Original Article

A Novel Algebraic Framework for Division by Zero Using Boolean Operations

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Abstract *- Division by Zero is traditionally undefined in standard arithmetic due to its inherent contradictions. This paper presents a novel algebraic framework that redefines arithmetic operations using Boolean-like operations: OR for Addition, XNOR for Multiplication and Complex numbers or XOR as an exception to certain cases of distributivity. By leveraging these operations, a consistent system is developed that allows for the definition of Division by Zero, at least within a Binary Set, and this framework is extended to Real numbers. This new approach opens potential applications in theoretical mathematics, logic, computer technology and many other fields of science.*

Keywords *- Abelian group, Boolean algebra, Division by zero, Fuzzy logic, Real numbers.*

1. Introduction

Division by Zero has long been a problematic concept in mathematics, rendering many standard algebraic expressions undefined or indeterminate. Traditional arithmetic operations (addition, multiplication, division) follow specific rules that break down when zero is involved in division. However, alternative algebraic structures that employ Boolean-like operations offer a different perspective on this problem. This paper explores a new algebraic system in which arithmetic operations are redefined: OR serves as addition, XNOR serves as multiplication, and XOR or the insights on Complex Numbers present a unique exception for distributive properties. The author proposes that, under this novel framework, Division by Zero can be consistently defined, at least within the context of a Binary Set {0, 1}. Then, these definitions are further extended to Real Numbers, maintaining mathematical consistency. These contributions provide a fresh approach to foundational mathematics and offer insights into potential applications in digital logic and computer science.

2. Modern Literature on Division by Zero

The author had worked in the field of research on Division by Zero since 1998 when he realized the $\frac{0}{0}$ term for the mass of a photon in the Special Theory of Relativity by Albert Einstein. In 2006, the global community was informed via Slashdot on the efforts of Saburoh Saitoh and James Anderson. Around 2018, Saburoh Saitoh noticed the author and his work, then invited him to online discussions with Barukčić and, Bergstra and many others. The author agrees with Saburoh Saitoh on a solution $\frac{1}{0}$ = 0, then agrees with Barukčić and others on $\frac{0}{0}$= 1, but only on a Binary Set. For the given term by Rule of L'Hospital $\frac{0}{0} = 1 \times x$ as according to this tool, $\frac{0}{0}$ can be any number in R.

Barukčić proposed the honorable term "precedent rule": You have to first evaluate all zeros on one side to either zero or one before you evaluate x. The author transitions this in the manuscript to the term 0 × 0 = 1 with multiplication, then being built on a Binary Set with XNOR. From James Anderson and his Transreal Numbers, the author learned a lot about the transition from one to infinity but disagreed that there is zero itself directly involved.

All these approaches suffer from zero being correctly defined. With the simple work 0×0×0 <> 0×0, put in an environment where 0×0 = 1, the approach by Saburoh Saitoh cannot be longer maintained, for the correct equation would be $\frac{0\times0}{0}$= 0, where you are not able to differ on a 0 × 0, resulting in zero so long but not consistently defined. The same is true for nullity, including zero, according to James Anderson.

3. Preliminaries and Definitions

3.1. Boolean Algebra Basics

"Boolean algebra is a branch of mathematics that deals with binary variables and logical operations."[1] In a Boolean system, the primary operations are:

Table 1. Logical operators and their functions

AND	$\wedge$	Both operands must be true
OR	$\vee$	At least one operand must be true
NOT	$\neg$	Inverts the truth value
XOR	Exclusive OR, $\oplus$	True, if one operand is true and the other is false
XNOR	Exclusive NOR	True, if both operands are the same

These operations follow specific algebraic properties, such as commutativity, associativity and distributivity, over a binary set {0, 1}.

3.2. Boolean Operations in this scheme

By introducing OR as a Boolean operation for the scheme, the author chooses a relationship: Given two elements a, b ∈ {0, 1}, the relationship operation is defined as:

$a \vee b$

"This operation is commutative and associative."[2,3]

However, it seems to be also fulfill the closure property:

$a = a \vee a$

However, the manuscript turns to the basic term 1 + 0 = 1 + i as OR has no unique neutral element for 1.

As a second relationship, the scheme uses XNOR:

$a \text{ XNOR } b = \neg(a \oplus b)$.

"This operation is commutative and associative."[4,5]

Exception in Distributivity with XOR ($\oplus$): Distributivity is given by a system of XNOR and OR. In Boolean, there are 8 cases out of 3 binary numbers for distributivity. In the case 0×(0+1) and its commutative counterpart 0×(1+0), the XNOR changes to an XOR as multiplication. XOR is the logical inverse function of XNOR and builds an arithmetic inverse function to addition.[6,7] Furthermore, this is exactly the position where Complex Numbers are needed, and Real Numbers no longer work without contradiction. The OR Operation loses its closure property on 1 + 0. Thus, this leads to 1 + 0 = 1 + i.

