Extension of Fixed-Point Theorem in Soft G-Metric Space

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Abstract: In the present paper we will extend some fixed-point theorem in soft G-metric space which is the combination of g metric and soft metric concept.

Keywords: Fixed point theorem, Generalised metric space, soft metric space

Introduction: In 1912 a dutch mathematician Brouwer introduce very first time a fresh concept of fixed point [3]. After this statement in 1922 S. Banach give this theorem a new path that make this theorem very popular and we all known this as Banach contraction principle [4]. In this principle he gave footprint for uniqueness of the fixed point, this is the reason why it being very popular. Later in 1930 J. Schauder again wrestle with brouwer's theorem and take them to new height with infinite dimensional Banach space [5].

Year by year cantor's set theory explore in many directions but it bound all the mathematician when some not clear information arrives and to resolve this problem some mathematician gave us new tools. Every tool is successor for the previous one and cover flaws and give solution for particular flaw. In this chain of research D. Molodtsov introduce a concept of soft set in 1999 which give revolution in research area of vagueness or unclear data [6]. It is taking the centre stage of research by its parameterization power. Soft set is nothing but a family of parameter which mapped on power set. After molodtsov many researcher doing experiments with this to utilize this concept in other or better manner and in sequence of experiments in 2012 and 2013 Sujoy das and S.K.Samanta put a ball on research area and it burst like bomb they introduce soft metric space[7][8]. They give a concept to the world which enable everyone to bridging uncertainties and distance. This concept work as a catalyst in understanding of distance and convergence in vague or unclear environment and simultaneously it helps in growing new branch in applied mathematics. After this concept researcher built fuzzy soft metric, soft s-metric space and many more. T. Beaula and C. Gunaseeli published a paper in 2014 which held the concept of fuzzy soft metric [9].

In 1992 Bapure Dhage find a new concept and he called as D-metric space, all the details of D-metric space gave in his ph. D. thesis [10][11][12][13]. B. Dhage’s research has many flaws which identify by Z. Mustafa and B. Sims in 2005, so they try to resolve these flaws and they get a new concept of G-metric space or generalized metric space which is generalization of metric space (X, d)[14]. After this it is became a hot topic to discuss for. In this we extend a research paper’s result in soft G-metric space of Sarika Jain, Rashmi Tiwari and Ramakant Bhardwaj [16]

Preliminary:

For a function $\mathcal{B}: Å \to Å$, there is a point $\wp$ in domain Å is fixed point if it unchanged in codomain also,

i.e.

$\mathcal{B}(\wp) = \wp$.

Example: $\mathcal{B}(\wp) = \wp^2,\ \forall\, \wp = 0,1$

Soft set: let Ʊ be a universal set and Å is the set of parameters then the pair of $(\mathcal{B}, Å)$ is said to be soft set over the universal set Ʊ when $\mathcal{B}: Å \to Å(Ʊ)$.

Soft metric space: let $\tilde{\varkappa}$ is a soft point, ∃ be set of parameters and $S\wp(\widetilde{\varkappa})$ is collection of all possible soft point then the mapping $\mathcal{B}: S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \to \mathcal{R}(\exists)^*$ is called soft metric space on the soft set $\tilde{\varkappa}$

1. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}) \geq 0, for\ all\ \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu} \in \tilde{\varkappa}$
2. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}) = 0, if\ and\ only\ if\ \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu},$
3. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}) = B(\widetilde{\mathbb{Q}_\mu}, \widetilde{\wp_\varepsilon}), for\ all\ \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu} \in \tilde{\varkappa}, symmetry\ property$
4. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}) \leq B(, \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{R}_\omega}) + B(\widetilde{\mathbb{R}_\omega}, \widetilde{\mathbb{Q}_\mu})$

G-Metric space: let $\varkappa$ be a non-empty set and $\mathcal{B}: \varkappa \times \varkappa \times \varkappa \to \mathcal{R}^+$ be a function which satisfying these conditions:

