

STUDY AND COMPARISON ON FRACTIONAL ORDER PID CONTROLLERS FOR FRACTIONAL ORDER TIME DELAY APPLICATIONS USING HARMONY SEARCH ALGORITHM

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ABSTRACT:

The Fractional-Order Proportional-Integral-Derivative (FOPID) Controller presents advancements over the conventional PID controller and serves as a crucial tool for accurately representing physical phenomena in real-world applications. However, current approaches to designing FOPID controllers are overly intricate and not well-suited for practical implementation. The new tuning methods are introduced to the conventional fractional order proportional-integral and derivative controller so that the system gives good performance. This paper introduces an effective approach to designing and tuning FOPID controllers for fractional-order controlled systems with time delays using Harmony Search Algorithm(HSA). The system's results are acquired through the utilization of the MATLAB/Simulink software.

Keywords: Proportional-Integral-Derivative (PID) Controller, Fractional Order PID (FOPID) Controller, fractional –order systems, time delay, Harmony Search Algorithm (HSA)

INTRODUCTION:

A fractional order PID Controller (FOPID - Controller) is a type of control system used to design the controller with the help of fractional calculus. Also, they offer several advantages over traditional PID controllers, such as improved performance in systems with non-integer dynamics, better handling of systems with long memory effects, and increased robustness to parameter variations. They can be particularly useful in applications where standard PID controllers fail to provide satisfactory results, such as in systems with highly nonlinear or time-varying dynamics. The Harmony Search Algorithm draws inspiration from the process of musical improvisation and serves as a metaheuristic optimization technique. When applied to tuning FOPID controllers in systems with fractional-order time delays, HSA enables efficient exploration of the solution space. It facilitates the discovery of optimal or near-optimal controller parameters that align with control objectives, all while accounting for the intricate dynamics of the system.

It offers superior performance compared to traditional PID controllers [1,2] due to the inclusion of two additional adjustable parameters. For instance, the fractional-order derivative has proven effective in handling systems like hydraulic servo systems with significant historical dependency and mechanical inertia [3]. Various time-domain employ optimization techniques as PSO [4], DE algorithms [5], and RBF neural networks [6] to optimize controller parameters. However, these techniques often demand significant time and may result in controllers lacking adequate robustness.

In [7,8], these were devised based on an ideal transfer function derived from Bode plots, entailing intricate computations like data fitting. Similarly, in [9], the controller's adjustable parameters were reduced from five to three by establishing a proportional relationship among them. Despite this, the tuning process remains intricate, involving both data fitting and optimization. Reference [10] proposed a simplified method for tuning FOPID controllers for systems with integer-order plants, albeit it still entails solving a set of implicit nonlinear equations. The Harmony Search Algorithm (HSA) tuned FOPID controllers for conventional systems without time delays were examined in [11]. This paper aims to explore the application of Fractional Order PID controllers tuned using the Harmony Search Algorithm for systems with fractional order time delays and its response is compared to that of with FOPID controller that is tuned using frequency domain specifications.

SYSTEM STUDIED:

This study examines a fractional-order controlled system with broad applicability.

$$P(s) = \frac{K}{T_n s^n + T_{n-1} s^{n-1} + \dots + T_0 s^0} e^{-Ls}$$

The typical structure of a controller is outlined as follows:

$$C_{FOPID}(s) = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu$$

Here, k_p , k_i and k_d represent the gains of the P,I and D components, respectively. Consequently, the controller can be written as:

$$C(s) = k_p + \frac{k_i}{s^\nu} + k_d s^\nu$$

By permitting $\lambda = \mu = \nu$, the enhanced relative stability induced by the derivative compensates for the diminished relative stability brought about by the integral component.

Specifications in the frequency domain:

Examine the closed-loop control system illustrated in Fig 1.

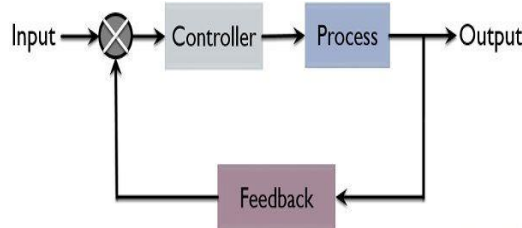


Fig. 1 Closed loop control system

The open-loop transfer function can be expressed as follows:

$$G(s) = P(s) C(s)$$

The controller is tuned using the gain crossover frequency, phase crossover frequency, gain margin, and phase margin simultaneously. These frequency-domain characteristics helps in plotting the Bode-plots. Modifying these frequency domain specifications allows for the attainment of curves with varying magnitudes and phases.

