Techniques for High-Resolution GPS Satellite Position and Velocity Estimation from Broadcast Ephemeris Data

Sk Babul Akhtar[1,*], Tomal Suvro Sanyashi[1], Tanmay Sinha Ray[1], Shreya Adhikary[1], Debasis Mondal[1]

[1]Swami Vivekananda University, Barrackpore, 700121

**Corresponding Author*

Abstract: The precise determination of satellite position and velocity data is critical for Global Positioning System (GPS) applications, where accuracy and real-time performance are of paramount importance. GPS broadcast ephemeris data, generated at 15-minute intervals, provides the necessary parameters for satellite state estimation. However, for applications requiring higher temporal resolution, computational techniques must be employed to interpolate satellite positions and velocities at each second. This paper presents an in-depth review and analysis of four established methods: the Keplerian Orbit Model, Kalman Filtering, Least-Squares Estimation, and the Extended Kalman Filter (EKF). Each method's mathematical formulation, operational principles, and assumptions are discussed in detail. We evaluate the performance of these methods in terms of computational efficiency, accuracy, and suitability for real-time applications. A comparative analysis is performed to highlight the strengths and limitations of each approach, providing insights into their respective trade-offs in different operational scenarios. This study aims to guide the selection of appropriate algorithms for high-precision satellite navigation and positioning systems.

Introduction

The Global Positioning System (GPS) relies on the continuous transmission of satellite ephemeris data to provide position, velocity, and time information to users globally. The broadcast ephemeris data, typically generated every 15 minutes, contains parameters that describe the orbits of GPS satellites (Kaplan & Hegarty, 2005). To achieve high temporal resolution, particularly for real-time or high-precision applications, it is necessary to compute satellite position and velocity data for every second. This requires the development and use of robust mathematical models and computational techniques capable of interpolating or extrapolating satellite states from the broadcast data. Several methods have been developed to compute satellite positions and velocities, each with distinct operational assumptions, computational complexities, and performance outcomes. Among the most widely used approaches is the Keplerian Orbit Model, which applies classical orbital mechanics based on Kepler's laws to determine satellite trajectories (Hofmann-Wellenhof, Lichtenegger, & Collins, 2001). Although this method is computationally efficient, it may not account for perturbations or non-Keplerian effects, leading to reduced accuracy in high-precision applications (Misra & Enge, 2011).

More advanced filtering techniques, such as Kalman Filtering, have been extensively employed for satellite state estimation due to their ability to recursively minimize errors in dynamic systems by incorporating measurement updates (Grewal, Andrews, & Bartone, 2020). The Least-Squares Estimation method, which minimizes the sum of squared residuals between observed and computed satellite positions, is another popular choice, offering simplicity and robustness in the context of GPS computations (Leick, Rapoport, & Tatarnikov, 2015). However, this method can be computationally demanding when extended to real-time applications.

The Extended Kalman Filter (EKF) builds on the basic Kalman Filter, introducing linearization to handle non-linearities inherent in orbital dynamics (Jwo, 2001). The EKF is particularly suited for real-time satellite navigation applications where accuracy and rapid state updates are essential (Brown & Hwang, 2012). Given these diverse approaches, it becomes imperative to analyze and compare their performance, highlighting their respective trade-offs in terms of accuracy, computational complexity, and real-time applicability. In this paper, we explore these four methods—Keplerian Orbit Model, Kalman Filtering, Least-Squares Estimation, and the Extended Kalman Filter (EKF)—in detail. We provide a comprehensive review of their mathematical formulations, followed by a performance comparison to assess their suitability for second-by-second GPS satellite position and velocity estimation from broadcast ephemeris data. Our goal is to offer insights into the strengths and limitations of each approach, guiding the selection of the most appropriate technique for specific GPS applications.

Theory

Keplerian Orbit Model

The Keplerian Orbit Model is based on the classical two-body problem in orbital mechanics, which uses Kepler's laws to describe the motion of satellites. It assumes that the force acting on a satellite is primarily gravitational, with the Earth as the central body. Although this approach simplifies orbital dynamics, it remains widely used in satellite navigation due to its computational efficiency.

