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ECCENTRICITY BASED TOPOLOGICAL INDICES OF POLYGONAL CYLINDER

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Abstract: *A topological index has a vital role in molecular chemistry. There are various topological descriptors in theoretical chemistry in particular; degree- based topological indices, distance-based, eccentricity- based and counting related indices of graphs. In this paper, we have obtained analytical expressions for various variants of eccentricity based indices such as first and second multiplicative Zagreb eccentricity indices, total eccentric index, connective eccentric index, Ediz eccentric connectivity index, modified and augmented eccentric connectivity indices and their respective polynomials of polygonal cylinder.*

Keywords: topological indices, eccentricity based indices, polygonal cylinder

1. INTRODUCTION

In recent years graph theory is substantially used in the branch of mathematical chemistry due to the fact this idea is associated with the realistic purposes of graph theory for solving the molecular problems. Over the years topological indices like Wiener index, Balaban index, Hosoya index, Randic index and so on have been studied significantly improved and currently the research and attention in this area has been accelerated exponentially. Throughout this paper we will focus on finite, simple and connected graphs. Let $G=(V(G),E(G))$ be a graph with $V(G)$ is a set of all vertices and $E(G)$ is a set of all edges. The degree of v, denoted by $\deg(v)$ or $d(v)$, is the number of edges incident with v in G. The eccentricity $\varepsilon(v)$ of a vertex $v\in V(G)$ is the maximum distance from v to any other vertex. The goal of this paper, to determine various eccentricity based indices of the polygonal cylinder. Nilanjan De[8] in 2012 defined first and second multiplicative Zagreb eccentricity indices as:

$$\prod E_1(G)=\prod_{v\in V(G)}\varepsilon(v)^2 \text{ and } \prod E_2(G)=\prod_{(u,v)\in V(G)}\varepsilon(u)\varepsilon(v). \qquad (1)$$

The total eccentricity of a graph G is denoted by $\zeta(G)$ is the sum of eccentricities of all vertices of a given graph G. Sharma et.al[9] introduced eccentric connectivity index and is interpreted as:

$$\zeta(G)=\sum_{v\in V(G)} d_u\varepsilon_u. \qquad (2)$$

In 2000, Gupta et.al. [6] defined connective eccentric index as:

$$C^{\xi}(G)=\sum_{u\in V(G)}\frac{d_u}{\varepsilon_u}. \qquad (3)$$

A. R. Ashrafi and M. Ghorbani[2] in 2010 defined modified eccentric connectivity index as:

$$\xi_c(G)=\sum_{v\in V(G)}(S_v\varepsilon_v), \qquad (4)$$

where S_v is the sum of degrees of all vertices adjacent to vertex v.

Ediz eccentric connectivity index of G is defined by S. Ediz et al. [5] in 2010. It is interpreted as:

$$ {}^{E}\xi^{c}(G)=\sum_{v\in V(G)}\frac{S_v}{\varepsilon_v}. \tag{5}$$

Dureja, Madan[4] introduced augmented eccentric connectivity index of graph *G*. It is defined as:

$$ A_{\xi}(G)=\sum_{v\in V(G)}\frac{M_v}{\varepsilon_v}. \tag{6}$$

where M_v is the product of degrees of all neighbors of vertex v of G.
Modified augmented eccentric connectivity index is proposed by M. Naeem et.al. [7] in 2018 and it is defined as:

$$ {}^{MA}\xi^{c}(G)=\sum_{v\in V(G)}(M_v\varepsilon_v). \tag{7}$$

The corresponding topological polynomials of Eq. (4) and Eq.(7) are displayed below: N. De et.al. [3] in 2014 defined modified eccentric connectivity polynomial as:

$$ \xi_c(G,x)=\sum_{v\in V(G)}S_u x^{\varepsilon_u}. \tag{8}$$

The first derivative of Eq. (8) at $x=1$ is the modified eccentric connectivity index.
M. Naeem et.al[7] in 2018 defined modified augmented eccentric connectivity polynomial as:

$$ {}^{MA}\xi^{c}(G,x)=\sum_{v\in V(G)}M_v x^{\varepsilon_v}. \tag{9}$$

2. POLYGONAL CYLINDER

Abdul Rauf Nizami[1] in 2010 defined polygonal cylinder $C_{n,m}$. Consider two copies of paths $P_n, n\geq 3$ with vertices $u_1,u_2,...,u_n$ and $v_1,v_2,...,v_n$ respectively. The Cartesian product $P_n\times P_n$ is defined by identify the vertices $(u_1,v_1),(u_1,v_2),...,(u_1,v_n)$ with the vertices $(u_n,v_1),(u_n,v_2),...,(u_n,v_n)$ and the edge $(u_1,v_i),(u_1,v_{i+1})$ with the edge $(u_n,v_i),(u_n,v_{i+1})$, where $1\leq i\leq n$. The resultant graph thus obtained is polygonal cylinder or $(n-1)$-gonal cylinder. It is denoted by $C_{n,n}$. The graph $C_{n,n}$ consists of $n(n-1)$ vertices and $[(2n-1)(n-1)]$ edges. *Figure 1.* is the 3-gonal cylinder.