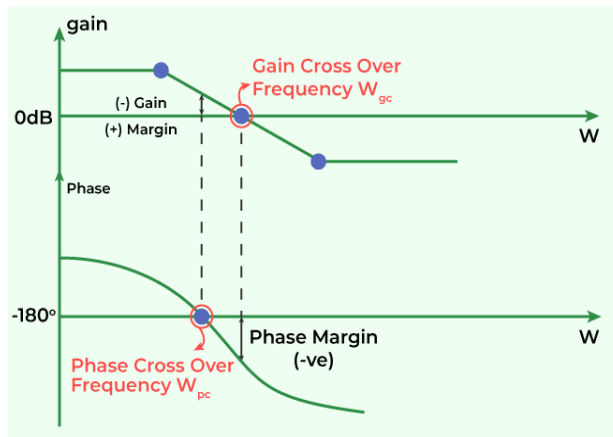


Fig.2 Bode plot showing specifications in frequency domain

FOPID controller can be established as follows:

(i) At ω_{gc} ,

$$\begin{aligned} \text{Magnitude} &= 1 \\ \text{Phase} &= -180^\circ + \phi_m. \end{aligned}$$

(ii) At ω_{pc} ,

$$\begin{aligned} \text{Magnitude} &= 1/A \\ \text{Phase} &= -180^\circ. \end{aligned}$$

However, these equations are nonlinear which results in considerable difficulty in the calculation., so they can be solved as presented.

Response of control systems in frequency domain:

By substituting s with $j\omega$ in $P(s)$, the response of the controlled plant with fractional order in frequency domain can be represented as:

$$P(s) = \frac{K e^{-j(L\omega + \theta(\omega))}}{D(\omega)}$$

Similarly, the controller can be written as:

$$C(j\omega) = C_1(\omega) + jC_2(\omega),$$

Subsequently, $G(j\omega)$ is formulated as:

$$G(j\omega) = \frac{C(\omega)}{D(\omega)} K e^{-j(L\omega + \theta(\omega))}$$

These expressions for $P(j\omega)$, $C(j\omega)$ and $G(j\omega)$, are utilized to define the following expressions for computing the controller parameters.

$$k_p = - \frac{k_d \omega_{gc}^v \sin(\pi v)}{\sin(\frac{\pi}{2} v)} - \frac{D(\omega_{gc}) \sin(\frac{\pi}{2} v + \theta_1)}{K \sin \frac{\pi}{2} v} \dots\dots(1)$$

$$k_i = k_d \omega_{gc}^{2v} + \omega_{gc}^v \frac{D(\omega_{gc}) \sin \theta_1}{K \sin \frac{\pi}{2} v} \dots\dots(2)$$

$$Kk_d \sin\left(\frac{\pi}{2} v\right) = \frac{\omega_{gc}^v D(\omega_{gc}) \sin \theta_1}{\omega_{pc}^{2v} - \omega_{gc}^{2v}} - \frac{\omega_{pc}^v D(\omega_{pc}) \sin \theta_2}{\omega_{pc}^{2v} - \omega_{gc}^{2v}} \dots\dots(3)$$

The process for designing the FOPID controller can be outlined as follows:

- 1) Select the parameters of ω_{gc} , ω_{pc} and φ_m .
- 2) Generate the curve of A in relation to v
- 3) Determine the v value corresponding to the specified A from the curve.
- 4) Compute k_p , k_i and k_d using equations (1), (2), and (3) respectively.

HARMONY SEARCH ALGORITHM:

The Harmony Search Algorithm is an optimization technique inspired by the collaborative improvisation of musicians playing together in harmony. In optimization, this algorithm iteratively refines a pool of potential solutions to a given problem, steadily enhancing their quality until it discovers an optimal or satisfactory solution. Employing this Algorithm to design a (FOPID) controller involves integrating differential evolution strategies to enhance exploration and exploitation of the search space, rendering it applicable across various domains.