1. $\mathcal{B}(\wp, \mathbb{Q}, \mathbb{R}) = 0, if \wp = \mathbb{Q} = \mathbb{R},$
2. $0 < \mathcal{B}(\wp, \wp, \mathbb{Q}), forall\ \wp, \mathbb{Q} \in \varkappa\ and\ \wp \neq \mathbb{Q},$
3. $\mathcal{B}(\wp, \mathbb{Q}, \mathbb{R}) \geq \mathcal{B}(\wp, \wp, \mathbb{Q}), forall\ \wp, \mathbb{Q}, \mathbb{R} \in \varkappa, with\ \mathbb{Q} \neq \mathbb{R}$
4. $\mathcal{B}(\wp, \mathbb{Q}, \mathbb{R}) = \mathcal{B}(\mathbb{Q}, \mathbb{R}, \wp) = \mathcal{B}(\mathbb{R}, \wp, \mathbb{Q}) = \cdots (symmetry\ in\ all\ the\ variables)$
5. $\mathcal{B}(\wp, \mathbb{Q}, \mathbb{R}) \leq \mathcal{B}(\wp, k, k) + \mathcal{B}(k, \mathbb{Q}, \mathbb{R}), for\ all\ \wp, \mathbb{Q}, \mathbb{R}, k \in \varkappa$

Then the function is said to be G-metric space or generalized metric on $\varkappa$ and the pair of $(\varkappa, \mathcal{B})$ is G-metric space.

Soft G-Metric space: let $\varkappa$ be a nonempty set and $\exists$ be set of parameters which is nonempty. A mapping $\mathcal{B}: S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \to \mathcal{R}(\exists)^*$ is called as soft G-metric space if it satisfying the following properties:

1. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}) = 0, if\ and\ only\ if\ \widetilde{\wp_\varepsilon} = \widetilde{\mathbb{Q}_\mu} = \widetilde{\mathbb{R}_\omega}$
2. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{Q}_\mu}) > 0, for\ all\ \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu} \in S\wp(\widetilde{\varkappa})\ with\ \widetilde{\wp_\varepsilon} \neq \widetilde{\mathbb{Q}_\mu}$
3. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}) \geq \mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu})\ for\ all\ \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega} \in S\wp(\widetilde{\varkappa})\ with\ \widetilde{\mathbb{Q}_\mu} \neq \widetilde{\mathbb{R}_\omega}$
4. $\mathcal{B}(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}) = \mathcal{B}\left(\widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}, \widetilde{\wp_\varepsilon}\right) = \mathcal{B}\left(\widetilde{\mathbb{R}_\omega}, \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}\right) = \cdots.$
5. $\mathcal{B}\left(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}\right) \leq B\left(\widetilde{\wp_\varepsilon}, \tilde{k}, \tilde{k}\right) + \mathcal{B}\left(\tilde{k}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}\right)\ for\ all\ \widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}, \tilde{k} \in S\wp(\widetilde{\varkappa})$

Main result:

Theorem: let $(\varkappa, \text{Å}, \exists)$ be a complete soft G-metric space and

let $\mathcal{B}: S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \to \mathcal{R}(\exists)^*$ be a mapping satisfies the following condition-

$$\alpha \text{Å}\left(\mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\right) + \beta\left[\text{Å}(\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}) + \text{Å}\left(\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}\right) + \text{Å}\left(\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\right)\right]$$
$$+ \delta\left[\frac{\text{Å}\left(\mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\right) + \text{Å}(\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}) + \text{Å}\left(\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}\right) + \text{Å}\left(\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\right)}{1 + \text{Å}\left(\mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\right)\{\text{Å}(\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}) + \text{Å}\left(\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}\right) + \text{Å}\left(\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\right)\}}\right]$$
$$\leq \gamma \text{Å}\left(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}\right)$$

…….(2.1)

For all $\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega} \in \varkappa$, where the constant $\alpha, \beta, \gamma, \delta$ satisfies $\alpha, \beta, \gamma, \delta > 0$;

$0 < \gamma < \alpha + \beta + \delta;\ \ \alpha \neq 0.$

Proof: Take an arbitrary and define a sequence $\kappa_{n+1} = \kappa_n,\ \ \kappa = 0,1,2,3, ...$

Put $\widetilde{\wp_\varepsilon} = \mathcal{K}_n,\ \widetilde{\mathbb{Q}_\mu} = \mathcal{K}_{n+1},\ \widetilde{\mathbb{R}_\omega} = \mathcal{K}_{n+2}$

