PBIB-DESIGNS ARISING FROM GEODETIC SETS IN CIRCULANT GRAPHS

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ABSTRACT. A $(v, g(G), \mu)$-design over regular graph $G = (V, E)$ of degree d is an ordered pair $D = (V, B)$, where $|V| = v$ and B is the set of geodetic sets in G called blocks such that if $i, j \in V$, $i \neq j$ and if i and j are not adjacent in G then there are exactly μ blocks containing i and j. In this paper, we obtain some partially balanced incomplete block (PBIB)-designs with association schemes arising from geodetic sets in circulant graphs with different jump sizes.

1. INTRODUCTION

A *balanced incomplete block design (BIBD)* is a set of v vertices arranged in b blocks of k vertices each in such a way that each vertex occurs in exactly r blocks and every pair of unordered vertices occurs in λ blocks. The combinatorial configuration so obtained is called a (v, b, r, k, λ)-design. Although BIBDs have many optimal properties, they do not fit well into most experimental situations as their repetition number is too large. To overcome this, a group of binary, equireplicate designs were introduced through, *Partially balanced incomplete block designs (PBIBDs).*

Walikar et al.[24] introduced a design called (v, β_o, μ)-design over a regular graph G. This design was somewhat similar to the (v, k, λ, μ)-design over a regular graph G introduced by Ionin and Shrikhande [15]. Ionin and Shrikhande [15] dealt the designs when the repetition was $r = 2\lambda - \mu$, whereas Walikar et al.[24] considered the designs with $r = \dfrac{(v-d-1)\mu}{\beta_o}$. Huilgol et al. [22] introduced a design called (geodetic) $(v, g(G), \mu)$-design arising from the geodetic sets of a graph, derived the governing results of a geodetic-design, if it exists, and then obtained such PBIB-designs for different products of graphs, viz., the cartesian product, the direct product, the lexicographic product, the corona product. The $(v, g(G), \mu)$-designs have $r = \dfrac{b \cdot g(G)}{v}$. In most cases, a geodetic set is usually independent, a comparative inevitable study of $(v, g(G), \mu)$ and (v, β_o, μ)-designs was considered in [22].

Beauty comes out of symmetry as well as asymmetry. Investigation of symmetries or asymmetries of

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structures yield powerful results in Mathematics. Circulant graphs form a class of highly symmetric graphical structures. Formally, an undirected graph acted on by a cyclic group of symmetries which takes any vertex to any other vertex is called a circulant graph. The properties, structures of these graphs have been investigated by many authors [4], [2], etc. The fact that these graphs have their adjacency matrices as circulant matrices, makes them to find many applications in far reaching fields where automorphisms and symmetries find applications. The circulant matrices were introduced by Catalan [8] way back in 1846. In 1994, the algebraic structures, properties and applications of circulant matrices have been summarized in the book "Circulant Matrices" by Davis [13] as well in [8], [14].

Inspired by all the above, in this paper we consider PBIB-designs arising in circulant graphs. Hence, these designs give layered structures based on the most fundamental parameters that is, geodetic sets. In themselves these two parameters have been studied in depth for graphs. But designs from these parameters are untouched. Although, other paraemeters like, minimum edge independent sets [10], minimum split dominating sets [16] are considered.

2. Definitions and Preliminary results

Throughout this paper, $G = (V, E)$ stands for a finite, undirected graph with neither loops nor multiple edges. The number of vertices is n and that of edges is m. The terms not defined here are used in the sense of Buckley and Harary [6].

