Totally β* - Continuous Functions in Topological Spaces

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Abstract

The aim of this paper is to define a new class of functions namely totally β*- continuous functions and slightly β*- continuous functions and study their properties . Additionally, we relate and compare these functions with some other functions in topological spaces.

Keywords and phrases: Totally β*- continuous and Slightly β*- continuous.

I. Introduction

Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years. Abd El- Monsef et al. introduced the notion of β - open sets and β -continuity in topological spaces. RC Jain introduced the concept of totally continuous functions and slightly continuous for topological spaces. In this paper, we define totally β*- continuous functions and slightly β*- continuous functions and basic properties of these functions are investigated and obtained.

II. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X). If A is β*-open and β*- closed , then it is said to be β*- clopen.

Definition 2.1: A subset A of a topological space X is said to be a β*-open [5] if A $\subseteq$ cl (int* (cl(A))).

Definition 2.2: A function f : (X, τ) ⟶(Y, σ) is called totally continuous [2] if f^{-1} (V) is clopen set in X for each open set V of Y .

Definition 2.3: A function f: (X, τ) ⟶ (Y, σ) is called a β*- continuous [8]if f^{-1} (O) is a β*-open set of (X,τ) for every open set O of (Y, σ).

Definition 2.4: A map f*:* (X, τ) ⟶ (Y, σ) is said to be perfectly β* - continuous [6]if the inverse image of every β***-**open set in (Y, σ) is both open and closed in (X, τ) .

Definition 2.5: A function f: (X, τ) ⟶ (Y, σ) is called a slightly continuous[2] if the inverse image of every clopen set in Y is open in X.

Definition 2.6: A function f: (X, τ) ⟶ (Y, σ) is called a contra continuous [1] if f^{-1} (O) is closed in (X, τ) for every open set O in (Y, σ).

Definition 2.7: A function f: (X, τ) ⟶ (Y, σ) is called Contra β*- continuous functions [7] if f^{-1} (O) is β***-** closed in (X, τ) for every open set O in (Y, σ).

Definition 2.8: A topological space X is called a β*- connected [9] if X cannot be expressed as a disjoint union of two non-empty β***-**open sets.

Definition 2.9: A map f : (X, τ) ⟶(Y, σ) is said to be pre β*-open [7] if the image of every β*- open set of X is β*-open in Y.

Definition 2.10: A topological space X is said to be connected [10] if X cannot be expressed as the union of two disjoint nonempty open sets in X.

Definition 2.11: A function f: (X, τ) ⟶ (Y, σ) is called a strongly β*- continuous [6] if the inverse image of every β* - open set in (Y, σ) is open in (X, τ).

Definition 2.12: A Topological space X is said to be β* -$T_{1/2}$ space or β*- space [8] if every β*- open set of X is open in X.

Definition 2.13: A space (X, τ) is called a locally indiscrete space [3] if every open set of X is closed in X.

Theorem 2.14[5]**:**

(i) Every open set is β*- open and every closed set is β*-closed set.

III. Totally β*- continuous functions

Definition 3.1: A function (X, τ) ⟶(Y, σ) is called totally β*- continuous functions if the inverse image of every open set of (Y, σ) is both β*- open and β*- closed subset of (X, τ).

Example 3.2: Let X = Y ={a, b, c}, τ = {ϕ,{a}, X}, σ = {ϕ, {a},{a, b},{a, c},Y} , β*O(X, τ) = {ϕ {a},{b},{c},{a, b},{a, c},{b, c}, X} and β*C(X,τ) = {ϕ , {a}, {b} , {c} , {a, b} ,{a, c} ,{b, c},X}. Let f*:* (X, τ) ⟶(Y, σ) be defined by f (a) = c , f(b) = a, f(c) = b . since , f^{-1} ({a}) = {b} , f^{-1} ({a, b}) = {b, c} and f^{-1} ({a, c}) = {a, b} is both β*- open and β*- closed in X. Therefore, f is totally β*- continuous.

Theorem 3.2: Every totally β*- continuous functions is β*- continuous.

Proof: Let O be an open set of (Y, σ). Since, f is totally β*- continuous functions, f^{-1}(O) is both β*- open and β*- closed in (X, τ). Therefore, f is β*- continuous.

Remark 3.3: The converse of above theorem need not be true.