STEPS:

- Initialize the population of FOPID controllers randomly or using a predefined strategy. Each controller consists of fractional-order parameters (e.g. P,I,D and fractional orders).
- Define an objective function that quantifies the effectiveness of the FOPID controller. This could be based on control system requirements such as overshoot, settling time, etc.
- In HSA, each candidate solution is generated by combining differences between two randomly chosen solutions with a third solution. At each iteration, generate a trial solution by applying differential evolution operators to selected candidate solutions from the harmony memory. The trial solution is a new candidate FOPID controller.
- HSA maintains a memory of candidate solutions known as the Harmony Memory (HM). This memory evolves over time as better solutions are discovered and replaces poorer solutions. The HM aid in achieving a balance between exploring and exploiting the search space. Initialise harmony memory: pick n random vectors.

$$X_1, X_2, X_3, \dots\dots\dots X_n$$

Also, Make a new vector x'

Determine the Harmony Memory Consideration Rate (HMCR), which controls the selection of components from memory. Randomly decide whether to choose from the HM or generate a modern solution

$$x_i^t = x_i^{random()}$$

Where P_{HMCR} : its value is 0.95.

- Determine the Pitch Adjustment Rate (PAR), which controls the pitch adjustment operation.
- Apply pitch adjustment to selected components from memory or newly generated solutions

$$x_i^t = x_i^t \pm bw \cdot random()$$

Where P_{PAR} : values range from 0.3 to 0.99.

BW : the 'distance bandwidth', the amount of change for pitch adjustments.

- Update the harmonies (solutions) based on the HMCR and PAR. If a new solution is generated, update the memory with it if it's better valued than the previous solution. Select the best solutions based on their fitness values.
- Repeat above Steps until a termination condition is met like rise time, overshoot, settling time, etc., along with the robustness of the controller.
- Select the best solution from the population based on fitness. Implement the best solution found as the FOPID controller.
- Evaluate the performance of system controlled by the designed FOPID controller using simulations. Terminate algorithm if the termination condition is met. The best FOPID controller obtained from the harmony memory represents the solution to the optimization problem

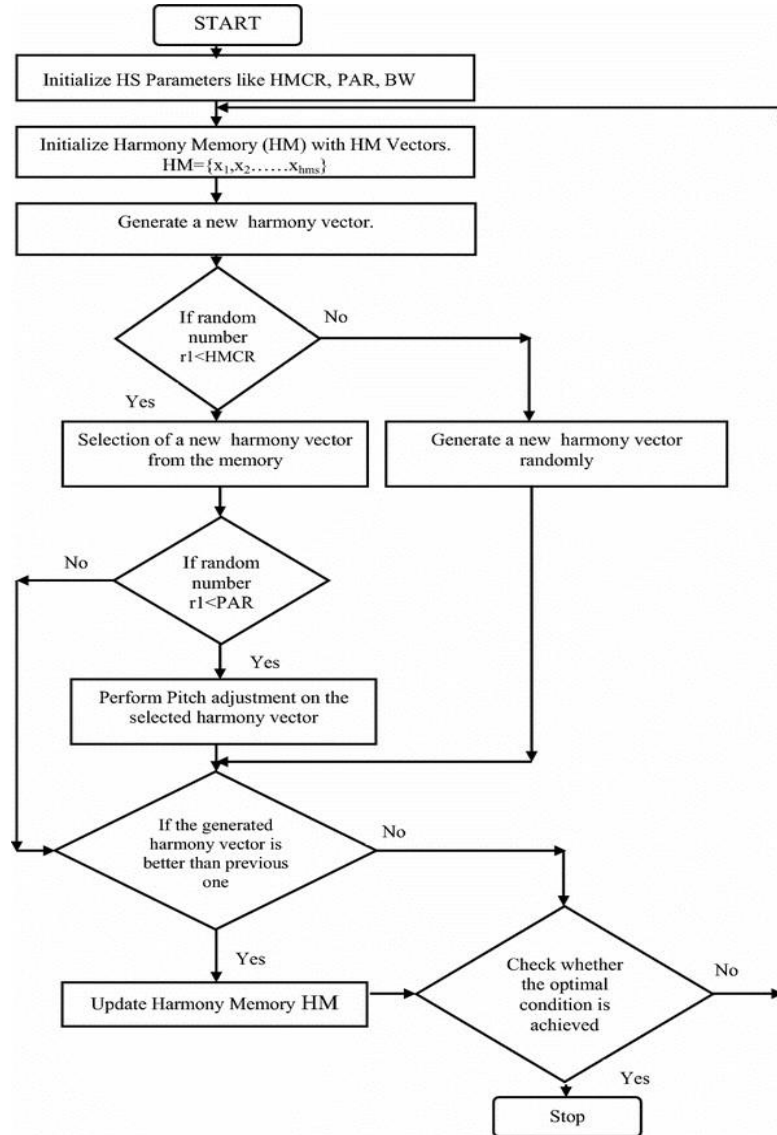


Fig.3 Flow chart of Harmony Search Algorithm

SIMULATION RESULTS:

Example 1: Let's examine a heat flow system, represented by the following system with a time delay [11].

$$P_1(s) = \frac{66.16 e^{-1.93s}}{12.72 s^{0.5} + 1}$$

The plant or system possesses a fractional time delay of 1.93 seconds, with its order being a fractional value of 0.5. The controller is devised utilizing frequency domain specifications. Let's assume the values $\omega_{gc} = 0.2$, $\omega_{pc} = 1$ and $\varphi_m = 65^\circ$, $A=5$. From this criteria, the curve of A in relation to v is plotted, as depicted in Fig. 3, yielding $v = 0.7632$. Subsequently, utilizing equations (1), (2), and (3), the calculated controller parameters are obtained.

$$k_p = 0.02504 ; k_i = 0.02523 \text{ and } k_d = 0.005127$$

FOPID controller is

$$C_1(s) = 0.02504 + \frac{0.02523}{s^{0.7632}} + 0.005127 s^{0.7632}$$

Simulation for above system is as shown in fig.4

Example_1: Simulation model (Open Loop)

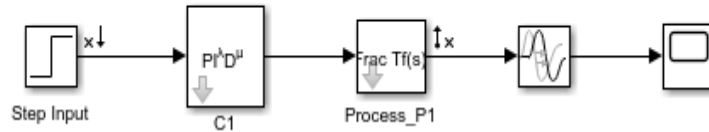


Fig.4 Simulation diagram of P₁C₁

The Bode diagram of P₁C₁ is as shown in fig.5

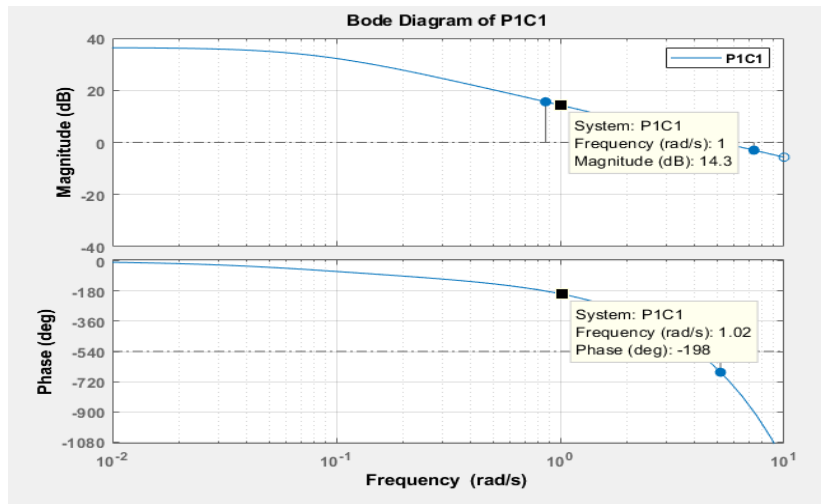


Fig.5 Bode diagram of P₁C₁

For the above process P₁(s), FOPID Controller C₁(s) is tuned using Harmony search algorithm. Its Simulation diagram observed from MATLAB simulation is shown in fig. 6

Extension_1: Simulation model

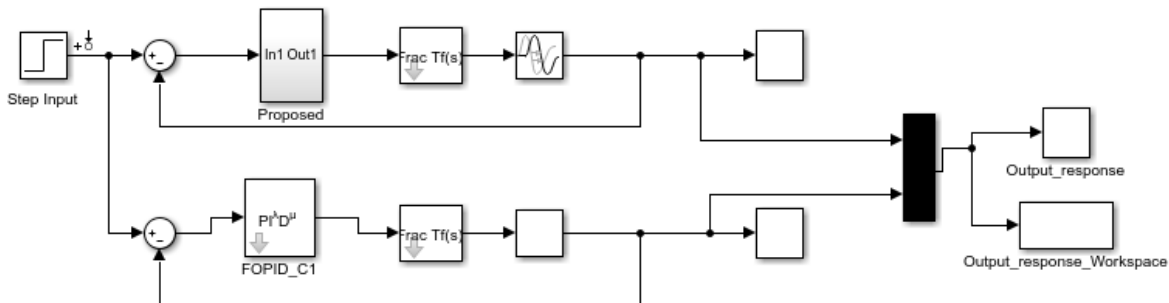


Fig. 6 Simulation model showing and HSA based FOPID and frequency tuned FOPID

Comparison of output results obtained from the process $P_1(s)$ and the FOPID controller $C_1(s)$ that is designed and tuned from frequency domain tuning method and Harmony Search Algorithm are shown in the fig.7