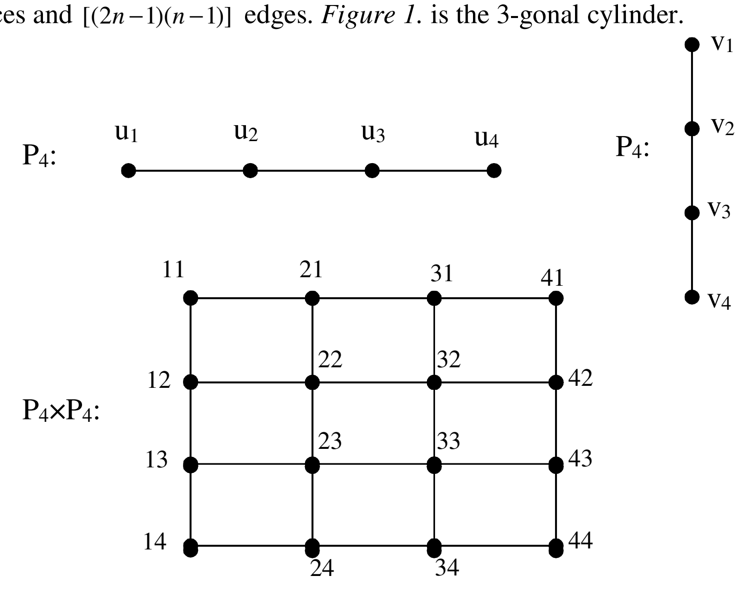

Figure 1. Grid $P_4\times P_4$

$C_{4,4,}$:

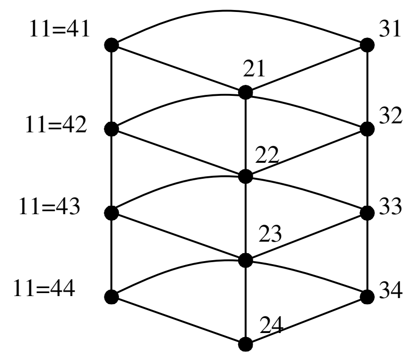

Figure. 2 Polygonal cylinder $C_{4,4}$

3. MAIN TITLE

We determine various eccentricity based indices and their respective polynomials of the polygonal cylinder.

2.1 First multiplicative Zagreb eccentricity index of the polygonal cylinder $C_{n,n}$

If n is even, $C_{n,n}$ consists of $n(n-1)$ vertices, in which $(2n-2)$ vertices; $\left(\frac{n}{2}\right)$ set of times has eccentricity $(n-1+k)$, for $k=0$ to $\left(\frac{n}{2}-1\right)$. Using Eq. (1) we have, $\prod E_1(G) = \prod_{v\in V(G)} \varepsilon(v)^2$

$$= \varepsilon(u_1)^2.\varepsilon(u_2)^2...\varepsilon(u_{n-1})^2.\varepsilon(v_1)^2....\varepsilon(v_n)^2$$

$$= (n-1)^2(n-1+1)^2(n-1+2)^2....\left(n-1+\left(\frac{n}{2}-1\right)\right)^2$$

$$= \prod_{k=0}^{\frac{n}{2}-1}\left((n-1+k)^2\right)^{2n-2}.$$

This result is true for all $n \geq 4$. If n is odd, $(n-1)$ vertices has eccentricity $(n-1)$; $(2n-2)$ vertices $\left\lfloor\frac{n}{2}\right\rfloor$ set of times has eccentricity $(n+k)$, for $k=0$ to $\left(\left\lfloor\frac{n}{2}\right\rfloor-1\right)$. Using Eq. (1) we have, $\prod E_1(G) = \prod_{v\in V(G)} \varepsilon(v)^2$

$$= \left((n-1)^2\right)^{n-1}(n)^2(n+1)^2(n+2)^2....\left(n+\left(\left\lfloor\frac{n}{2}\right\rfloor-1\right)\right)^2$$

$$= \left((n-1)^2\right)^{n-1}\prod_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\left((n+k)^2\right)^{2n-2}.$$

This result is true for all $n \geq 5$.