Let x and y be vertices of a connected graph G. A shortest x, y-path is also called a x, y-geodesic. The distance between two vertices x and y is defined as the length of a x, y-geodesic in G and is denoted by $d_G(x, y)$ or $d(x, y)$, in short if the graph G is clear from context. For a vertex x in G, its *eccentricity*, $ecc(x)$ is defined as the distance to a farthest vertex from x in G. For a vertex x, a vertex y is an *eccentric vertex* if, $d(x, y) = ecc(x)$. In a graph G, the *diameter* of G, denoted as $diam(G)$, is the maximum eccentricity in G and the *radius* of G, denoted as $rad(G)$, is the minimum eccentricity in G. A graph G is called a *self-centered graph* if $diam(G) = rad(G)$. If each vertex of a graph G has exactly one eccentric vertex, then G is called a *unique eccentric vertex graph.* For a vertex x in a connected graph G, let $d_i(x)$ be the number of vertices at distance i from x. The *distance degree sequence* of a vertex x is $dds(x) = (d_0(x), d_1(x), \ldots, d_{ecc(x)}(x))$. The *distance degree sequence DDS(G)* of a graph G consists of the collection of sequences $dds(x)$ of all its vertices, listed in numerical order. The *distance degree regular (DDR) graph* is a graph in which all vertices have the same distance degree sequence.

Next we list some definitions and results related to circulant graphs and geodeticity.
Given a set $S \subseteq V(G)$, its *geodetic closure* $I[S]$ is the set of all vertices lying on some shortest path joining two vertices of S that is,

$$I[S] = \{v \in V(G) | v \in I(x,y), x, y \in S\} = \bigcup_{x,y \in S} I(x,y).$$

A set $S \subseteq V(G)$ is called a *geodetic set* in G if $I[S] = V(G)$; that is, every vertex in G lies on some geodesic between two vertices from S. The *geodetic number* $g(G)$ of a graph G is the minimum cardinality of a geodetic set in G.

Following are two simple (of course, not trivial) results on geodetic number.

Theorem 2.1. [6] *Let G be a connected graph. Then $g(G) = n$ if and only if $G = K_n$.*

Theorem 2.2. [6] *For a connected graph G, $g(G) = 2$ if and only if there exist two vertices u and v with $d(u,v) = diam(G)$ and every vertex of G lies on a geodesic between vertices u and v.*

Definition 2.1. *For a given positive integer p, let $n_1, n_2, \ldots, n_k$ be a sequence of integers where*

$$0 < n_1 < n_2 < \ldots < n_k < \frac{(p+1)}{2}.$$

Then the Circulant graph $C_p(n_1, n_2, \ldots, n_k)$ is the graph on p vertices $v_1, v_2, \ldots, v_p$ with a vertex v_i adjacent to each vertex $v_{i \pm n_j}(mod p)$. The values of n_i, are called jump sizes.

Next we list some definitions related to designs.

Definition 2.2. [23] *Given a set $\{1, 2, \ldots, v\}$ a relation satisfying the following conditions is said to be an association scheme with m classes.*

(1) *Any two symbols α and β are i^{th} associates for some i, with $1 \leq i \leq m$ and this relation of being i^{th} associates is symmetric.*
(2) *The number of i^{th} associates of each symbol is n_i .*
(3) *If α and β are two symbols which are i^{th} associates, then the number of symbols which are j^{th} associates of α and k^{th} associates of β is p^i_{jk} and is independent of the pair of i^{th} associates α and β.*

Definition 2.3. [23] *Consider a set of symbols $V = \{1, 2, \ldots, v\}$ and an association scheme with m classes on V. A Partially balanced incomplete block design (PBIBD) is a collection of b subsets of V called blocks, each of them containing k symbols $(k < v)$, such that every symbol occurs in r blocks*

and two symbols α and β which are i^{th} associates occur together in λ_i blocks, the numbers λ_i being independent of the choice of the pair α and β.
The numbers v, b, r, k, λ_i $(1, 2, \ldots, m)$ are called the parameters of first kind and the numbers $n_i's$ and p^i_{jk} are called the parameters of second kind.

