New class of closed sets in Nano topological spaces

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Abstract

In the present work a novel type of closed sets were established in Nano topological spaces and investigated their characteristics. The relationship between these sets with other types of closed sets was also discussed.

Key Words

Nano closed set, Nano closure, Nano open set, Nano interior, Nano generalized closed set, Nano generalized star closed set.

1. Introduction

The term 'generalized closed set' was first used by N.Levine [4] in 1970.This idea has given good initiation to many authors to introduce new concepts in topological spaces. Nano topological spaces were introduced by Lellis Thivagar [5]. Nano open sets are the elements of a Nano topological space. In a similar way Nano interior, Nano closed set and Nano closure were also introduced. Nano generalized closed set, Nano α generalized closed set, Nano gα closed set, Nano gr closed set and Nano rg closed set in Nano topological spaces were introduce by Bhuvaneswari [1,2,3] in their research work. V.Rajendran, P.Santhish Mohan & K.Indirani [7, 8] introduced and studied Nano generalized star closed sets in Nano topological spaces. With these concepts as the inspiration, new type of closed sets namely Nano g**-closed sets were introduced in Nano topological spaces. Their characteristics and relationship with other types of closed sets were studied in this paper.

2. Preliminaries

Definition 2.1 [4]

Let A be a subset of a topological space (X,τ).Then A is called a generalized closed set (briefly g-closed) if $cl(A)\subseteq V$ whenever $A\subseteq V$ and V is open in (X,τ). g-open set is the complement of a g-closed set.

Definition 2.2 [6]

Let A be a subset of a topological space (X,τ).Then A is called a generalized star closed set (briefly g*-closed) if $cl(A)\subseteq V$ whenever $A\subseteq V$ and V is g-open in (X,τ). g*-open set is the complement of a g*-closed set.

Definition 2.3 [5]

Let U be a non-empty finite set called the universe and R be an equivalence relation on U. It is called an indiscernibility relation. This relation partitions U into disjoint equivalence classes. Elements in the same equivalence class are said to be indiscernible with one another. We call (U,R) as the approximation space.

Let X be any subset of U. Then,

(i) The set $L_R(X)=\bigcup_{x\in U}\{R(x):R(x)\subseteq X\}$ is called the lower approximation of X with respect to R.

(ii) The set $U_R(X)=\bigcup_{x\in U}\{R(x):R(x)\cap X\neq\varnothing\}$ is called the upper approximation of X with respect to R.

(iii) The set $B_R(X)=U_R(X)-L_R(X)$ is called the boundary region of X with respect to R.

Here $R(x)$ denotes the equivalence class of $x\in U$.

Property 2.4 [5]

Let X,Y be any two subsets of the approximation space (U,R).Then,

1. $L_R(X)\subseteq X\subseteq U_R(X))$
2. $L_R(\varnothing)=U_R(\varnothing)=\varnothing$
3. $L_R(U)=U_R(U)=U$
4. $U_R(X\cup Y)=U_R(X)\cup U_R(Y)$
5. $U_R(X\cap Y)\subseteq U_R(X)\cap U_R(Y)$
6. $L_R(X\cup Y)\supseteq L_R(X)\cup L_R(Y)$
7. $L_R(X\cap Y)=L_R(X)\cap L_R(Y)$
8. If $X\subseteq Y$ then, $L_R(X)\subseteq L_R(Y)$ and $U_R(X)\subseteq U_R(Y)$
9. $U_R(X^c)=[L_R(X)]^c$ and $L_R(X^c)=[U_R(X)]^c$
10. $U_R(U_R(X))=L_R(U_R(X))=U_R(X)$
11. $L_R(L_R(X))=U_R(L_R(X))=L_R(X)$

Definition 2.5 [5]

Let U be the universe, R be an equivalence relation on U, $X \subseteq U$ and $\tau_R(X) = \{U, \varnothing, U_R(X), L_R(X), B_R(X)\}$.Then, $\tau_R(X)$ satisfies the following.

(i) $U, \varnothing \in \tau_R(X)$

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

The set $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X.
We call $(U, \tau_R(X))$ as a Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets. The complement of Nano open sets are called Nano closed sets.

Definition 2.6 [5]

Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$, then

(i) The Nano interior of A is defined as the union of all Nano open subsets of A and is denoted by $N\operatorname{int}(A)$. That is, $N\operatorname{int}(A)$ is the largest Nano open subset of A.

(ii) The Nano closure of A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. That is, $Ncl(A)$ is the smallest Nano closed set containing A.

Definition 2.7 [1]

A subset A of a Nano topological $(U, \tau_R(X))$ is called,

(i) Nano generalized closed set (briefly Ng-closed) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is a Nano open set.

(ii) Nano g*-closed set (briefly Ng*-closed) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is a Nano generalized open set.

3. Nano g**-closed sets

Throughout this paper $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U and U/R denotes the family of equivalence classes of U by R.

We define the following closed set in a Nano topological space $(U, \tau_R(X))$.

Definition 3.1

A subset A of a Nano topological space $(U, \tau_R(X))$ is called a Nano g**-closed set (briefly Ng**-closed set) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is a Ng*-open set.

Example 3.2: Consider $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b,d\},\{c\}\}$. Let $X=\{a,b\}$ and $\tau_R(X)=\{U,\varnothing,\{a\},\{b,d\},\{a,b,d\}\}$.Then, $(U,\tau_R(X))$ is a Nano topological space.
Consider the subset $A=\{a,b,c\}$.The Ng*-open set containing A is only U and $Ncl(A)=U$. Hence $A=\{a,b,c\}$ is a Ng**−closed set.

Theorem 3.3: In a Nano Topological space $(U,\tau_R(X))$ if A is Ng**−closed set, then $Ncl(A)-A$ contains no nonempty Ng*−closed set.