Example 3.4: Let X = Y = {a, b, c}, τ = {ϕ, {a},{a, b},{a, c}, X}, σ = {ϕ, {a, b},Y}. Let f:(X, τ) ⟶(Y, σ) be defined by f (a) = a , f(b) = b, f(c) = c. β*O(X, τ) = { ϕ,{a}, {a, b},{a, c},X} and β* C(X, τ) = {ϕ,{b},{c},{b, c},X} . Clearly, f is not totally β*-continuous since f^{-1} ({a, b}) = {a, b} is β*-open in X but not β*- closed. However, f is β*- continuous.

Theorem3.5: Every totally continuous function is totally β*- continuous.

Proof: Let O be an open set of (Y, σ). Since, f is totally continuous functions, f^{-1} (O) is both open and closed in (X, τ). Since every open set is β*- open and every closed set is β*- closed. f^{-1} (O) is both β*- open and β*- closed in (X, τ). Therefore, f is totally β*- continuous.

Remark 3.6: The converse of above theorem need not be true.

Example 3.7: Let X = Y = {a, b, c}, τ = {ϕ, {a}, X}, $τ^c$ = {ϕ, {b, c}, X} , σ = {ϕ,{a},{a, b},{a, c},Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f (a) = a , f(b) = b, f(c) = c . β*O(X, τ) = { ϕ,{a}, {b},{c}, {a, b},{a, c}, {b, c},X}, β* C(X, τ) = { ϕ,{a}, {b},{c}, {a, b},{a, c}, {b, c},X} . Clearly, f is totally β*-continuous but f^{-1} ({a, b}) = {a, b}, f^{-1} ({a, c}) = {a, c} is not open and closed in X. Therefore, f is not totally continuous.

Theorem 3.8: Every perfectly β*- continuous map is totally β*- continuous.

Proof: Let f : (X, τ) ⟶(Y, σ) be a perfectly β*- continuous map. Let O be an open set of (Y, σ). Then O is β*- open in (Y, σ). Since f is perfectly β*- continuous, f^{-1} (O) is both open and closed in (X, τ), implies f^{-1} (O) is both β*- open and β*- closed in (X, τ). Therefore, f is totally β*- continuous.

Remark 3.9: The converse of above theorem need not be true.

Example3.10: Let X = Y = {a, b, c, d}, τ = {ϕ, {a, b}, {a, b, c},X}, $τ^c$ = {ϕ, {c, d}, {d},X} , σ = {ϕ, {a},{b, c, d},Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f (a) = a , f(b) = b, f(c) = c , f(d) = d. β*O(X, τ) = { ϕ, {a},{b},{c},{a, b}, {a, c},{a, d},{b, c}, {b, d},{c, d}, {a, b, c}, {a, b, d},{a, c, d},{b, c, d},X}, β* C(X, τ) = ϕ, {a},{b},{c}, {d},{a, b}, {a, c},{a, d},{b, c}, {b, d},{c, d}, {a, b, d},{a, c, d},{b, c, d},X}. β* O(Y, σ) = {ϕ,{a},{b},{c}, {d},{a, b}, {a, c},{a, d},{b, c}, {b, d},{c, d}, {a, b, c},{a, b, d}, {a, c, d},{b, c, d},Y}.Clearly, f is totally β*-continuous but f^{-1} ({a}) = {a} , f^{-1} ({b}) = {b} , f^{-1} ({c}) = {c} , f^{-1} ({d}) = {d} , f^{-1} ({a, c}) = {a, c} , f^{-1} ({a, d}) = {a, d},f^{-1} ({b, c}) = {b, c} , f^{-1} ({b, d}) = {b, d} , f^{-1} ({c, d}) = {c, d} , f^{-1} ({a, b, d}) = {a, b, d} , f^{-1} ({a, c, d}) = {a, d},f^{-1} ({b, c, d}) = {b, c, d} is not open and closed in X. Therefore, f is not perfectly β*-continuous.

Remark 3.11: The concept of totally β*- continuous and strongly β*- continuous are independent of each other.