Then

$$\alpha Å(\mathcal{BK}_n, \mathcal{BK}_{n+1}, \mathcal{BK}_{n+2})$$
$$+ \beta\left[Å(\mathcal{K}_n, \mathcal{BK}_n, \mathcal{BK}_n) + Å(\mathcal{K}_{n+1}, \mathcal{BK}_{n+1}, \mathcal{BK}_{n+1}) + Å(\mathcal{K}_{n+2}, \mathcal{BK}_{n+2}, \mathcal{BK}_{n+2})\right]$$
$$+ \delta[\frac{Å(\mathcal{BK}_n, \mathcal{BK}_{n+1}, \mathcal{BK}_{n+2}) + Å(\mathcal{K}_n, \mathcal{BK}_n, \mathcal{BK}_n) + Å(\mathcal{K}_{n+1}, \mathcal{BK}_{n+1}, \mathcal{BK}_{n+1}) + Å(\mathcal{K}_{n+2}, \mathcal{BK}_{n+2}, \mathcal{BK}_{n}}{1 + Å(\mathcal{BK}_n, \mathcal{BK}_{n+1}, \mathcal{BK}_{n+2})\{Å(\mathcal{K}_n, \mathcal{BK}_n, \mathcal{BK}_n) + Å(\mathcal{K}_{n+1}, \mathcal{BK}_{n+1}, \mathcal{BK}_{n+1}) + Å(\mathcal{K}_{n+2}, \mathcal{BK}_{n+2}, \mathcal{BK}_{n}}$$
$$\leq \gamma Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})$$

$$\alpha Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3})$$
$$+ \beta\left[Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}) + Å(\mathcal{K}_{n+2}, \mathcal{K}_{n+3}, \mathcal{K}_{n+3})\right]$$
$$+ \delta[\frac{Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}) + Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}) + Å(\mathcal{K}_{n+2}, \mathcal{K}_{n+3}, \mathcal{K}_{n+3})}{1 + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3})\{Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}) + Å(\mathcal{K}_{n+2}, \mathcal{K}_{n+3}, \mathcal{K}_{n+3})\}}]$$
$$\leq \gamma Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})$$

$$\alpha Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}) + \beta Å(\mathcal{K}_n, \mathcal{K}_{n+3}, \mathcal{K}_{n+3})$$
$$+ \delta[\frac{Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})}{1 + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}).A(\mathcal{K}_n, \mathcal{K}_{n+3}, \mathcal{K}_{n+3})}] \leq \gamma Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})$$

$$\alpha Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3})$$
$$\leq \gamma Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2}) - \beta Å(\mathcal{K}_n, \mathcal{K}_{n+3}, \mathcal{K}_{n+3})$$
$$- \delta[\frac{Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})}{1 + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}).Å(\mathcal{K}_n, \mathcal{K}_{n+3}, \mathcal{K}_{n+3})}]$$

$$\alpha Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}) \leq (\gamma - \beta)Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})$$

$$Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}) \leq \frac{(\gamma - \beta - \delta)}{\alpha} Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})$$

$$0 < \gamma < \alpha + \beta + \delta$$

$$0 < \gamma - \beta - \delta < \alpha$$

$$0 < \frac{\gamma - \beta - \delta}{\alpha} < 1$$

Assume that $\frac{\gamma-\beta-\delta}{\alpha} = \mathbb{z}$ then

$$Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}) \leq \mathbb{z} Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})$$

Similarly,

$$Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2}) \leq ℤÅ(\mathcal{K}_{n-1}, \mathcal{K}_n, \mathcal{K}_{n+1})$$

Processing n times

$$Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+3}) \leq ℤ^{n+1}Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+2})$$

Now we show that $\mathcal{K}_n$ is Cauchy sequence. Without loss of generality assume that $n > m$

Then

$$Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_{m+1}) \leq Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(\mathcal{K}_{n+1}, \mathcal{K}_m, \mathcal{K}_m)$$

$$Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_{m+1}) \leq Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}) + Å(\mathcal{K}_{n+2}, \mathcal{K}_m, \mathcal{K}_{m+1})$$

$$Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_{m+1}) \\ \leq Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+3}, \mathcal{K}_{n+3}) + \cdots + Å(\mathcal{K}_{m-1}, \mathcal{K}_m, \mathcal{K}_{m+1})$$

$$Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_{m+1}) \leq \mathcal{L}^n Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2) + \mathcal{L}^{n-1}Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2) + \cdots + \mathcal{L}^m Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2)$$

$$Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_{m+1}) \leq \mathcal{L}^n(1 + \mathcal{L} + \mathcal{L}^2 + .. + \mathcal{L}^{n-m})Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2)$$

$$Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_{m+1}) \leq \frac{\mathcal{L}^n}{1 - \mathcal{L}^{n-m}} Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2)$$

Hence, limit $m, n \to \infty$

$$\lim_{m,n\to\infty} Å\,(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_{m+1}) = 0$$

i.e. $\{\mathcal{K}_n\}$ is Cauchy sequence.