Proof: Suppose A is a Ng**−closed set in the Nano Topological space $(U,\tau_R(X))$.
Let Z be a Ng*−closed set contained in $NCl(A)-A$.Then, $Z\subseteq Ncl(A)$ and $Z\not\subset A$. This implies $A\subseteq Z^c$. But Z^c is a Ng*−open set. Since A is a Ng**−closed set, $Ncl(A)\subseteq Z^c$ which implies that $Z\subseteq[Ncl(A)]^c$.
Now, $Z\subseteq Ncl(A)$ and $Z\subseteq[Ncl(A)]^c\Rightarrow Z\subseteq Ncl(A)\cap[Ncl(A)]^c\Rightarrow Z=\varnothing$.
This shows that $Ncl(A)-A$ contains no nonempty Ng*−closed set.

Theorem 3.4: If A and B are Ng**−closed sets in a Nano Topological space $(U,\tau_R(X))$, then $A\cup B$ is a Ng**−closed set.

Proof: Let V be a Ng*−open set containing $A\cup B$.It is clear that $A\subseteq A\cup B$ and $B\subseteq A\cup B$ and hence $A\subseteq V, B\subseteq V$.Since, A and B are Ng**−closed sets, $Ncl(A)\subseteq V$ and $Ncl(B)\subseteq V$. So, $Ncl(A)\cup Ncl(B)\subseteq V$.But, $Ncl(A\cup B)=Ncl(A)\cup Ncl(B)$ and hence $Ncl(A\cup B)\subseteq V$. This proves that $A\cup B$ is a Ng**−closed set in $(U,\tau_R(X))$.

Theorem 3.5: If A is a Ng**−closed set and $A\subseteq B\subseteq Ncl(A)$ then, B is also a Ng**−closed set.

Proof: Let V be a Ng*−open set containing B.Since, $A\subseteq B$, V is a Ng*−open set containing A also. But, A is a Ng**−closed set and hence, $Ncl(A)\subseteq V$.
Now, $B\subseteq Ncl(A)\Rightarrow Ncl(B)\subseteq Ncl(A)\Rightarrow Ncl(B)\subseteq V$.This proves the theorem.

Theorem 3.6: Every Nano closed set is a Ng**−closed set.

Proof: Let A be a Nano closed set and V be a Ng*−open set containing A.Since, A is a Nano closed set, $Ncl(A)=A$.This implies, $Ncl(A)\subseteq V$ proving that A is a Ng**−closed set.

Remark 3.7: The converse of the above theorem need not be true as shown in the following example.

Example 3.8: Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$. Let $X=\{a,b\}$ and $\tau_R(X)=\{U,\varnothing,\{a\},\{b,d\},\{a,b,d\}\}$. Then, $(U,\tau_R(X))$ is a Nano topological space. In this space,

- ✓ The Nano closed sets are $\{U,\varnothing,\{b,c,d\},\{a,c\},\{c\}\}$.
- ✓ The Ng*-open sets are $\{U,\varnothing,\{d\},\{b\},\{a\},\{a,b\},\{b,d\},\{a,b,d\},\{a,d\}\}$.

Consider the subset $A=\{a,b,c\}$.The $\text{Ng}*-$open set containing A is only U and $Ncl(A)=U$. Hence $A=\{a,b,c\}$ is a $\text{Ng}**-$closed set . But it is not a Nano closed set.

Theorem 3.9: Every $\text{Ng}*-$closed set is a $\text{Ng}**-$closed set .

Proof: Let A be a $\text{Ng}*-$closed set and V be a $\text{Ng}*-$open set containing A.Since, every $\text{Ng}*-$open set is a Nano generalized open set, V is a Nano generalized open set containing A. But, A is a $\text{Ng}*-$closed set set. So, $Ncl(A)\subseteq V$ .This shows that A is a $\text{Ng}**-$closed set.

Remark 3.10: The converse of the above theorem need not be true as shown in the following example.

Example 3.11: Let $U=\{a,b,c\}$ with $U/R=\{\{a\},\{b,c\}\}$. Let $X=\{a,c\}$. By definition consider $\tau_R(X)=\{U,\varnothing,\{a\},\{b,d\},\{a,b,d\}\}$.Then, $(U,\tau_R(X))$ is a Nano topological space. In this space,

- ✓ The Nano closed sets are $\{U,\varnothing,\{a,c\},\{b,c\},\{a,b\},\{a\},\{b\}\}$
- ✓ The Nano generalized closed sets are $\{U,\varnothing,\{a,c\},\{b,c\},\{a,b\},\{a\},\{b\}\}$
- ✓ The Nano g*- closed sets are $\{U,\varnothing,\{a\},\{b,c\},\{a,b\}\}$
- ✓ The Nano g**- closed sets are $\{U,\varnothing,\{a,c\},\{b,c\},\{a,b\},\{a\},\{b\}\}$

The subset $\{a,c\}$ is a $\text{Ng}**-$closed set., but it is not a $Ng*-$closed set .

Theorem 3.12: For each $a\in U$, either $\{a\}$ is a $\text{N}g*-$closed set or $\{a\}^c$ is a $\text{Ng}**-$closed set in $\tau_R(X)$.

Proof: Suppose for each $a\in U$, $\{a\}$ is not a $\text{N}g*-$closed set in $\tau_R(X)$.Then, $\{a\}^c$ is not a $\text{N}g*-$open set in $\tau_R(X)$.This implies, the only $\text{N}g*-$open set containing $\{a\}^c$ is U. Hence, $Ncl(\{a\}^c)\subseteq U$. So, $\{a\}^c$ is $\text{Ng}**-$closed set.

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