3. $(v, g(G), \mu)$-DESIGNS OVER CIRCULANT GRAPHS

The general set up of designs arising from a particular graph parameter along with all the associates is an interesting one as seen in the previous section. Considering sets of vertices corresponding to different graph invariants serves the purpose and finds many applications. Particularly, in this section, we determine designs having blocks as the vertices belonging to geodetic sets in circulant graphs and the association schemes referring to the vertices at different distances. Hence, the graphs for which these designs exist are highly regular in nature to keep their repetition number fixed.
The $(v, g(G), \mu)$-designs were defined by Huilgol et al. [22] and construct PBIB-designs with different association schemes corresponding to the circulant graphs with different jump sizes. Formally, we first recollect the definition of $(v, g(G), \mu)$-design as follows:

Definition 3.1. [22] *A $(v, g(G), \mu)$-design, called a geodetic design (in short) over a regular, self-centered graph $G = (V, E)$ of degree d, is an ordered pair $D = (V, B)$, where $V = V(G)$ and B, the set of all geodetic sets in G, called blocks, containing the vertices belonging to the minimum geodetic sets, of size $g(G)$ and every pair of non-adjacent vertices appearing in exactly μ blocks.*

The next result of Huilgol et al. [22] gives a sufficient condition for the existence of a $(v, g(G), \mu)$-design. We state it here for the purpose of motivation of the present paper.

Proposition 3.1. [22] *Let G be a regular graph of degree d and let $D = (V, B)$ be a geodetic $(v, g(G), \mu)$-design over G. Then there exists an integer r, called the repetition number. Furthermore the following three conditions hold:*

$$g(G)(v + b)\mu = v(r + g(G)) \tag{3.1}$$

$$vr = bg(G) \tag{3.2}$$

$$r \geq \mu. \tag{3.3}$$

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Remark 1. *Proposition 3.1 is a fundamental one. Its three conditions work as governing results for $(v, g(G), \mu)$-designs. Thus the above result is a crucial one in proving our main results. In the following sections, we consider circulant graphs and construct $(v, g(G), \mu)$-designs with different association schemes.*

Theorem 3.1. [20] *The collection of all geodetic sets in a regular, self-centered, unique eccentric vertex graph forms a PBIB-design.*

Theorem 3.2. [20] *The collection of all geodetic sets in a complete k-partite graph $K_{n,n,\ldots,n}$, for $n \geq 2$, $k \geq 2$ forms a PBIB-design with parameters given as follows:*

- $v = 2k$, $b = k$, $g(K_{2,2,\ldots,2}) = 2$, $r = 1$, $\lambda_1 = 0$, $\lambda_2 = 1$, *when* $n = 2$*;*
- $v = 3k$, $b = k$, $g(K_{3,3,\ldots,3}) = 3$, $r = 1$, $\lambda_1 = 0$, $\lambda_2 = 1$, *when* $n = 3$*;*
- $v = 4k$, $b = 36\binom{k}{2} + k$, $g(K_{4,4,\ldots,4}) = 4$, $r = 18k - 17$, $\lambda_1 = 10$, $\lambda_2 = 6k - 5$, *when* $n = 4$*;*
- $v = nk$, $b = \binom{k}{2}\frac{n^2(n-1)^2}{4}$, $g(K_{n,n,\ldots,n}) = 4$, $r = \frac{(k-1)n(n-1)^2}{2}$, $\lambda_1 = (n-1)^2$, $\lambda_2 = (k-1)\binom{n}{2}$, *when* $n \geq 5, k \geq 2$.

Lemma 3.1. *The collection of all geodetic sets in $C_n\left(1, 2, \ldots, \left\lfloor \frac{n}{2} \right\rfloor\right)$, $n \geq 3$ vertices form $(v, g(G), \mu)$-design with 1-association scheme with parameters $v = n$, $b = 1$, $g(G) = n$, $r = 1$ and $\mu = 0$. The parameter of second kind is given by $n_1 = n - 1$.*

Proof. Clearly, $C_n\left(1, 2, \ldots, \left\lfloor \frac{n}{2} \right\rfloor\right) \cong K_n$, $n \geq 3$. And we know that $g(K_n) = n$. So there exists a single geodetic set forming a PBIB-design with parameters as in the statement. Also, the parameters of second kind are given by $n_1 = n - 1$ and $P_1 = [p_{11}^1] = [n-2]$. □

Remark 2. *If the jump size of circulant graphs is one, then it is a cycle C_n for $n \geq 3$ vertices. That is, $C_n(1) \cong C_n$; $n \geq 3$.*

