, b, c\}$ be a non-empty fuzzy set, consider five topologies

$$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}, \tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}\}, \tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,c\}}\}, \tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\}, \tau_5 = \{\tilde{1}_X, \tilde{0}_X\}$$

Fp-open sets are $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}\}$

Fp-closed sets are $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{b,c\}}, \chi_{\{c\}}, \chi_{\{b\}}\}$

$$Fp-cl\chi_{\{a\}} = \tilde{1}_X$$

$$Fp-cl\chi_{\{a,b\}} = \tilde{1}_X$$

$$Fp-cl\chi_{\{a,c\}} = \tilde{1}_X$$

Every non-empty Fp-open set is Fp-dense in X.

Hence X is Fp-hyper connected Fp-topological space.

Theorem 5.3: A Fp-topological space X is Fp-hyper connected if and only if any two non-empty Fp-open sets intersect.

Proof: Let X be Fp-hyper connected.

Let χ_{δ_1} and χ_{δ_2} are two non-empty Fp-open sets.

Claim: $\chi_{\delta_1} \wedge \chi_{\delta_2} \neq \tilde{0}_X$

Suppose not then, Hence $\chi_{\delta_1} \leq \tilde{1}_X - \chi_{\delta_2}$

$$Fp-cl\chi_{\delta_1} \prec Fp-cl(\tilde{1}_X - \chi_{\delta_2})$$

Since X is Fp-hyper connected, $Fp-cl\chi_{\delta_1} = \tilde{1}_X$.

Hence $\tilde{1}_X \prec Fp-cl(\tilde{1}_X - \chi_{\delta_2})$

Now $\tilde{1}_X - \chi_{\delta_2}$ is Fp-closed $\Rightarrow Fp-cl(\tilde{1}_X - \chi_{\delta_2}) = \tilde{1}_X - \chi_{\delta_2}$

$\tilde{I}_X \prec (\tilde{I}_X - \chi_{\delta_2}) \Rightarrow \tilde{I}_X = (\tilde{I}_X - \chi_{\delta_2})$ Which implies $\chi_{\delta_2} = \tilde{0}_X$

Since $\chi_{\delta_2} \neq \tilde{0}_X$ $\chi_{\delta_1} \wedge \chi_{\delta_2} \neq \tilde{0}_X$.

Hence any two non-empty Fp-open sets intersect.

Conversely if any two non-empty Fp-open sets intersect.

Claim: X is Fp-hyper connected.

Let χ_{δ_1} be a non-empty Fp-open set.

Claim: $Fp-cl\chi_{\delta_1} = \tilde{I}_X$

If not, $Fp-cl\chi_{\delta_1} \neq \tilde{I}_X$

Then, $1-(Fp-cl\chi_{\delta_1}) \neq \tilde{0}_X$

Let $\chi_{\delta_2} = 1-(Fp-cl\chi_{\delta_1})$

$\chi_{\delta_1} = Fp-cl\chi_{\delta_1}$

Hence $\chi_{\delta_1} \wedge \chi_{\delta_2} = \tilde{0}_X$

But χ_{δ_1} and χ_{δ_2} are non-empty Fp-open sets.

Hence $Fp-cl\chi_{\delta_1} = \tilde{I}_X$

Hence χ_{δ_1} is Fp-dense in X.

Theorem 5.4: X is Fp-hyper connected if and only if any Fp-closed set not equal to X has empty Fp-interior.

Proof: X is Fp-hyper connected.

Let χ_{δ_2} be a P-closed set where $\chi_{\delta_2} \neq \tilde{I}_X$

Claim: $Fp-int\chi_{\delta_2} = \tilde{0}_X$

If not, Let $\chi_{\delta_1} = Fp-int\chi_{\delta_2} = \tilde{0}_X, \chi_{\delta_1} \neq \tilde{0}_X$

Now χ_{δ_1} is a non-empty Fp-open set because $Fp-int\chi_{\delta_2}$ is Fp-open and X is Fp-hyper connected. Hence $Fp-cl\chi_{\delta_1} = \tilde{I}_X$ now χ_{δ_2} is an Fp-closed set containing χ_{δ_1}

$$\Rightarrow Fp-cl\chi_{\delta_1} \prec \chi_{\delta_2}$$

$$\Rightarrow \tilde{I}_X \prec \chi_{\delta_2}$$

Hence $\tilde{I}_X = \chi_{\delta_2}$. Since $\tilde{I}_X \neq \chi_{\delta_2}$ Hence $Fp-int\chi_{\delta_2} = \tilde{0}_X$

Hence any Fp-closed set not equal to X has empty Fp-interior.

Conversely,

Now every Fp-closed set not equal to X has empty Fp-interior.

Claim: X is Fp-hyper connected.

Let χ_{δ_1} be a non-empty q-open set.

Claim: $Fp-cl\chi_{\delta_1} = \tilde{I}_X$

If not, $Fp-cl\chi_{\delta_1}$ is an Fp-closed set not equal to X.

Hence $Fp-int(Fp-cl\chi_{\delta_1}) = \tilde{0}_X$

Now $\chi_{\delta_1} \prec (Fp-cl\chi_{\delta_1})$

$$Fp-int\chi_{\delta_1} \prec Fp-int(Fp-cl\chi_{\delta_1})$$

Since χ_{δ_1} is Fp-open, $Fp-int\chi_{\delta_1} = \chi_{\delta_1}$

Hence $Fp-int\chi_{\delta_1} \prec Fp-int(Fp-cl\chi_{\delta_1}) = \tilde{0}_X$

Hence $\chi_{\delta_1} \prec \tilde{0}_X$

Since χ_{δ_1} is non-empty.

Hence $Fp-cl\chi_{\delta_1} = \tilde{I}_X$

Hence X is Fp-hyper connected.

VI. CONCLUSION

In this paper the idea of fuzzy connectedness and fuzzy separated sets in fuzzy penta topological space were introduced and studied.

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