Since $(\varkappa, Å, \exists)$ is complete, so there exists $w \in X$ such that $\mathcal{K}_n \to w$, which implies,

$$\lim_{n\to\infty} Å\,(\mathcal{K}_n, \mathcal{K}_n, w) = 0$$

Next, we will show that w is fixed point of $\mathcal{B}$. we take $\widetilde{\wp_\varepsilon} = \mathcal{K}_n$ and

$\widetilde{ℚ_\mu} = \widetilde{ℝ_\omega} = w$ then

$$\alpha Å(\mathcal{B}\mathcal{K}_n, \mathcal{B}w, \mathcal{B}w) + \beta\left[Å(\mathcal{K}_n, \mathcal{B}\mathcal{K}_n, \mathcal{B}\mathcal{K}_n) + Å(w, \mathcal{B}w, \mathcal{B}w) + Å(w, \mathcal{B}w, \mathcal{B}w)\right] \\ + \delta\left[\frac{Å(\mathcal{B}\mathcal{K}_n, \mathcal{B}w, \mathcal{B}w) + Å(\mathcal{K}_n, \mathcal{B}\mathcal{K}_n, \mathcal{B}\mathcal{K}_n) + Å(w, \mathcal{B}w, \mathcal{B}w) + Å(w, \mathcal{B}w, \mathcal{B}w)}{1 + Å(\mathcal{B}\mathcal{K}_n, \mathcal{B}w, \mathcal{B}w)\{Å(\mathcal{K}_n, \mathcal{B}\mathcal{K}_n, \mathcal{B}\mathcal{K}_n) + Å(w, \mathcal{B}w, \mathcal{B}w) + Å(w, \mathcal{B}w, \mathcal{B}w)\}}\right] \\ \leq \gamma Å(\widetilde{\wp_\varepsilon}, w, w)$$

$$a Å(\mathcal{K}_{n+1}, Bw, Bw) + b\left[Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(w, Bw, Bw) + Å(w, Bw, Bw)\right] + \frac{Å(\mathcal{K}_{n+1}, Bw, Bw) + Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(w, Bw, Bw) + Å(w, Bw, Bw)}{1 + Å(\mathcal{K}_{n+1}, Bw, Bw)\{Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) + Å(w, Bw, Bw) + Å(w, Bw, Bw)\}} \leq c Å(\mathcal{K}_n, w, w)$$

$$a Å(x_{n+1}, Bw, Bw) + b\left[Å(x_n, x_{n+1}, x_{n+1}) + 2Å(w, Bw, Bw)\right] + d\left[\frac{Å(x_{n+1}, Bw, Bw) + Å(x_n, x_{n+1}, x_{n+1}) + 2Å(w, Bw, Bw)}{1 + Å(x_{n+1}, Bw, Bw)\{Å(x_n, x_{n+1}, x_{n+1}) + 2Å(w, Bw, Bw)\}}\right] \leq c Å(x_n, w, w)$$

$$a Å(x_{n+1}, Bw, Bw) + b\left[Å(x_n, x_{n+1}, x_{n+1}) + 2Å(w, Bw, Bw)\right] + d\left[\frac{2Å(x_n, x_{n+1}, x_{n+1}) + 2Å(w, Bw, Bw)}{1 + Å(x_{n+1}, Bw, Bw)\{Å(x_n, x_{n+1}, x_{n+1}) + 2Å(w, Bw, Bw)\}}\right] \leq c Å(x_n, w, w)$$

As $n \to \infty$ we have

$$\alpha Å(w, \mathcal{B}w, \mathcal{B}w) + \beta[2Å(w, \mathcal{B}w, \mathcal{B}w)] + \delta\left[\frac{2Å(w, \mathcal{B}w, \mathcal{B}w)}{1 + Å(w, \mathcal{B}w, \mathcal{B}w).2Å(w, \mathcal{B}w, \mathcal{B}w)}\right] \leq 0$$

$$(\alpha + 2\beta)Å(w, \mathcal{B}w, \mathcal{B}w) \leq 0$$

Which is contradiction, so $\mathcal{B}w = w$

i.e. w is fixed point of $\mathcal{B}$.