Hence $\prod E_1(G) = \begin{cases} \prod_{k=0}^{\frac{n}{2}-1} \left((n-1+k)^2\right)^{2n-2}; & if\ n\ is\ even \\ \left((n-1)^2\right)^{n-1} \prod_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor -1} \left((n+k)^2\right)^{2n-2}; & if\ n\ is\ odd. \end{cases}$

2.2 Second multiplicative Zagreb eccentricity index of the polygonal cylinder $C_{n,n}$

For even n, second multiplicative Zagreb eccentricity index can be computed as follows: (2n–2) vertices $\left\lfloor \frac{n-1}{2} \right\rfloor$ set of times has eccentricity $(n+k-1)$, for $k=1$ to $\left\lfloor \frac{n-1}{2} \right\rfloor$; $(3n-3)$ vertices has eccentricity $(n-1)$; (2n–2) vertices $\left\lfloor \frac{n-1}{2} \right\rfloor$ set of times has eccentricity of the form $\varepsilon(u)\varepsilon(v) = (n+k-1)(n+k-2)$, for $k=1$ to $\left\lfloor \frac{n-1}{2} \right\rfloor$. Using Eq. (1) we have,

$$\begin{aligned} \prod E_2(G) &= \prod_{(u,v)\in V(G)} \varepsilon(u)\varepsilon(v) \\ &= \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n+k-1)(n+k-1)\right)^{2n-2} \left((n-1)(n-1)\right)^{3n-3} \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n+k-1)(n+k-2)\right)^{2n-2} \\ &= \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n+k-1)^2\right)^{2n-2} \left((n-1)^2\right)^{3n-3} \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n+k-1)(n+k-2)\right)^{2n-2} \end{aligned}$$

This is true for all $n \geq 4$. Similarly the result follows for odd $n \geq 5$.

Hence,

$$\prod E_2(G) = \begin{cases} \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n-1+k)^2\right)^{2n-2} \left((n-1)^2\right)^{3n-3} \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n+k-1)(n+k-2)\right)^{2n-2}; & if\ n\ is\ even \\ \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n-1+k)^2\right)^{2n-2} \left((n-1)^2\right)^{n-1} \prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left((n+k-1)(n+k-2)\right)^{2n-2}; & if\ n\ is\ odd. \end{cases}$$

2.3 Total eccentricity index of polygonal cylinder $C_{n,n}$

In $C_{n,n}$, $(2n-2)$ vertices $\left(\frac{n}{2}\right)$ times has eccentricity $(n+1-k); k=0$ to $\left(\frac{n}{2}-1\right)$,for even n. Similarly for odd n, $(2n-2)$ vertices $\left\lfloor \frac{n}{2} \right\rfloor$ times has eccentricity $(n+k); k=0$ to $\left(\left\lfloor \frac{n}{2} \right\rfloor -1\right)$, and $(n-1)$ vertices has eccentricity $(n-1)$. By the definition of total eccentricity, we have

$$\zeta(G)=\sum_{v\in V(G)}\varepsilon_u=\begin{cases}(2n-2)\displaystyle\sum_{k=0}^{\frac{n}{2}-1}(n+k-1); & if\ n\ is\ even\\ (n-1)^2+(2n-2)\displaystyle\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}(n+k); & if\ n\ is\ odd.\end{cases}$$

Hence the result.

2.4 Eccentric connectivity index and connective eccentric index of polygonal cylinder $C_{n,n}$

Using Eq. (2) we have, $\xi(G)=\sum_{v\in V(G)}d_u\varepsilon_u$. In this graph, $(2n-2)$ vertices have degree 3 and eccentricity $\left(n+\frac{n}{2}-2\right)$; again $(2n-2)$ vertices $\left(\frac{n}{2}-1\right)$times has degree 4 and eccentricity $(n+k-1), k=0$ to $\left(\frac{n}{2}-2\right)$ for even n. Similarly, for odd n: $(2n-2)$ vertices have degree 3 and eccentricity $\left(n+\left\lfloor\frac{n}{2}\right\rfloor-1\right)$; again $(2n-2)$ vertices $\left(\left\lfloor\frac{n}{2}\right\rfloor-1\right)$times has degree 4 and eccentricity $(n+k), k=0$ to $\left(\left\lfloor\frac{n}{2}\right\rfloor-2\right)$; 4 vertices have degree $(n-1)$ and eccentricity $(n-1)$. Hence,

$$\xi(G)=\begin{cases}(2n-2)\left[3.\left(n+\frac{n}{2}-2\right)\right]+(2n-2)\displaystyle\sum_{k=0}^{\frac{n}{2}-2}[4.(n+k-1)]; & if\ n\ is\ even\\ (2n-2)\left[3.\left(n+\left\lfloor\frac{n}{2}\right\rfloor-1\right)\right]+4.(n-1)(n-1)+(2n-2)\displaystyle\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-2}[4.(n+k)]; & if\ n\ is\ odd.\end{cases}$$