Example 3.12: Let X = Y = {a, b, c, d}, τ = {ϕ, {a, b}, X}, $τ^c$ = {ϕ, {c, d}, X} , β*O(X, τ) = {ϕ,{a},{b}, {c},{d},{a, b},{a, c}, {a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}, X}, β*C(X, τ) = {ϕ,{a},{b}, {c},{d},{a, b},{a, c}, {a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}, X}, σ = {ϕ, {a},{abc},Y}. β*O(Y, σ) = {ϕ,{a},{b}, {c},{d},{a, b},{a, c}, {a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}, Y} Let f: (X, τ) ⟶(Y, σ) be defined by f (a) = a , f(b) = b, f(c) = c , f(d) = d. Clearly, f is totally β*continuous but f^{-1} ({b}) = {b}, f^{-1} ({c}) = {c},f^{-1} ({d}) = {d}, f^{-1} ({a, c}) = {a, c},f^{-1} ({a, d}) = {a, d}, f^{-1} ({b, c}) = { b, c},f^{-1} ({b, d}) = {b, d}, f^{-1} ({c, d}) = {c, d} , f^{-1} ({a, b, c}) = {a, b, c}, f^{-1} ({a, b, d}) = {a, b, d} , f^{-1} ({a, c, d}) = {a, c, d},f^{-1} ({b, c, d}) = {b, c, d} is not open in X .Therefore, f is not strongly β*- continuous.

Example 3.13: Let X = Y = {a, b, c}, τ = {ϕ, {c},{a, c},{b, c}, X}, $τ^c$ = {ϕ, {a},{b},{a, b}, X}, σ = {ϕ, {a},{a, b},{a, c },Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f (a) = c, f(b) = b, f(c) = a . β* O(X, τ) = {ϕ, {c},{a, c},{b, c}, X}, β* C(X, τ) = {ϕ,{a},{b},{a, b}, X} and β* O(Y, σ) = {ϕ, {a},{a, b},{a, c },Y} . Clearly, f is strongly β*-continuous but f^{-1} ({a}) = {c}, f^{-1} ({a, b}) = {b, c},f^{-1} ({a, c}) = {a, c} is β*- open in X but not β*- closed .Therefore, f is not totally β*-continuous.

Theorem 3.14: If f: X × Y is a totally β*- continuous map, and X is β*- connected, then Y is an indiscrete space.

Proof: Suppose that Y is not an indiscrete space. Let A be a non-empty open subset of Y. Since, f is totally β*- continuous map, then f^{-1} (A) is a non-empty β*- clopen subset of X. Then X = f^{-1} (A) ∪ (f^{-1} (A))^c . Thus,X is a union of two non-empty disjoint β*- open sets which is contradiction to the fact that X is β*-connected. Therefore, Y must be an indiscrete space

Theorem 3.15: Let f: X ⟶ Y and g: Y ⟶ Z be functions. Then g ∘ f : X ⟶ Z

(i) If f is β*- irresolute and g is totally β*- continuous then g ∘ f is totally β*- continuous

(ii) If f is totally β*- continuous and g is continuous then g ∘ f is totally β*- continuous.

Proof:

(i) Let O be an open set in Z. Since g is totally β*- continuous, g^{-1} (O) is β*- clopen in Y. Since f is β* irresolute, f^{-1} (g^{-1} (O)) is β*- open and β*- closed in X. Since, (g ∘ f) -1 (O) = f^{-1} (g^{-1} (O)). Therefore, g ∘ f is totally β*- continuous.

(ii) Let O be an open set in Z. Since g is continuous, g^{-1} (O) is open in Y. Since, f is totally β*- continuous, f^{-1} (g^{-1} (O)) is β*- clopen in X. Hence, g ∘ f is totally β*- continuous.

IV. Slightly β*- continuous functions.

Definition 4.1: A function (X, τ) ⟶(Y, σ) is called slightly β*-continuous at a point x ∈ X if for each clopen subset V of Y containing f(x), there exists a β*- open subset U in X containing x such that f(U) ⊆ V. The function f is said to be slightly β*- continuous if f is slightly β*- continuous at each of its points.

Definition 4.2: A function (X, τ) ⟶(Y, σ) is said to be slightly β*- continuous if the inverse image of every clopen set in Y is β*- open in X.

Example 4.3: Let X = Y = {a, b, c}, τ = {ϕ, {a}, X} , σ = {ϕ, {b},{c},{a, b},{b, c},Y}, $σ^c$ = { ϕ, {a},{c},{a, b},{a, c},Y}, and β* O(X, τ) = {ϕ,{a},{b},{c},{a, b},{a, c }, {b, c}, X}, β* O(Y, σ) =

{ϕ,{a},{b},{c},{a, b},{a, c },Y}.Let f: (X, τ) ⟶(Y, σ) be defined by f (a) = a , f(b) = b, f(c) = c , Clearly, f is slightly β*- continuous .

Proposition 4.4: The definition 4.1 and 4.2 are equivalent.