uniqueness:

let f & g are two more fixed point of B these are distinct from w

then we take $\widetilde{\wp_\varepsilon} = w, y = f, z = g$

$$\alpha Å(\mathcal{B}w, \mathcal{B}f, \mathcal{B}g) + \beta\left[Å(w, \mathcal{B}w, \mathcal{B}w) + Å(f, \mathcal{B}f, \mathcal{B}f) + Å(g, \mathcal{B}g, \mathcal{B}g)\right] + \delta\left[\frac{Å(\mathcal{B}w, \mathcal{B}f, \mathcal{B}g) + Å(w, \mathcal{B}w, \mathcal{B}w) + Å(f, \mathcal{B}f, \mathcal{B}f) + Å(g, \mathcal{B}g, \mathcal{B}g)}{1 + Å(\mathcal{B}w, \mathcal{B}f, \mathcal{B}g)\{Å(w, \mathcal{B}w, \mathcal{B}w) + Å(f, \mathcal{B}f, \mathcal{B}f) + Å(g, \mathcal{B}g, \mathcal{B}g)\}}\right] \leq \gamma Å(w, f, g)$$

$$Å(w, f, g) \leq \frac{\gamma}{\alpha} Å(w, f, g)$$

Which is contradiction so,

$w = f = g,$

hence w is unique fixed point of $\mathcal{B}$.

So complete the proof of the theorem.

Theorem: Let $(\varkappa, \mathring{A}, \exists)$ be a complete soft G-metric space, and let

$\mathcal{B}: S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \to \mathcal{R}(\exists)^*$ be a mapping satisfies the following condition

$$min\{\mathring{A}\big(\mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\big), \mathring{A}\big(\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}, \mathcal{B}\widetilde{\wp_\varepsilon}\big), \mathring{A}\big(\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}, \mathcal{B}\widetilde{\mathbb{Q}_\mu}\big), \mathring{A}\big(\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}, \mathcal{B}\widetilde{\mathbb{R}_\omega}\big)\} \leq \alpha\mathring{A}\big(\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega}\big) \ldots..(2.1.1)$$

for all $\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega} \in \varkappa, where\ 0 \leq \alpha < 1.$

Proof: take an arbitrary and define a sequence $\mathcal{K}_{n+1} = \mathcal{K}_n, n = 0,1,2, \ldots$

Substituting $\widetilde{\wp_\varepsilon} = \mathcal{K}_n,\ \ \widetilde{\mathbb{Q}_\mu} = \mathcal{K}_{n+1},\ \ \widetilde{\mathbb{R}_\omega} = \mathcal{K}_{n+1}$

$$min\{\mathring{A}(\mathcal{B}\mathcal{K}_n, \mathcal{B}\mathcal{K}_{n+1}, \mathcal{B}\mathcal{K}_{n+1}), \mathring{A}(\mathcal{K}_n, \mathcal{B}\mathcal{K}_{n+1}, \mathcal{B}\mathcal{K}_{n+1}), \mathring{A}(\mathcal{K}_{n+1}, \mathcal{B}\mathcal{K}_{n+1}, \mathcal{B}\mathcal{K}_{n+1}), \mathring{A}(\mathcal{K}_{n+1}, \mathcal{B}\mathcal{K}_{n+1}, \mathcal{B}\mathcal{K}_{n+1})\} \leq \alpha\mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})$$

mi n $\mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}), \mathring{A}(\mathcal{K}, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}), \mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}), \mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2})$
$\leq a\mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})$

$$min\big\{\mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}), \mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})\big\} \leq a\mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})$$

if we take

$$min\big\{\mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}), \mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})\big\} = \mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}) \ldots.(2.1.2)$$

then from above equation

$$\mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}) \leq \alpha\mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})$$

And if we take

$$min\{\mathring{A}(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}), \mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})\} = \mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})$$

then

$$\mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) \leq \alpha\mathring{A}(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})$$

Which is contradiction, so that

$$Å(\mathcal{K}_{n+1}, \mathcal{K}_{n+2}, \mathcal{K}_{n+2}) \leq \alpha Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1})$$

Similarly, we can show that

$$Å(\mathcal{K}_n, \mathcal{K}_{n+1}, \mathcal{K}_{n+1}) \leq \alpha Å(\mathcal{K}_{n-1}, \mathcal{K}_n, \mathcal{K}_n)$$

Next, we show that $\{\mathcal{K}_n\}$ is Cauchy sequence. Without loss of generality assume that $n > m$,