Lemma 3.2. *The collection of all geodetic sets in C_{2n}, $n \geq 2$ forms a $(v, g(G), \mu)$-design with n-association schemes with the parameters $v = 2n$, $b = n$, $g(G) = 2$, $r = 1$ and $\mu = 1$. The parameters of second kind are given by*

$$n_i = \begin{cases} 1, & if \quad i = n, \\ 2, & if \quad 1 \leq i \leq n-1. \end{cases}$$

Proof. Since C_{2n} is a self-centered, unique eccentric graph of diameter n, parameters of first kind follow from Theorem 3.1.
Since C_{2n} is a distance degree regular (DDR) graph with distance degree sequence (dds) of each vertex being $(1, 2, 2, \dots, 2, 1)$. The parameters of second kind are given by Theorem 7.1[9]. □

Remark 3. *The collection of all geodetic sets in circulant graph C_{2n+1}, $n \geq 2$ does not form a $(v, g(G), \mu)$-design, as the repetition number r is not unique.*

Lemma 3.3. *The collection of all geodetic sets in $C_{2n}(1, 2, \dots, n-1)$ forms a $(v, g(G), \mu)$-design with 2-association schemes and parameters are given by, $v = 2n$, $b = n$, $g(G) = 2$, $r = 1$, $\mu = 1$. The parameters of second kind are given by $n_1 = 2(n-1)$, $n_2 = 1$.*

Proof. We know that $C_{2n}(1, 2, \dots, n-1) \cong K_{2,2,\dots,2}$, proof follows from Theorem 3.2.
It is easy to see that the parameters of second kind are entries of distance degree sequence (dds) of each vertex, as G is a DDR graph with dds of each vertex being $(1, 2(n-1), 1)$. The parameters of second kind are given by $n_1 = 2(n-1)$, $n_2 = 1$. And,

$$P^1 = \begin{pmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{pmatrix} = \begin{pmatrix} 2(n-2) & 1 \\ 1 & 0 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{pmatrix} = \begin{pmatrix} 2(n-1) & 0 \\ 0 & 0 \end{pmatrix}.$$

□

Theorem 3.3. *The collection of all geodetic sets in $C_{2n}(2, n)$ for $n \geq 5$ odd, forms a $(v, g(G), \mu)$-design with $\left\lceil \frac{n}{2} \right\rceil$-association schemes and parameters are given by $v = 2n$, $b = 2n$, $g(G) = 3$, $r = 3$ and $\mu = 2$. The parameters of second kind are given by*

$$n_i = \begin{cases} 2, & if \quad i = \left\lceil \frac{n}{2} \right\rceil; \\ 3, & if \quad i = 1; \\ 4, & if \quad 2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1. \end{cases}$$

Proof. We know that the circulant $C_{2n}(2, n) \cong K_2 \times C_n$, whenever n is odd. This graph is also known as prism. Let $G = K_2 \times C_n$ for $n \geq 5$ and label its vertices as $v_1, v_2, \dots, v_{2n}$. This graph is obtained by taking two copies of C_n and joining corresponding vertices by prism edges. Now choose a vertex from first copy of C_n, say v_1 whose eccentric vertices, say, v_q, v_{q+1} lie in the second copy

of C_n. Hence, $d(v_1, v_q) = diam(G) = n + 1 = d(v_1, v_{q+1})$. Now $S = \{v_1, v_q, v_{q+1}\}$ is a minimum geodetic set, implying that $g(G) = 3$. Since v_1 is an arbitrarily chosen vertex from one copy of C_n, we get, n geodetic sets from first copy and similarly, n from the second copy, making the number of blocks equal to $2n$. Then, by using the relation $vr = bg(G)$ we get $r = 3$ and using the first equality in Condition (i) of Proposition 3.1, we get $\mu = 2$.
Thus, we have a PBIB-design with parameters $v = 2n$, $b = 2n$, $g(G) = 3$, $r = 3$ and $\mu = 2$.