Proof: Suppose the definition 4.1 holds. Let O be a clopen set in Y and $x \in f^{-1}(O)$. Then $f(x) \in O$ and thus there exists a β*- open set U_x such that $x \in U_x \subseteq f^{-1}(O)$ and $f^{-1}(O) = \cup U_x$. Since, arbitrary union of β*- open set is β*- open. $f^{-1}(O)$ is β*- open in X and therefore, f is slightly β*-continuous Suppose, the definition 4.2 holds. Let $f(x) \in O$ where, O is a clopen set in Y. Since, f is slightly β*- continuous, $x \in f^{-1}(O)$ where $f^{-1}(O)$ is β*- open in X. Let $U = f^{-1}(O)$. Then U is β*- open in X, $x \in X$ and $f(U) \subseteq O$.

Theorem 4.5: For a function f: (X, τ) ⟶(Y, σ) , the following statements are equivalent.

(i) f is slightly β*- continuous.

(ii) The inverse image of every clopen set O of Y is β*- open in X.

(iii) The inverse image of every clopen set O of Y is β*- closed in X.

(iv) The inverse image of every clopen set O of Y is β*- clopen in X.

Proof:

(i) ⇒ (ii): Follows from the proposition 4.4

(ii) ⇒ (iii): Let O be a clopen set in Y which implies O^c is clopen in Y. By (ii), $f^{-1}(O^c) = (f^{-1}(O))^c$ is β*- open in X. Therefore, $f^{-1}(O)$ is β*- closed in X.

(iii) ⇒ (iv): By (ii) and (iii), $f^{-1}(O)$ is β*-clopen in X.

(iv) ⇒ (i): Let O be a clopen set in Y containing f(x), by (iv) $f^{-1}(O)$ is β*- clopen in X. Take $U = f^{-1}(O)$, then $f(U) \subseteq O$. Hence, f is slightly β*-continuous.

Theorem 4.6: Every slightly continuous function is slightly β*- continuous.

Proof: Let f: (X, τ) ⟶(Y, σ) be a slightly continuous function. Let O be a clopen set in Y. Then, $f^{-1}(O)$ is open in X. Since, every open set is β*- open. Hence, f is slightly β*- continuous .

Remark 4.7: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.8: Let X = Y = {a, b, c, d}, τ = {ϕ,{a}, {a, b, c}, X} , τ^c = {ϕ, {d}, {b, c, d}, X} σ = {ϕ,{a},{b, c, d},Y}, σ^c { {ϕ,{a},{b, c, d},Y} and β* O(X, τ) = {ϕ,{a}, {b},{c},{d},{a, b},{a, c},{a, d},{b, c}, {b, d},{c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}, X}.Let f: (X, τ) ⟶(Y, σ) be defined by f(a)= a , f(b)= b, f(c)= c, f(d)= d. Clearly, f is slightly β*-continuous but not slightly continuous. Since, $f^{-1\{}$(b, c, d)}= {b, c, d} is not open in X.

Theorem 4.9: Every β*- continuous function is slightly β*- continuous.

Proof: Let f: (X, τ) ⟶(Y, σ) be a β*- continuous function. Let O be a clopen set in Y. Then, $f^{-1}(O)$ is β*- open in X and β*- closed in X. Hence, f is slightly β*- continuous.

Remark 4.10: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.11: Let X = { a, b, c} ,τ = {ϕ, {a},{b},{a, b}, X} , $τ^c$ = {ϕ, {c},{a, c},{b, c}, X}, σ = {ϕ, {b},{c}, {a, b},{b, c},Y}, $σ^c$ = {ϕ, {a},{c}, {a, b}, {a, c},Y} and β* O(X, τ) ={ϕ, {a},{b},{a, b},{a, c}, {b, c},X}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) =c , f(b) = b, f(c) =a , The function f is slightly β*-continuous but not β*-continuous, since, f^{-1} { b} = { c} is not β*- open in X.

Theorem 4.12: Every contra β*- continuous function is slightly β*-continuous.

Proof: Let f: (X, τ) ⟶(Y, σ) be a contra β*- continuous function. Let O be a clopen set in Y. Then, f^{-1} (O) is β*- open in X. Hence, f is slightly β*- continuous.