Then

$$Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_m) \leq Å(\mathcal{K}_n, \mathcal{K}_{n-1}, \mathcal{K}_{n-1}) + Å(\mathcal{K}_{n-1}, \mathcal{K}_m, \mathcal{K}_m)$$

$$\leq Å(\mathcal{K}_n, \mathcal{K}_{n-1}, \mathcal{K}_{n-1}) + \ldots + Å(\mathcal{K}_{m-1}, \mathcal{K}_m, \mathcal{K}_m)$$

$$\leq \mathcal{L}^n Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2) + \mathcal{L}^{n-1} Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2) + \ldots + \mathcal{L}^{m-1} Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2)$$

$$\leq \mathcal{L}^n (1 + \mathcal{L} + \mathcal{L}^2 + \ldots + \mathcal{L}^{n-m})\, Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2)$$

$$\leq \frac{\mathcal{L}^n}{1 - \mathcal{L}^{n-m}} Å(\mathcal{K}_0, \mathcal{K}_1, \mathcal{K}_2)$$

Hence, limit$m, n \to \infty$

$$\lim_{m,n\to\infty} Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_m) = 0$$

i.e.$\{\mathcal{K}_n\}$ is Cauchy sequence.

Since $(\varkappa, Å, \exists)$ is complete soft G- metric space which gives

$w \in \varkappa \text{ such that } \{\mathcal{K}_n\} \to w, \text{as } n \to \infty$

Next, we will show that w is fixed point of $\mathcal{B}$. for this we take

$\widetilde{\wp_\varepsilon} = \mathcal{K}_n$ and $\widetilde{\mathbb{Q}_\mu} = \widetilde{\mathbb{R}_\omega} = w$,

$$min\{Å(\mathcal{B}\mathcal{K}_n, \mathcal{B}w, \mathcal{B}w), Å(\mathcal{K}_n, \mathcal{B}\mathcal{K}_n, \mathcal{B}\mathcal{K}_n), Å(w, \mathcal{B}w, \mathcal{B}w), Å(w, \mathcal{B}w, \mathcal{B}w)\} \leq \alpha Å(\mathcal{K}_n, w, w)$$

$$min\{Å(\mathcal{B}\mathcal{K}_n, \mathcal{B}w, \mathcal{B}w), Å(\mathcal{K}_n, \mathcal{B}x_n, \mathcal{B}x_n), Å(w, \mathcal{B}w, \mathcal{B}w)\} \leq \alpha Å(\mathcal{K}_n, w, w)$$

As $n \to \infty$ we have

$$min\{Å(\mathcal{B}w, \mathcal{B}w, \mathcal{B}w), Å(w, \mathcal{B}w, \mathcal{B}w), Å(w, \mathcal{B}w, \mathcal{B}w)\} \leq \alpha Å(w, w, w)$$

which is contradiction, so $\mathcal{B}w = w$ i.e. w is fixed point of $\mathcal{B}$.

uniqueness:

let f and g are two more fixed points of $\mathcal{B}$, different from w, $i.e. w \neq f \neq g$.

we take $\widetilde{\wp_\varepsilon} = w,\ \widetilde{\mathbb{Q}_\mu} = f, \widetilde{\mathbb{R}_\omega} = g$ now put this in 2.1.1

$$min\{Å(\mathcal{B}w,\mathcal{B}w,\mathcal{B}w), Å(w,\mathcal{B}w,\mathcal{B}w), Å(f,\mathcal{B}f,\mathcal{B}f), Å(g,\mathcal{B}g,\mathcal{B}g)\} \leq \alpha Å(w,f,g)$$

$$Å(w,f,g) \leq \alpha Å(w,f,g)$$

Which is contradiction, so $w = f = g$, i.e. w is unique fixed point of $\mathcal{B}$.

Hence proved of the theorem.

Theorem: let $(\varkappa, Å, \exists)$ be a complete soft G-metric space, and let $\mathcal{B}: S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \times S\wp(\widetilde{\varkappa}) \to \mathcal{R}(\exists)^*$ be a mapping satisfies the following condition