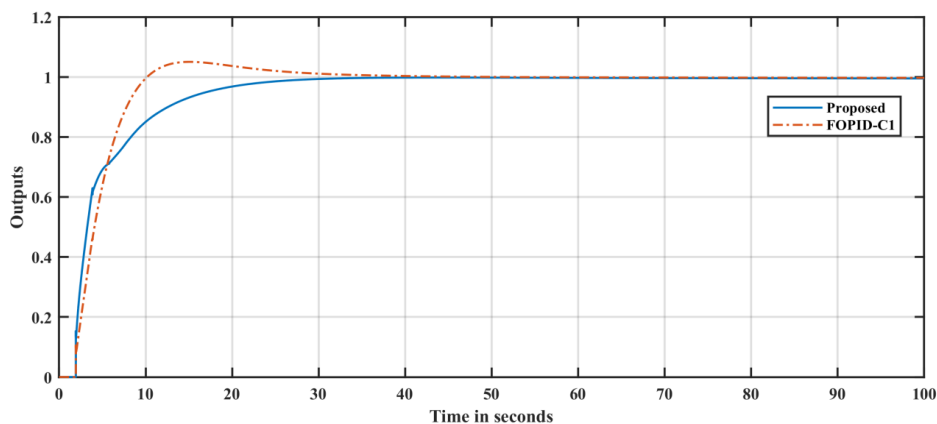


Fig.7 Step responses of closed loop system of P_1C_1

Some of the parameters like over shoot, settling time, IAE and ISE are compared from the output obtained by tuning controller in both the methods and tabulated as in Table-1.

Table-1: Comparative analysis between and HSA based FOPID and frequency tuned FOPID

Control method	Overshoot	Settling time	Integral Absolute Error	Integral Square Error
Proposed (HSA) based FOPID	1.58	26.32	5.307	3.110
Frequency controlled FOPID- C_1	12.71	29.05	5.416	3.367

From fig., It can be observed that settling time (T_s) can be reduced from 29.05 sec to 26.32 sec; IAE is reduced from 5.416 to 5.307 and ISE is reduced from 3.367 to 3.110. Thus overshoot, settling time and errors like (IAE) and (ISE) are decreased when this system’s FOPID controller is tuned by Harmony Search Algorithm (HSA).

Example 2: Examine a one-degree-of-freedom helicopter characterized by system with inadequate damping and time delay [12].

$$P_2(s) = \frac{4.2313 e^{-0.6s}}{0.2 s^{2.3208} + 0.41683 s^{0.96} + 1}$$

This process has fractional time delay of 0.6 sec, and its order is fractional value of 2.3208. Controller is designed using frequency domain specifications. By varying ϕ_m a graph is plotted btw A and v . Assume values of $\omega_{gc} = 04, \omega_{pc} = 2$ and $\phi_m = 65^\circ$. From these values, The plot of A against v can be followed, as illustrated in the figure. As v approaches 1.033, the value of A tends toward infinity. To guarantee $A > 0, v$ must surpass 1.033. For $A = 6$, the corresponding v value is 1.0655; then using (1),(2),(3); calculated controller parameters are: $k_p = 0.01737$; $k_i = 0.09193$ and $k_d = 0.01565$