Remark 4.13: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.14: Let X = Y ={ a, b, c, d}, τ = {ϕ, {a},{b},{a, b}, {a, b, c}, X} , σ = {ϕ, {a}, {b},{c},{a, b},{a, c}, {b, c},{a, b, c},{a, b, d},Y}and $σ^c$ = {ϕ,{c}, {d},{a, d},{b, d},{c, d},{a, b, d},{a, c. d},{b, c, d},Y} and β* O(X, τ) = {ϕ,{ a}, {b},{c},{a, b},{a, c},{a, d}, {b, c}, {b, d}, {c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d},X}, β* C(X, τ) = {ϕ, {a}, {b},{c}, {d},{a, b},{a, c},{a, d}, {b, c}, {b, d}, {c, d},{a, b, d},{a, c, d},{b, c, d},X}Let f: (X, τ) ⟶(Y, σ) be defined by f(a)=a , f(b)= b ,f(c)=c, f(d) = d. The function f is slightly β*-continuous but not contra β*-continuous, since, f^{-1} {(a, b, c)}= { a, b, c} is not β*- closed in X.

Remark 4.15: Composition of two slightly β*-continuous need not be slightly β*-continuous as it can be seen from the following example.

Example 4.16: Let X=Y= Z ={a, b, c, d} ,and the topologies are τ = { ϕ ,{a} , {b},{c},{a, b},{a, c},{b, c},{a, d},{a, b, c},{a, b, d},{a, c, d} ,X} and σ = { ϕ , {a} , {b},{c},{a, b},{a, c},{b, c},{a, b, c},{a, b, d},Y }, $σ^c$ = { ϕ , {c} , {d}, {a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}, Y} and η = {ϕ ,{a},{b, c, d},Z}, η^c = {ϕ ,{a},{b, c, d}. β* O(X, τ) = {ϕ, {a}, {b},{c},{a, b},{a, c},{a, d}, {b, c}, {a, b, c},{a, b, d},{a, c, d}, X}, β* O(Y, σ) = {ϕ,{ a}, {b},{c},{a, b},{a, c},{a, d}, {b, c}, {b, d}, {a, b, c},{a, b, d},{a, c, d},{b, c, d}, Z}, Define f: (X, τ) ⟶(Y, σ) by f(a) = a ,f(b) = b, f(c) = c, f(d)=d. Clearly, f is slightly β*-continuous. Define g: (Y, σ) ⟶ (Z , η) by g(a) = a, g(b) = b , g (c) = c, g(d) = d. Clearly, g is slightly β*-continuous. But (g ∘ f): (X, τ) ⟶ (Z , η) is not slightly β*-continuous, since $(g \circ f)^{-1}$ ({b, c, d}) = f^{-1} (g^{-1}{b, c, d}) = f^{-1} ({ b, c, d}) = {b, c, d} is not a β*-open in (X, τ).

Theorem 4.17: Let f: X ⟶ Y and g: Y ⟶ Z be functions. Then the following properties hold:

(i) If f is β*- irresolute and g is slightly β*-continuous then (g ∘ f) is slightly β*-continuous.

(ii) If f is β*- irresolute and g is β*-continuous then (g ∘ f) is slightly β*-continuous.

(iii) If f is β*- irresolute and g is slightly continuous then (g ∘ f) is slightly β*-continuous.

(iv) If f is β*-continuous and g is slightly continuous then (g ∘ f) is slightly β*-continuous.

(v) If f is strongly β*-continuous and g is slightly β*- continuous then (g ∘ f) is slightly continuous.

(vi) If f is slightly β*-continuous and g is perfectly β*- continuous then (g ∘ f) is β*- irresolute.

(vii) If f is slightly β*-continuous and g is contra continuous then (g ∘ f) is slightly β*-continuous.

(viii) If f is β*- irresolute and g is contra β*-continuous then (g ∘ f) is slightly β*-continuous.

Proof:

(i) Let O be a clopen set in Z. Since, g is slightly β*-continuous, g^{-1} (O) is β*- open in Y. Since, f is β*-β*- irresolute, f^{-1} (g^{-1} (O)) is β*- open in X. Since, $(g \circ f)^{-1}$ (O) = f^{-1} (g^{-1} (O)), g ∘ f is slightly β*- continuous .

(ii) Let O be a clopen set in Z. Since, g is β*-continuous, g^{-1} (O) is β*-open in Y. Since, f is β*-irresolute, f^{-1} (g^{-1} (O)) is β*-open in X. Hence, g ∘ f is slightly β*- continuous.

(iii) Let O be a clopen set in Z. Since, g is slightly continuous, g^{-1}(O) is open in Y . Since, f is β*-irresolute, f^{-1} (g^{-1} (O)) is β*-open in X. Hence, g ∘ f is slightly β*- continuous.