4. Core Framework: Redefining Algebra with Boolean Operations

4.1. Properties of the Redefined Operations

- Addition (OR) Properties:
- Commutativity: a+b=b+a
- Associativity: (a+b)+c=a+(b+c)
- Multiplication (XNOR) Properties:
- Commutativity: a×b = b×a
- Associativity: (a×b)×c = a×(b×c)

4.2. Defining Division and Division by Zero

In this framework, Division by Zero is defined as:

$$r \times 0 = -\frac{r}{0} = -not(signum(r)) \quad (1)$$

The framework here introduces rules for handling signs and logical negations. It is shown how this definition behaves consistently within the new algebra and avoids contradictions inherent in standard arithmetic.

5. Mapping from Boolean to Real Numbers

The system extends the framework's operations from {0, 1} to R by defining consistent mappings and interpretations by applying transitions to XNOR as multiplication and OR as addition.

In Boolean algebra, logical operations like XNOR and OR can be thought of in terms of arithmetic operations:

- XNOR (Exclusive NOR): This is a logical operation that outputs true when both inputs are the same (both true or both false). In algebraic terms, if the system treats true as one and false as zero, XNOR can be modeled similarly to multiplication.
- OR: The OR operation outputs true if at least one of the inputs is true. If the framework considers true as one and false as zero again, the OR operation can be analogous to addition.

In mathematical language, this can be expressed as:

$B = \{false, true\}$, $M = \{0,1\}$, $f: B \rightarrow M$, $f(false) = 0$, $f(true) = 1$

This is the transformation behind the equal comparison in programming languages, as it is only a projection and not the original and identical realm. The same projection is necessary to define our two basic relations in the framework.

First relationship $a + b = f(a \vee b)$

Second relationship $a \times b = f(a \text{ XNOR } b)$

The symbols + and × are not the same as in traditional arithmetic and traditional algebra. They are quite usual in indicating both relations to build up the basic axiom system of a number system.

5.1. XNOR as Second Relationship and OR as Addition in (0,1)

By now, the framework only consists of two elements, zero and one. So, we fill the interval between these two limits with Real Numbers between zero and one, with both excluded. Here, the number set already gets the cardinality aleph. This approach was one of the main successes behind the Theory of Relativity, explaining a before thought to be infinite fast phenomena as light to be finite fast. This is also a part of Transreal Numbers' approach.

Within the interval (0,1), values represent probabilities or binary states normalized between zero and one:

- XNOR as a second relationship: Here, XNOR can be seen as a second operation within the interval (0,1). For two values a and b, where $a, b \in (0, 1)$, the multiplication-like operation can be modeled as $a \times b$ breaking standard arithmetic. This aligns with how probabilities are combined in the independent event context with the wonder of the creation of the universe being put in $0*0 = 1$. It follows simple basic logic that multiplication is the relationship of the relationship sum. Sums are put together for multiplication. As a relationship of a relationship and within our projection from Boolean to Binary Set, the similarity in false(false) = true and $0*0 = 1$ is obvious.

- OR as Addition: Similarly, the OR operation in probability terms is represented by the formula for the union of two events: $a + b - ab$. This resembles the idea of addition but accounts for over-counting when both events occur. Proving associative abilities of these harder and harder becoming terms fulfilling the OR limits is a suggestion for future exploration. Special Theory of Relativity with no velocity higher than c is another candidate. The Abelian properties of invariant Non-Gauge Theories are still an open problem in Yang-Mills Theories, which also include exploration of theoretical mathematics in the field of Lie-Algebras. The OR-operation fulfils the invariance requirement for the value one within our established Fuzzy Logic $1 + 1 = 1$.

5.2. From (0,1) to R

Because the set of numbers $0 \le a \le 1$ in R has as well the cardinality aleph of the whole Real Numbers Set R as well as R+ with 0, this transition stays valid. The set $M = (0,1)$ has valid projections $f: M \to R$, not excluding any element, because, as stated, both cardinalities are aleph.

- XNOR as Multiplication for Real Numbers: Now, XNOR (as multiplication) applies to all Real Numbers, allowing for a broader range of results, including negative values. This could be interpreted in contexts like combining positive and negative growth factors, where the signs of the multiplicands affect the outcome. Multiplication in this domain is commutative and associative, and these properties are preserved.