Using the above index, we can compute connective eccentric index. Since by Eq. (3), we have $C^{\xi}(G)=\sum_{v\in V(G)}\frac{d_u}{\varepsilon_u}$

$$=\begin{cases}(2n-2)\dfrac{3}{\left(n+\frac{n}{2}-2\right)}+(2n-2)\displaystyle\sum_{k=0}^{\frac{n}{2}-2}\left[\frac{4}{(n+k-1)}\right]; & if\ n\ is\ even\\ (2n-2)\dfrac{3}{\left(n+\left\lfloor\frac{n}{2}\right\rfloor-1\right)}+4\left[\dfrac{(n-1)}{(n-1)}\right]+(2n-2)\displaystyle\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-2}\left[\frac{4}{(n+k)}\right]; & if\ n\ is\ odd.\end{cases}$$

Hence the result.

2.5 The modified eccentric connectivity index and Ediz eccentric connectivity index of polygonal cylinder $C_{n,n}$

From the definition of $\xi_c(G)$, S_v is the sum of degrees of all vertices adjacent to vertex v. Here $(2n-2)$ vertices have $S_v=3+3+4=10$ and eccentricity $\left(n+\left(\frac{n-5}{2}\right)+1\right)$;

$(2n-2)$ vertices have $S_v = 3+4+4+4+4 = 15$ and eccentricity $\left(\frac{n-5}{2}\right)$; $(n-1)$ vertices has $S_v = 4+4+4+4+4 = 16$ and eccentricity $(n-1)$, for odd n. Similarly for even n, $(2n-2)$ vertices have $S_v = 3+3+4 = 10$ and eccentricity $\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor\right)$; $(2n-2)$ vertices have $S_v = 3+4+4+4+4 = 15$ and eccentricity $\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor-1\right)$; $(2n-2)$ vertices $(n-5)$ times has $S_v = 4+4+4+4+4 = 16$ and eccentricity $\left(\frac{3n-8}{2}-k\right), k=0$ to $(n-6)$. Hence the result. The corresponding topological polynomial of Eq. (4) is Eq. (8). We have $\xi_c(G,x) = \sum_{v\in V(G)} S_v x^{\varepsilon_v}$.

Using the above datas we get,

$$\xi^c(G,x) = \begin{cases} (2n-2)\left(10x^{\left(n+\left(\frac{n-5}{2}\right)+1\right)}\right) + (2n-2)\left(15x^{\left(n+\left(\frac{n-5}{2}\right)\right)}\right) + (n-1)\left(16x^{(n-1)}\right); & \text{if } n \text{ is odd} \\ (2n-2)\left(10x^{\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor\right)}\right) + (2n-2)\left(15x^{\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor-1\right)}\right) + (2n-2)\sum_{k=0}^{n-6}\left(16x^{\left(\frac{3n-8}{2}-k\right)}\right); & \text{if } n \text{ is even.} \end{cases}$$

We can compute Ediz eccentric connectivity index ${}^E\xi^c(G)$ by using modified eccentric connectivity index of $C_{n,n}$. We have by Eq (5), ${}^E\xi^c(G) = \sum_{v\in V(G)} \frac{S_v}{\varepsilon_v}$

$$= \begin{cases} (2n-2)\left(\frac{10}{\left(n+\left(\frac{n-5}{2}\right)+1\right)}\right) + (2n-2)\left(\frac{15}{\left(n+\left(\frac{n-5}{2}\right)\right)}\right) + (n-1)\left(\frac{16}{(n-1)}\right); & \text{if } n \text{ is odd} \\ (2n-2)\left(\frac{10}{\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor\right)}\right) + (2n-2)\left(\frac{15}{\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor-1\right)}\right) + (2n-2)\sum_{k=0}^{n-6}\left(\frac{16}{\frac{3n-8}{2}-k}\right); & \text{if } n \text{ is even.} \end{cases}$$

Hence the Proof.