(iv) Let O be a clopen set in Z. Since, g is slightly continuous, g^{-1} (O) is open in Y . Since, f is β*-continuous, f^{-1} (g^{-1} (O)) is β*- open in X. Hence, g ∘ f is slightly β*- continuous.

(v) Let O be a clopen set in Z. Since, g is slightly β*-continuous, g^{-1} (O) is β*-open in Y. Since, f is strongly β*- continuous, f^{-1} (g^{-1} (O)) is open in X. Therefore, g ∘ f is slightly continuous.

(vi) Let O be a β*-open in Z. Since, g is perfectly β*-continuous, g^{-1} (O) is open and closed in Y. Since, f is slightly β*-continuous, f^{-1} (g^{-1} (O)) is β*- open in X. Hence, g ∘ f is β*- irresolute.

(vii) Let O be a clopen set in Z. Since, g is contra continuous, g^{-1} (O) is open and closed in Y. Since, f is slightly β*- continuous, f^{-1} (g^{-1} (O)) is β*- open in X. Hence, g ∘ f is slightly β*- continuous.

(viii) Let O be a clopen set in Z. Since, g is contra β*- continuous, g^{-1} (O) is β*- open and β*- closed in Y.Since, f is β*- irresolute, f^{-1} (g^{-1} (O)) is β*- open and β*- closed in X. Hence, g ∘ f is slightly β*-continuous.

Theorem 4.18: If the function f: (X, τ) ⟶(Y, σ) is slightly β*-continuous and (X, τ) is β*- T1/2 space, then f is slightly continuous.

Proof: Let O be a clopen set in Y. Since, g is slightly β*-continuous, f^{-1} (O) is β*-open in X. Since, X is β*-T1/2 space, f^{-1} (O) is open in X. Hence, f is slightly continuous.

Theorem 4.19: Let f: (X, τ) ⟶(Y, σ) and g: (Y, σ) ⟶ (Z , η) be functions. If f is surjective and pre β*-open and (g ∘ f): (X, τ) ⟶ (Z , η) is slightly β*-continuous, then g is slightly β*- continuous.

Proof: Let O be a clopen set in (Z , η). Since, (g ∘ f): (X, τ) ⟶ (Z , η) is slightly β*- continuous, f^{-1} (g^{-1}(O)) is β*- open in X. Since, f is surjective and pre β*- open f(f^{-1} (g^{-1} (O))) = g^{-1} (O) is β*- open in Y. Hence, g is slightly β*- continuous.

Theorem 4.20: Let f: (X, τ) ⟶(Y, σ) and g: (Y, σ) ⟶ (Z , η) be functions. If f is surjective , pre β*-open and β*- irresolute, then (g ∘ f): (X, τ) ⟶ (Z , η) is slightly β*- continuous if and only if g is slightly β*- continuous.

Proof: Let O be a clopen set in (Z , η). Since, (g ∘ f): (X, τ) ⟶ (Z , η) is slightly β*- continuous, f^{-1} (g^{-1}(O)) is β*- open in X. Since, f is surjective and pre β*-open f(f^{-1} (g^{-1} (O))) = g^{-1} (O) is β*- open in Y. Hence, g is slightly β*- continuous.

Conversely, let g is slightly β*- continuous. Let O be a clopen set in (Z , η), then g^{-1} (O) is β*- open in Y. Since, f is β*- irresolute, f^{-1} (g^{-1} (O)) is β*- open in X. Hence, (g ∘ f): (X, τ) ⟶ (Z , η) is slightly β*-continuous.

Theorem 4.21: If f: (X, τ) ⟶(Y, σ) is a slightly β*- continuous and (Y, σ) is a locally indiscrete space then f is β*- continuous.

Proof: Let O be an open subset of Y. Since, (Y, σ) is a locally indiscrete space, O is closed in Y. Since, f is slightly β*- continuous, f^{-1} (O) is β*- open in X. Hence, f is β*- continuous.

Theorem 4.22: If f: (X, τ) ⟶(Y, σ) is a slightly β*-continuous and A is an open subset of X then the restriction f |A : (A, τ_A) ⟶(Y, σ) is slightly β*-continuous.

Proof: Let V be a clopen subset of Y. Then $(f|A)^{-1}$ (V) = f^{-1} (V) ∩ A. Since f^{-1} (V) is β*-open and A is open, $(f|A)^{-1}$ (V) is β*- open in the relative topology of A. Hence, f |A : (A, τ_A) ⟶(Y, σ) is slightly β*-continuous.

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