It is easy to see that the parameters of second kind are the distance degree sequence (dds) of each vertex, as prism is a DDR graph. Hence the parameters of second kind are given by

$$n_i = \begin{cases} 2, & if \quad i = \left\lceil \frac{n}{2} \right\rceil; \\ 3, & if \quad i = 1; \\ 4, & if \quad 2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \end{cases}$$

The parameters of second kind can be represented as the following matrix

$$P^k = \begin{pmatrix} p^k_{11} & p^k_{12} \cdots & p^k_{1\lceil\frac{n}{2}\rceil} \\ p^k_{21} & p^k_{22} \cdots & p^k_{2\lceil\frac{n}{2}\rceil} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ p^k_{\lceil\frac{n}{2}\rceil 1} & p^k_{\lceil\frac{n}{2}\rceil 2} \cdots & p^k_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil} \end{pmatrix}$$

Now we give all the entries of above matrix by considering different values of k.
Case 1. If $k = 1$, then
$p^1_{ij} = 1$ for $i = j = \left\lceil \frac{n}{2} \right\rceil$, $i = j = \left\lceil \frac{n}{2} \right\rceil - 1$ and $i = \left\lceil \frac{n}{2} \right\rceil - 1$, $j = i + 1$; $j = \left\lceil \frac{n}{2} \right\rceil - 1$, $i = j + 1$;
$p^1_{ij} = 2$ for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 2$, $j = i + 1$ and $1 \leq j \leq \left\lceil \frac{n}{2} \right\rceil - 2$, $i = j + 1$.

Case 2. If $k = 2$, then
$p^k_{ij} = 1$ for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 2$, $j = i + 2$ and $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1$, $j = i + 1$;
$p^k_{ij} = 1$ for $1 \leq j \leq \left\lceil \frac{n}{2} \right\rceil - 2$, $i = j + 2$ and $1 \leq j \leq \left\lceil \frac{n}{2} \right\rceil - 1$, $i = j + 1$;
$p^k_{ij} = 2$ for $1 \leq i = j \leq \left\lceil \frac{n}{2} \right\rceil - 1$.

Case 3. If $3 \leq k \leq \left\lceil \frac{n}{2} \right\rceil - 2$, then
$p_{ij}^k = 1$ for $i+j=k$, $i+j=n-k$, $i+j=n-k+1$ and $i+j=n-k+2$;
$p_{ij}^k = 2$ for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$, $j=i+k$ and $1 \leq j \leq \left\lceil \frac{n}{2} \right\rceil$, $i=j+k$.

Case 4. If $k = \left\lceil \frac{n}{2} \right\rceil - 1$, then
$p_{ij}^k = 1$ for $i+j=k$, $i+j=k+1$, $i+j=k+2$ and $i+j=k+3$.

Case 5. If $k = n_{\lceil \frac{n}{2} \rceil}$, then
$p_{ij}^k = 1$ for $i=1$, $j = i + \left\lceil \frac{n}{2} \right\rceil - 1$ and $j=1$, $i = j + \left\lceil \frac{n}{2} \right\rceil - 1$;
$p_{ij}^k = 2$ for $1 \leq i \leq \left\lfloor \frac{k}{2} \right\rfloor$, $j=k-i$ and $1 \leq j \leq \left\lfloor \frac{k}{2} \right\rfloor$, $i=k-j$ and the anti-diagonal entries are dds of C_{n-1} and the remaining entries are zero.

□

Theorem 3.4. *The collection of all geodetic sets in* $C_n\left(1,2,\ldots,\frac{n-2}{2}\right)$ *for* $n \geq 4$ *even, form* $(v, g(G), \mu)$*-design with* 2*-association schemes and parameters* $v=n$, $b=\frac{n}{2}$, $g(G)=2$, $r=1$ *and* $\mu=1$. *The parameters of second kind are given by* $n_1 = 2(n-1)$ *and* $n_2=1$.