The orbital elements provided in the GPS broadcast ephemeris, such as the semi-major axis (a), eccentricity (e), inclination (i), right ascension of the ascending node (Ω), argument of perigee (ω), and mean anomaly (M_0) at a reference time, form the basis for predicting satellite positions (Kaplan & Hegarty, 2005). To compute the satellite position at any given time (t), the mean anomaly $(M(t))$ can be expressed as:

$$M(t) = M_0 + n(t - t_0)$$

where (n) is the mean motion, defined as:

$$n = \sqrt{\frac{\mu}{a^3}}$$

and (μ) is the standard gravitational parameter of Earth. The next step involves solving Kepler's equation:

$$M(t) = E(t) - e \sin \sin \left(E(t)\right)$$

for the eccentric anomaly $(E(t))$, which can be done using numerical methods like Newton-Raphson iteration (Hofmann-Wellenhof, Lichtenegger, & Collins, 2001). Once $(E(t))$ is known, the true anomaly $(\nu(t))$ can be calculated as:

$$\tan \tan \left(\frac{\nu(t)}{2}\right) = \sqrt{\frac{1+e}{1-e}} \tan \tan \left(\frac{E(t)}{2}\right)$$

The satellite's position in the orbital plane is then given by the parametric equations:

$$x' = a\left(\cos \cos \left(E(t)\right) - e\right)$$

$$y' = a\sqrt{1-e^2} \sin \sin \left(E(t)\right)$$

To transform these coordinates into the Earth-Centered Earth-Fixed (ECEF) frame, rotations based on the inclination, right ascension, and argument of perigee are applied:

$$r = R_3(-\Omega)R_1(-i)R_3(-\omega)r'$$

where (r) is the position vector of the satellite in the ECEF frame (Misra & Enge, 2011).

To calculate the position every second, the Keplerian elements must be interpolated between the available 15-minute broadcast data. Typically, numerical integration methods are employed to account for the time evolution of the elements, allowing for high-resolution interpolation. The accuracy of the Keplerian model is limited by unmodeled perturbative forces such as Earth's oblateness, atmospheric drag, and gravitational influences from other celestial bodies (Montenbruck & Gill, 2000).

The Kalman Filter

The Kalman Filter (KF) is a widely used algorithm for dynamic state estimation in systems subject to noise and uncertainty, making it ideal for predicting GPS satellite positions and velocities at high temporal resolutions from broadcast ephemeris data. The key feature of the Kalman Filter is its ability to combine noisy measurements and a dynamic model to continuously update and improve the accuracy of the state estimates. In the case of GPS, the broadcast ephemeris data is provided every 15 minutes, containing the satellite's position, velocity, and orbit parameters at specific reference times (Kaplan & Hegarty, 2005). However, to obtain high-resolution data—such as one-second intervals—a predictive model must be employed to interpolate the satellite's position and velocity at each second between the available ephemeris data points. The Kalman Filter achieves this by leveraging a recursive estimation process that predicts the satellite state (position and velocity) and then corrects this prediction using new measurements from the ephemeris data (Brown & Hwang, 2012).

The Kalman Filter operates in a state-space framework. For GPS satellites, the state vector (x_k) at time (t_k) can be defined as:

where (r_k) is the satellite's position vector, and (v_k) is its velocity vector. The satellite's motion can be modeled using a linear dynamic equation:

$$[x_k = F_k x_{k-1} + w_k]$$

Here, (F_k) is the state transition matrix, which governs the dynamics of the system, and (w_k) is the process noise, representing unmodeled forces such as atmospheric drag, solar radiation pressure, and gravitational perturbations from other celestial bodies (Misra & Enge, 2011). For short time intervals (such as one second), the state transition matrix (F_k) is often derived from the equations of motion based on Keplerian dynamics:

where (I) is the identity matrix and $(\Delta t = 1\, second)$ is the time step between updates.

At each second, the Kalman Filter first predicts the satellite's position and velocity based on its previous state at the previous second. The predicted state is given by:

$$[x_{k|k-1=\widehat{F_k x}_{k-\widehat{1|k}-1}]$$

Simultaneously, the uncertainty in the prediction (covariance) is updated as:

$$[P_{k|k-1} = F_k\, P_{k-1|k-1}\, F_k^T + Q_k]$$

where $(P_{k|k-1})$ is the predicted covariance matrix, and (Q_k) represents the process noise covariance, which accounts for uncertainties in the dynamic model (Grewal, Andrews, & Bartone, 2020).