$$\frac{min\{Å\left(\mathcal{B}\widetilde{\wp_\varepsilon},\mathcal{B}\widetilde{\mathbb{Q}_\mu},\mathcal{B}\widetilde{\mathbb{R}_\omega}\right), Å(\widetilde{\wp_\varepsilon},\mathcal{B}\widetilde{\wp_\varepsilon},\mathcal{B}\widetilde{\wp_\varepsilon}), Å\left(\widetilde{\mathbb{Q}_\mu},\mathcal{B}\widetilde{\mathbb{Q}_\mu},\mathcal{B}\widetilde{\mathbb{Q}_\mu}\right), Å\left(\widetilde{\mathbb{R}_\omega},\mathcal{B}\widetilde{\mathbb{R}_\omega},\mathcal{B}\widetilde{\mathbb{R}_\omega}\right)\}}{min\{Å(\widetilde{\wp_\varepsilon},\mathcal{B}\widetilde{\wp_\varepsilon},\mathcal{B}\widetilde{\wp_\varepsilon}), Å\left(\widetilde{\mathbb{Q}_\mu},\mathcal{B}\widetilde{\mathbb{Q}_\mu},\mathcal{B}\widetilde{\mathbb{Q}_\mu}\right), Å\left(\widetilde{\mathbb{R}_\omega},\mathcal{B}\widetilde{\mathbb{R}_\omega},\mathcal{B}\widetilde{\mathbb{R}_\omega}\right)\}} \leq \alpha Å\left(\widetilde{\wp_\varepsilon},\widetilde{\mathbb{Q}_\mu},\widetilde{\mathbb{R}_\omega}\right)$$

For all $\widetilde{\wp_\varepsilon}, \widetilde{\mathbb{Q}_\mu}, \widetilde{\mathbb{R}_\omega} \in \varkappa, where\ 0 \leq \alpha \leq 1$.

Proof: take an arbitrary and define a sequence $\mathcal{K}_{n+1} = \mathcal{K}_n,\ \ n = 0,1,2.\ldots.$

Substituting $\widetilde{\wp_\varepsilon} = \mathcal{K}_n,\ \ \widetilde{\mathbb{Q}_\mu} = \mathcal{K}_{n+1},\ \ \mathbb{R}_\omega = \mathcal{K}_{n+1}$, then we have

$$\frac{min\{Å(\mathcal{B}\mathcal{K}_n,\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1}), Å(\mathcal{K}_n,\mathcal{B}\mathcal{K}_n,\mathcal{B}\mathcal{K}_n), Å(\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1}), Å(\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1})\}}{min\{Å(\mathcal{K}_n,\mathcal{B}\mathcal{K}_n,\mathcal{B}\mathcal{K}_n), Å(\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1}), Å(\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1})\}}$$

$$\leq \alpha Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

$$\frac{min\{Å(\mathcal{B}\mathcal{K}_n,\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1}), Å(\mathcal{K}_n,\mathcal{B}\mathcal{K}_n,\mathcal{B}\mathcal{K}_n), Å^2(\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1})\}}{min\{Å(\mathcal{K}_n,\mathcal{B}\mathcal{K}_n,\mathcal{B}\mathcal{K}_n), Å(\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1}), Å(\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1},\mathcal{B}\mathcal{K}_{n+1})\}} \leq \alpha Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

$$\frac{\mathrm{mi\ n}\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}), Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1}), Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})\}}{min\{Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1}), Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}), Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})\}} \leq \alpha Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

$$\frac{\min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1}),Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})\}}{min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})\}} \leq \alpha Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

Case1: if we take

$$\frac{\min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1}),Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})\}}{min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})\}} = \frac{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})}{Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})}$$

$$\frac{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})}{Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})} \leq \alpha Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

$$Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}) \leq \alpha Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

Case2: if we take

$$\frac{\min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1}),Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})\}}{min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})\}} = \frac{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})}{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})}$$

$$\frac{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})}{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})} \leq \alpha Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

$$Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1}) \leq \alpha Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

Which is contradiction, so that

Case3:

$$\frac{\min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1}),Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})\}}{min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}),Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})\}} = \frac{Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})}{Å(\mathcal{K}_{n},\mathcal{K}_{n+1},\mathcal{K}_{n+1})}$$

$$\frac{Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})}{Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})} \leq \alpha Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

$$Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}) \leq bÅ(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1}), \qquad where\ b = \sqrt{\alpha}$$

Case4:

$$\frac{\min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}), Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1}), Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})\}}{min\{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}), Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})\}}$$
$$= \frac{Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})}{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})}$$

$$\frac{Å^2(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})}{Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2})} \leq \alpha Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

$$Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}) \leq \alpha Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

From case1,2,3 and 4, we have

$$Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}) \leq \alpha Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1})$$

By induction we have

$$Å(\mathcal{K}_{n+1},\mathcal{K}_{n+2},\mathcal{K}_{n+2}) \leq \alpha^{n+1} Å(\mathcal{K}_0,\mathcal{K}_1,\mathcal{K}_1)$$

Similarly, we can show that

$$Å(\mathcal{K}_n,\mathcal{K}_{n+1},\mathcal{K}_{n+1}) \leq \alpha Å(\mathcal{K}_{n-1},\mathcal{K}_n,\mathcal{K}_{n+2})$$

Next, we show that $\{\mathcal{K}_n\}$ is Cauchy sequence. Without loss of generalitt assume the $n > m$.