$$C_4(s) = 0.01737 + \frac{0.09193}{s^{1.0655}} + 0.01565 s^{1.0655}$$

The Bode diagram of the open loop system P_2C_4 as shown in fig.9

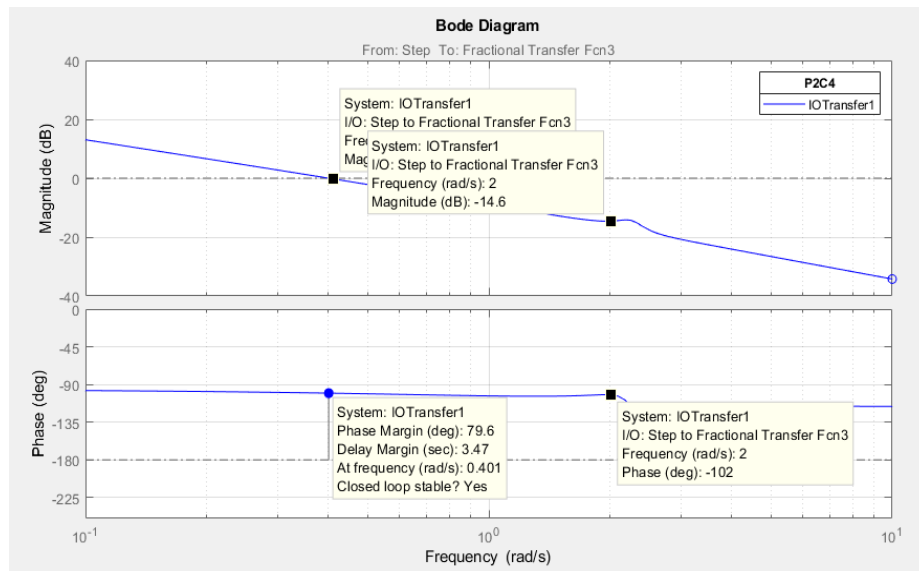
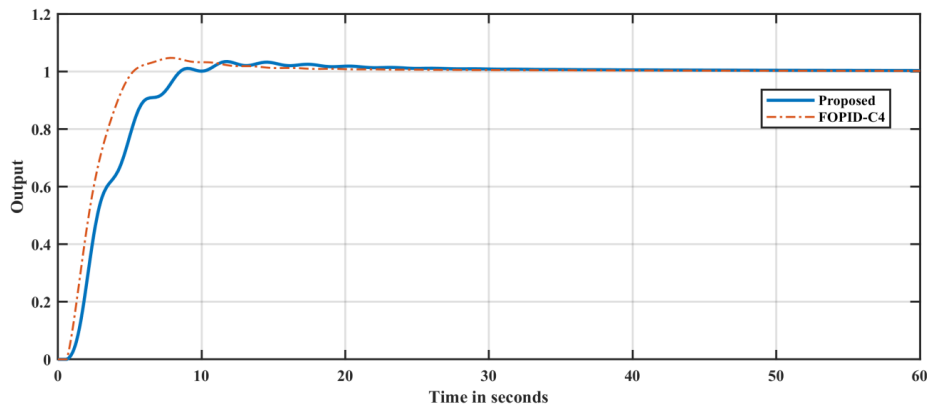


Fig.9 Bode plot of P_2C_4

Comparison of output results obtained from the process $P_2(s)$ and the FOPID controller $C_4(S)$ that is designed and tuned from frequency domain tuning method and Harmony Search Algorithm are shown in the fig.11

Fig.11 Step responses of closed loop system of P_1C_4

The parameters such as Over shoot, settling time, IAE and ISE are compared from the output obtained by tuning FOPID controller in both the methods and tabulated as in Table-2.

Table-2: Comparative analysis between and HSA based FOPID and frequency tuned FOPID

Control method	Overshoot	Settling time	Integral Absolute Error	Integral Square Error
Proposed (HSA) based FOPID	2.54	12.28	2.852	1.729
Frequency controlled FOPID-C₄	4.43	15.66	3.978	2.105

From fig., It can be observed that Overshoot is reduced from 15.66 to 12.28; Settling time (T_s) can be reduced from 15.66 sec to 12.28 sec; IAE is reduced from 3.978 to 2.852 and ISE is reduced from 2.105 to 1.729. Thus overshoot, settling time and errors like (IAE) and (ISE) are decreased when this system's FOPID Controller is tuned by Harmony Search Algorithm .

Conclusion:

This paper has offered valuable insights into the performance of fractional-order PID controller designs in systems with fractional-order delays. The study demonstrated the effectiveness of FOPID controllers in managing systems characterized by fractional-order time delays. Utilizing the Harmony Search Algorithm for tuning the parameters of FOPID controllers proved superior in terms of transient response, steady-state error, and resistance to disturbances. It led to reduced peak overshoot, shorter settling time, and decreased error. The system's robustness was validated by adjusting the plant gain, K . In conclusion, the FOPID controller fine-tuned by the Harmony Search Algorithm emerges as the optimal choice.

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