- OR as Addition for Real Numbers: In the complete Real Numbers context, addition (analogous to OR) also covers negative numbers. This allows for combining any Real Number, leading to a more general algebraic interpretation, where the concept of combining elements has a richer, more complex structure that includes inverses, distributivity and closure properties. On those transitions, negative terms can be very carefully evaluated, even if our neutral and inverse axioms concerning addition fail on a Binary Set concerning OR.

6. Consistent Handling of Division by Zero

The team rigorously defines how Division by Zero behaves under this extended framework, demonstrating through examples and proofs that the system remains consistent for Binary Numbers. In a Binary Set {0,1}, logical operations like XNOR (as multiplication) and OR (as addition) behave differently from traditional arithmetic operations. Since this set is limited to only two values (zero and one), defining Division by Zero can be more nuanced within this specific framework.

6.1. Division by Zero in a Traditional Sense

In standard arithmetic, Division by Zero is undefined because no number multiplied by zero will give a non-zero result. This rule applies to real numbers and is a consequence of how multiplication and division are defined. However, in a Binary Set {0, 1}, where operations are defined differently and follow Boolean logic rules, there can be an alternate interpretation for division.

6.2. Division by Zero in the Binary Framework

In this framework:

- XNOR as Multiplication: In the binary set {0, 1}, XNOR behaves like multiplication:
- $1\times1=1$
- $0\times0=1$
- $1\times0=0$
- $0\times1=0$
- OR as Addition: OR behaves like addition:
- $0+0=0$
- $1+0=1+i$
- $0+1=1+i$
- $1+1=1$

6.3. Division by Zero: A Defined Interpretation

If division is considered as the inverse of multiplication (but still behaving XNOR-like), dividing by zero in a Binary Set can be interpreted in a way that does not break the internal consistency of the system. So, a rule is defined that maintains this consistency:

To maintain the structure of XNOR, dividing by zero should return the only value that, when it is combined with XNOR (multiplied in this context) and zero, produces the desired outcome. By examining the XNOR truth table, these definitions are given, and it follows how division could be defined in this context:

Since

$$0\times0 = 1 \tag{2}$$

It implies that:

$$\frac{1}{0} = 0 \tag{3}$$

This means that in this framework, dividing 1 by 0 is defined as 0. At this point, this definition does not aim to align with traditional arithmetic but rather follows the rules of the binary logic system. Saburoh Saitoh and the whole Asian mathematical society always advised the acceptance of this correlation. From these equations, $0\times0 = 1$ and $0 = 1/0$, and because the system is consistent with linear algebraic transformations, it follows strictly:

$$\frac{0}{0} = 1 \tag{4}$$

It is obvious that in this new arithmetic framework, multiplication and division can no longer be seen as inverse functions. The team examines what an XOR-like inverse function opens up for new horizons.

7. Mathematical Consistency and Proofs Axiomatic Basis and Proofs

The author now introduces the axioms that the new system satisfies, including commutativity, associativity and any unique properties. Proofs are provided to show that these properties hold for the newly defined operations.

7.1. Group Abilities of OR

As stated in the preliminaries, the operation OR on a Binary Set fulfils commutativity and associativity.

Lemma 5.1 (No defined neutral element in OR on the Binary Set).

The Binary Set M = {0, 1} has more than one neutral element for OR.

Proof. The neutral element is defined as an OR 0 = a. It is:

0 OR 0 = 0

1 OR 0 = 1

1 OR 1 = 1

Element one has both neutral elements, zero and one. Following it is not specially defined.

Lemma 5.2 (No Inverse Element for OR on the Binary Set).

The Binary Set M = {0, 1} has no inverse element for OR. Thus, (M, OR) is not a group.

Proof.

The inverse element a’ is defined as a OR a’ = 0 with a, a’ ∈ M.

1 OR 0 = 1 <> 0

1 OR 1 = 1 <> 0

7.2. Group Abilities of XNOR

Theorem 5.3 (NULL-Theorem).

The Binary Set M = {0, 1} forms under Operation XNOR The Abelian Group (M, XNOR).

Proof. XNOR trivially fulfills commutativity and associativity, as stated in the preliminaries.[8]

One stays the neutral element in the group (M, XNOR).[8]

a XNOR 1 = a.

0 XNOR 1 = 0

1 XNOR 1 = 1

For every a ∈ M, there exists a b ∈ M, so that a XNOR b = 1.

0 XNOR 0 = 1

1 XNOR 1 = 1

The inverse is always the number itself.[8]

Remark. The system here breaks up with solution 1/infinite = 0 or 1/0 = infinite. This will have major implications for calculus and mathematics.