2.6 Augmented eccentric connectivity index and Modified Augmented eccentric connectivity index and its polynomial of $C_{n,n}$

By Eq. (6), we have $A_\xi(G) = \sum_{v\in V(G)} \frac{M_v}{\varepsilon_v}$. For odd n, $(2n-2)$ vertices has $M_v = 3.3.4 = 36$ and eccentricity $\left(n+\left(\frac{n-5}{2}\right)+1\right)$; $(2n-2)$ vertices has $M_v = 3.4.4.4 = 192$ and eccentricity $\left(n+\left(\frac{n-5}{2}\right)\right)$; $(2n-2)$ vertices $\left(\frac{n-5}{2}\right)$ times has $M_v = 4.4.4.4 = 256$ and eccentricity $(n+k); k=0$ to $\left(\frac{n-7}{2}\right)$; $(n-1)$ vertices has $M_v = 4.4.4.4 = 256$ and eccentricity

$(n-1)$. Similarly for even n, $(2n-2)$ vertices has $M_v = 3.3.4 = 36$ and eccentricity $\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor\right)$; $(2n-2)$ vertices has $M_v = 3.4.4.4 = 192$ and eccentricity $\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor-1\right)$; $(2n-2)$ vertices $(n-5)$ times has $M_v = 4.4.4.4 = 256$ and eccentricity $\left(\frac{3n-8}{2}-k\right)$; $k=0$ to $(n-6)$. Hence,

$$A_\xi(G)=\begin{cases}(2n-2)\left(\dfrac{36}{n+\left(\frac{n-5}{2}\right)+1}\right)+(2n-2)\left(\dfrac{192}{n+\left(\frac{n-5}{2}\right)}\right)+(2n-2)\displaystyle\sum_{k=0}^{\frac{n-7}{2}}\left(\frac{256}{n+k}\right)+(n-1)\left(\frac{256}{n-1}\right); & if\ n\ is\ odd\\ (2n-2)\left(\dfrac{36}{n+\left\lfloor\frac{n-3}{2}\right\rfloor}\right)+(2n-2)\left(\dfrac{192}{n+\left\lfloor\frac{n-3}{2}\right\rfloor-1}\right)+(2n-2)\displaystyle\sum_{k=0}^{n-6}\left(\frac{256}{\left(\frac{3n-8}{2}-k\right)}\right); & if\ n\ is\ even\end{cases}$$

Using the above result, we can compute modified augmented eccentric connectivity index. By Eq. (7), we have ${}^{MA}\xi^c(G)=\sum_{v\in V(G)} M_v\varepsilon_v$

$$=\begin{cases}(2n-2)\left(36\left(n+\left(\frac{n-5}{2}\right)+1\right)\right)+(2n-2)\left(192\left(n+\left(\frac{n-5}{2}\right)\right)\right)+(2n-2)\displaystyle\sum_{k=0}^{\frac{n-7}{2}}\left(256(n+k)\right)+(n-1)\left(256(n-1)\right); & if\ n\ is\ odd\\ (2n-2)\left(36\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor\right)\right)+(2n-2)\left(192\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor-1\right)\right)+(2n-2)\displaystyle\sum_{k=0}^{n-6}\left(256\left(\frac{3n-8}{2}-k\right)\right); & if\ n\ is\ even\end{cases}$$

Also, we can compute modified augmented eccentric connectivity polynomial ${}^{MA}\xi^c(G,x)$ using its index ${}^{MA}\xi^c(G)$ .Therefore, ${}^{MA}\xi^c(G,x)=\sum_{v\in V(G)} M_v x^{\varepsilon_v}$

$$=\begin{cases}(2n-2)\left(36x^{\left(n+\left(\frac{n-5}{2}\right)+1\right)}\right)+(2n-2)\left(192x^{\left(n+\left(\frac{n-5}{2}\right)\right)}\right)+(2n-2)\displaystyle\sum_{k=0}^{\frac{n-7}{2}}\left(256x^{(n+k)}\right)+(n-1)\left(256x^{(n-1)}\right); & if\ n\ is\ odd\\ (2n-2)\left(36x^{\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor\right)}\right)+(2n-2)\left(192x^{\left(n+\left\lfloor\frac{n-3}{2}\right\rfloor-1\right)}\right)+(2n-2)\displaystyle\sum_{k=0}^{n-6}\left(256x^{\left(\frac{3n-8}{2}-k\right)}\right); & if\ n\ is\ even\end{cases}$$

Hence the result.

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