Proof. Let $G = C_n\left(1,2,\ldots,\frac{n-2}{2}\right)$ where n is even. Since G is a self-centered, unique eccentric vertex graph of diameter 2 and regularity $(n-2)$, parameters of first kind are as in the statement followed by Theorem3.1.
Since G is a distance degree regular (DDR) graph with distance degree sequence (dds) of each vertex being $(1, 2(n-1), 1)$, we get, the parameters of second kind as entries of the distance degree sequence itself. Hence the parameters of second kind are given by $n_1 = 2(n-1)$, $n_2 = 1$ with

$$P^1 = \begin{pmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{pmatrix} = \begin{pmatrix} n-4 & 1 \\ 1 & 0 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{pmatrix} = \begin{pmatrix} n-2 & 0 \\ 0 & 0 \end{pmatrix}.$$

□

Remark 4. *The collection of all geodetic sets in* $C_n\left(1,2,\ldots,\frac{n-2}{2}\right)$ *for odd* n, *does not form* $(v, g(G), \mu)$*-design as the repetition number* r *is not unique.*

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Theorem 3.5. *The collection of all geodetic sets in* $C_n\left(1,2,\ldots,\left\lceil\frac{n-2}{2}\right\rceil\right)$ *for odd* n*, forms a* $(v,g(G),\mu)$*-design.*

Proof. For odd n, $C_n\left(1,2,\ldots,\left\lceil\frac{n-2}{2}\right\rceil\right)\cong K_n$. Hence the result follows from Lemma 3.1. □

Remark 5. *The collection of all geodetic sets in* $C_{2n+1}(1,n)$*, where* $n\geq 3$ *and odd, does not form* $(v,g(G),\mu)$*-design, as the value of* μ *is not unique. But, for* $n=2$*,* $C_{2n+1}(1,2)\cong K_5$*, hence, we get a design followed by Lemma 3.1.*

Next we consider circulants having only odd jump sizes. These are particularly different from the above, as they are isomorphic to complete bipartite graphs. That is, $C_n\left(1,3,\ldots,\left\lfloor\frac{n}{2}\right\rfloor\right)$ = $C_{2n}(1,3,\ldots,n)\cong K_{n,n}, n\geq 6$. We give a result on geodetic number of a complete bipartite graph, that helps in getting the next result.

Theorem 3.6. [11] *For the complete bipartite graph* $G=K_{m,n}$*,* $g(K_{m,n})=min\{m,n,4\}$.

Theorem 3.7. *The collection of all geodetic sets in* $C_{2n}(1,3,\ldots,n)$ *for* $n\geq 3$ *forms a* $(v,g(G),\mu)$*-design with* 2*-association schemes and parameters are given as follows:*

- $v=6$, $b=2$, $g(K_{3,3})=3$, $r=1$, $\mu=1$, *when* $n=3$*;*
- $v=8$, $b=38$, $g(K_{4,4})=4$, $r=19$, $\mu=7$, *when* $n=4$*;*
- $v=2n$, $b=\frac{n^2(n-1)^2}{4}$, $g(K_{n,n})=4$, $r=\frac{n(n-1)^2}{2}$, $\mu=\binom{n}{2}$, *when* $n\geq 5$.

Proof. We have $C_{2n}(1,3,\ldots,n)\cong K_{n,n}$, for $n\geq 3$, the first parameter set follows from Theorem 3.2. Since complete bipartite graph $K_{n,n}$ is a distance degree regular (DDR) graph with distance degree sequence (dds) of each vertex being $(1,n,n-1)$, the parameters of the second kind are given by $P^1=\begin{pmatrix}p_{11}^1 & p_{12}^1\\ p_{21}^1 & p_{22}^1\end{pmatrix}=\begin{pmatrix}0 & n-1\\ n-1 & 0\end{pmatrix}$

$$P^2=\begin{pmatrix}p_{11}^2 & p_{12}^2\\ p_{21}^2 & p_{22}^2\end{pmatrix}=\begin{pmatrix}n & 0\\ 0 & n-2\end{pmatrix}.$$ □

4. Conclusion

In this paper we have determined geodetic designs for circulant graphs with different jump sizes. Also non-existence of these two types of designs are addressed.

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