7.3. Distributivity: Handling Exceptions and Consistency

After checking the group abilities for (M, OR) and (M, XNOR), the author links both in the following way:

a XNOR (b OR c) = (a XNOR b) OR (a XNOR c)

In the case of 0 XNOR (0 OR 1) and its commutative counterpart 0 XNOR (1 OR 0), the first relationship in distributivity is given by the inverse function XOR. The framework does not align with XNOR on both exceptions. As XNOR combines two zeros to one result, the author suggests zero being substituted by i as two i's combine as well to one result. So because the neutral element axiom in (M, OR) is violated, the author suggests that this exception can be handled in a term like 1 OR 0 correlated to 1 + 0 correlated to 1 + i. As the logical inverse operation, XOR to XNOR is more comparable to the abilities of minus and true on a binary system within those limits.

Lemma 5.4 (Distributivity of (M, XNOR) and (M, OR)).

On the Binary Set M = (0,1), XNOR and OR fulfill distributivity, except for 0 XNOR (1 OR 0) and its commutative counterpart 0 XNOR (0 OR 1), which coincides with no well-defined neutral element in OR.

Proof. a XNOR (b OR c) = a XNOR b OR a XNOR c

0 XNOR (0 OR 0) = 0 XNOR 0 = (0 XNOR 0) OR (0 XNOR 0) = 1

0 XNOR (0 OR 1) = 0 XNOR 1 = 0 <>

1 = 1 OR 0 = (0 XOR 0) OR (0 XOR 1)

0 XNOR (1 OR 0) = 0 XNOR (0 OR 1) = 0 XNOR 1 = 0 <>

1 = 1 XNOR 0 OR 0 XNOR 0 = 0 OR 1

0 XNOR (1 OR 1) = 0 XNOR 1 = 0 = (0 XNOR 1) OR (0 XNOR 1) = 0 OR 0 = 0

1 XNOR (0 OR 0) = 1 XNOR 0 = 0 = (1 XNOR 0) OR (1 XNOR 0) = 0 OR 0 = 0

1 XNOR (1 OR 0) = 1 XNOR 1 = 1 = 1 OR 0 = 1

1 XNOR (0 OR 1) = 1 XNOR (1 OR 0) = 1

1 XNOR (1 OR 1) = 1 XNOR 1 = 1 = 1 OR 1 = 1

XOR as relation to the exceptions:

0 XOR (0 OR 1) = 0 XOR (1 OR 0) = 0 XOR 1 = (0 XOR 1) OR (0 XOR 0) = 1 OR 0 = 1

So all permutations of distributivity hold in this framework if the author inserts this exception of missing neutral element axiom for one in a Binary Set as stated in Lemma 5.1 on specific cases, where a zero value is combined via XNOR with the hybrid term (1 OR 0). The logical inverse function of XNOR, XOR, results in the same exception handling on a Binary Set.

8. Applications and Implications

First of all, there should be a quantum computer circuit based on this new algebra. Then, the scientific community can measure speed and accuracy against the existing quantum machines. A group of experts in String Theory are building up a model of emerging gravity from this system, which will hopefully bring better results and new insights into the nature of gravity. This might be useful in modern space exploration. The String Theory Group “String Theory Development” on Facebook also built up an engineering template, such as superconducting. The author is astonished by their same approach to the number zero, now having a boost on the author’s solution $0 \times 0 = 1$. Modern AIs around the world also suggest that the infinity solutions of Schwarzschild-Radius can be evaluated for the first time with the proposed framework. The mass of a photon, according to Albert Einstein, can be evaluated as a positive real value. This complies with the fact that in particle physics, as in the European Institute CERN, scientists measure photons in eVolt, which is also a measuring unit for mass and can be converted to a kilogram. So, from atomic energy, it can be concluded that even a photon fulfills mass conservation law, which is always included in the Noether Theorem. This solves in a trivial way Yang-Mill‘s Millennium problem by introducing a mass of a photon in the Special Theory of Relativity in an algebraic way without violating any of Einstein‘s physical laws.