Then

$$Å(\mathcal{K}_n,\mathcal{K}_m,\mathcal{K}_m) \leq Å(\mathcal{K}_n,\mathcal{K}_{n-1},\mathcal{K}_{n-1}) + Å(\mathcal{K}_{n-1},\mathcal{K}_m,\mathcal{K}_m)$$

$$\leq Å(\mathcal{K}_n,\mathcal{K}_{n-1},\mathcal{K}_{n-1}) + \ldots.. + Å(\mathcal{K}_{m-1},\mathcal{K}_m,\mathcal{K}_m)$$

$$\leq \mathcal{L}^n Å(\mathcal{K}_0,\mathcal{K}_1,\mathcal{K}_2) + \mathcal{L}^{n-1} Å(\mathcal{K}_0,\mathcal{K}_1,\mathcal{K}_2) + \ldots. + \mathcal{L}^{m-1} Å(\mathcal{K}_0,\mathcal{K}_1,\mathcal{K}_2)$$

$$\leq \mathcal{L}^n (1 + \mathcal{L} + \mathcal{L}^2 + \ldots. + \mathcal{L}^{n-m}) Å(\mathcal{K}_0,\mathcal{K}_1,\mathcal{K}_2)$$

$$\leq \frac{\mathcal{L}^n}{1 - \mathcal{L}^{n-m}} Å(\mathcal{K}_0,\mathcal{K}_1,\mathcal{K}_2)$$

Hence, $limit\ m, n \to \infty$

$$\lim_{m,n\to\infty} Å(\mathcal{K}_n, \mathcal{K}_m, \mathcal{K}_m) = 0$$

i.e. $\{\mathcal{K}_n\}$ is cauchy sequence.

Since $(\varkappa, Å, \exists)$ is complete soft G-metric space which gives $w \in \varkappa$ such that $\{\mathcal{K}_n\} \to w\ as\ n \to \infty$.

Next, we will show that w is fixed point of R. for this we take

$\widetilde{\wp_\varepsilon} = \mathcal{K}_n$ and $\widetilde{\mathbb{Q}_\mu} = \widetilde{\mathbb{R}_\omega} = w$

$$min\{Å(\mathcal{B}\mathcal{K}_n, \mathcal{B}w, \mathcal{B}w), Å(\mathcal{K}_n, \mathcal{B}\mathcal{K}_n, \mathcal{B}\mathcal{K}_n), Å(w, \mathcal{B}w, \mathcal{B}w), Å(w, \mathcal{B}w, \mathcal{B}w)\} \leq \alpha Å(\mathcal{K}_n, w, w)$$

$$min\{Å(\mathcal{B}\mathcal{K}_n, \mathcal{B}w, \mathcal{B}w), Å(\mathcal{K}_n, \mathcal{B}x_n, \mathcal{B}x_n), Å(w, \mathcal{B}w, \mathcal{B}w)\} \leq \alpha Å(\mathcal{K}_n, w, w)$$

As $n \to \infty$ we have

$$min\{Å(\mathcal{B}w, \mathcal{B}w, \mathcal{B}w), Å(w, \mathcal{B}w, \mathcal{B}w), Å(w, \mathcal{B}w, \mathcal{B}w)\} \leq \alpha Å(w, w, w)$$

which is contradiction, so $\mathcal{B}w = w$ i.e. w is fixed point of $\mathcal{B}$.

uniqueness:

let f & g are two another fixed points of $\mathcal{B}$, different from w, i.e. $w \neq f \neq g$.

we take $\widetilde{\wp_\varepsilon} = w,\ \widetilde{\mathbb{Q}_\mu} = f, \widetilde{\mathbb{R}_\omega} = g$ now put this in 2.1.1

$$min\{Å(\mathcal{B}w, \mathcal{B}w, \mathcal{B}w), Å(w, \mathcal{B}w, \mathcal{B}w), Å(f, \mathcal{B}f, \mathcal{B}f), Å(g, \mathcal{B}g, \mathcal{B}g)\} \leq \alpha Å(w, f, g)$$

$$Å(w, f, g) \leq \alpha Å(w, f, g)$$

Which is contradiction, so $w = f = g$, i.e. w is unique fixed point of $\mathcal{B}$.

Hence proved of the theorem.

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