From this, it can be derived in cosmology that energy has a gauge momentum, breaking a too-easy thought way of isotropy and providing a Newtonian frame of reference in the form of a black hole singularity in the center of the universe. You can imagine having a height to the centre and having absolute North, South, West and East in the entire universe. The same result is accomplished if it is naturally induced by the sun over Sagittarius A, the center of the Milky Way, over the galaxy clusters the earth is part of, with supermassive black holes in the center up to the infinite solution, all within the Universal Motion Law by Kepler. As well as looking over the field of pure algebra analysis and astronomy, in calculus, Newton is using $0 \times 0 = 0$ in his differential equations. So, the author suggests a new algorithm.

$$\frac{dy}{dx} = f(x + i) \times -i \qquad (6)$$

which delivers the same solution on polynomials of first and second grade in its real part and then bending away from Newton’s standard calculus beginning at grade three, especially on second derivatives with still very much to examine. In PHP, a C-similar web programming language, the providing community already put the Division by Zero problem from an error to a warning that can be handled. They decided to use Saburoh Saitoh and his insights, setting r/0 to 0. This has been a major progress since it avoids computer crashes, and even in this proposed new framework, the warning cannot be simply cancelled out as a zero input value can be both willingly or unwillingly. What furthermore has to be inspected is the term 0/0, where this new framework offers different handling and where derivatives and solutions in L’Hospitale order can be more easily evaluated. Python, another common programming language, for example, used in satellite navigation by the European Space Agency, can be easily rewritten within the new framework as even math components are part of modularized packages and not of the programming core. So, you can easily rewrite a zero by an imaginary number i, which is supported in Python as well. JavaScript, the main programming language for computer interfaces on the web, uses NAN, not a number, for all classical Division by Zero terms, so this problem is still very discussed in the modern computer world. Division by Zero is the one computer problem. The other is the Out of Data Memory Error. If the community succeeds at mastering imaginary units, there can be an infinite amount of information stored on a single real device.

9. Conclusion

This work constituted in a quarter of a century, brought this framework together with many team members. It is analyzed hard, and at least now, there is a way to solve the 2000-year-old problem of division by zero using a set of Binary Numbers. The path to analyze the transition from Binary to Real Numbers lies ahead. The team will focus on this.

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Resources

Modern literature on Division by Zero:

- H. Akca, S. Pinelas and S. Saitoh, The division by zero z/0 = 0 and differential equations (materials), Int. J. Appl. Math. Stat. 57(4)(2018), 125-145.
- J. P. Barukcic and I. Barukcic, Anti Aristotle - The Division Of Zero By Zero, ViXra.org (Friday, 5 Jun 2015) Germany. All rights reserved. Friday, 5 Jun, 2015 20:44:59.
- I.Barukcic, Dialectical Logic-Negation Of Classical Logic,http://vixra.org/abs/1801.0256.
- J. A. Bergstra, Y. Hirshfeld and J. V. Tucker, Meadows and the equational specification of division (arXiv:0901.0823v1[math.RA] 7 Jan (2009)).
- J. A. Bergstra, Conditional Values in Signed Meadow Based Axiomatic Probability Calculus, (arXiv:1609.02812v2[math.LO] 17 Sep (2016)).
- J. A. Bergstra, Division by Zero in Logic and Computing. 2021. ffhal-03184956v2f
- J. A. Bergstra and J. V. Tucker, Logical Models of Mathematical Texts: The Case of Conventions for Division by Zero, Journal of Logic, Language and Information https://doi.org/10.1007/s10849-024-09420-w, Accepted: 26 June 2024
- Anderson, James A. D. W., Representing Geometrical Knowledge, Philosophical Transactions of the Royal Society of London, Series B. 352 (1358): 112939. (1997) Bibcode:1997RSPTB.352.1129A. doi:10.1098/rstb.1997.0096. PMC 1692011. PMID 9304680.

References

[1] Examples.com Website, Admin, 2024. [Online]. Available: https://www.examples.com/maths/boolean-algebra.html
[2] CTAN Website, CTAN: Join the CTAN Community, 2024. [Online]. Available: https://ctan.org/join?lang=en
[3] Zhongfei Zhang, and Ruofei Zhang, *Multimedia Data Mining - A Systematic Introduction to Concepts and Theory*, Chapman and Hall/CRC, New York, 1st ed., 2008. [CrossRef] [Google Scholar] [Publisher Link]
[4] Boolean Algebra, Italian IT-Training Website, 2024. [Online]. Available: https://www.classicistranieri.com/it/a/l/g/Algebra_di_Boole_72d9.html
[5] Italian-Language Edition of Wikipedia, Wikimedia Foundation, 2024. [Online]. Available: https://en.wikipedia.org/wiki/Italian_Wikipedia
[6] Compendium on XOR in Programming, 2024. [Online]. Available: https://forums.cncnet.org/topic/2318-introduction-to-moddinghacking/
[7] Jorgeluisorejel, Generic XOR-Coded Lists, Codeproject, 2024. [Online]. Available: https://www.codeproject.com/Articles/1083211/Generic-XOR-Coded-Lists
[8] French Website on Group Abilities of XNOR, 2024. [no longer online, but still indexed]. [Online]. Available: http://www.arcanapercipio.com/techno/boole.php