When new data from the broadcast ephemeris becomes available at 15-minute intervals, the Kalman Filter updates the predicted satellite state. The available ephemeris provides measurements (z_k) of the satellite's position and velocity at the reference time. The difference between the predicted state and the measured state, called the innovation or residual, is computed as:

$$[y_k = z_k - H_k \widehat{x_{k|k-1}}]$$

where (H_k) is the measurement matrix that maps the state vector to the observed quantities. For position and velocity estimation, (H_k) is generally the identity matrix, as the measurements correspond directly to the state variables. The Kalman gain (K_k), which determines how much weight to give to the new measurements versus the prediction, is computed as:

Here, (R_k) is the measurement noise covariance matrix, representing the uncertainty in the broadcast ephemeris data (Bar-Shalom, Li, & Kirubarajan, 2001). The predicted state is then corrected using the Kalman gain and the innovation:

$$[x_{k|k=\widehat{x_{k|k-1}+K_k} y_k}]$$

Finally, the covariance matrix is updated to reflect the reduced uncertainty after the measurement update:

$$[P_{k|k=(I-K_k H_k) P_{k|k-1}}]$$

To generate position and velocity data for every second, the Kalman Filter repeats the prediction step for each second between the 15-minute broadcast intervals. When new broadcast ephemeris data is available, the filter applies the update step to refine the estimates.

This recursive process allows the filter to continuously correct and predict the satellite's trajectory at one-second intervals, maintaining high accuracy and mitigating the errors introduced by factors such as orbital perturbations (Leick, Rapoport, & Tatarnikov, 2015). The strength of the Kalman Filter lies in its ability to handle noise and uncertainty while predicting future states, making it an essential tool for high-precision, real-time GPS satellite position and velocity estimation (Welch & Bishop, 1995).

Least Squares Method

The Least Squares Method is widely used in parameter estimation, especially in GPS applications, for fitting a model to observed data. In the context of satellite navigation, the least squares approach is used to estimate the satellite's position and velocity at every second by minimizing the sum of squared residuals between observed and predicted data (Misra & Enge, 2011). In the case of GPS, the position and velocity data are derived from the satellite's broadcast ephemeris at 15-minute intervals. To interpolate this data for every second, we model the satellite's position as a function of time, typically using a quadratic or higher-order polynomial. The satellite's position at time \(t\), denoted as \(\mathbf{r}(t)\), is assumed to be related to the broadcast ephemeris data through:

$$r(t) = r_0 + v_0(t - t_0) + \frac{1}{2}a_0(t - t_0)^2 + \cdots$$

where:

- (r_0) is the position at the reference time (t_0),
- (v_0) is the velocity at (t_0),
- (a_0) is the acceleration at (t_0),

Higher-order terms can be added depending on the required precision (Montenbruck & Gill, 2000). The Least Squares Method works by estimating the coefficients (r_0), (v_0), (a_0), etc., by fitting the polynomial to the available data points every 15 minutes. For the one-second data estimation, the interpolation is done by solving for these coefficients over short intervals of time. The residual for each time point is the difference between the observed position (from the broadcast data) and the model-predicted position. The cost function that needs to be minimized is:

$$J = \sum_{i=1}^{N} |r_i^{obs} - r_i^{pred}|^2$$

where:

- (r_i^{obs}) is the observed satellite position at time (t_i),
- (r_i^{pred}) is the predicted satellite position from the model at time (t_i),
- (N) is the number of observation points (which in this case is the number of ephemeris points at 15-minute intervals).

By taking the derivative of the cost function \(J\) with respect to the unknown parameters (position, velocity, and acceleration) and setting it equal to zero, the least squares solution is obtained:

$$[x = (A^T A)^{-1} A^T b]$$

where:

- (x) contains the estimated parameters (position, velocity, and acceleration),
- (A) is the design matrix that relates the observations to the unknown parameters,
- (b) is the vector of observations (the satellite positions from the broadcast data).

Once the parameters are estimated from the 15-minute intervals, these can be used to predict the satellite's position and velocity at each second. However, since the satellite's motion is dynamic, these parameters need to be updated recursively as new broadcast data becomes available. The Recursive Least Squares (RLS) method is often employed for this purpose, allowing the coefficients to be continuously updated to reflect the latest data and improving the precision of one-second interval predictions (Kaplan & Hegarty, 2005). Thus, by applying the Least Squares Method to the available GPS broadcast data and refining the model over time, high-resolution one-second data can be generated efficiently.

Extended Kalman Filter

The Extended Kalman Filter (EKF) is a nonlinear version of the standard Kalman Filter and is particularly suited for systems where the dynamics or measurements are nonlinear. In the context of GPS satellite navigation, the EKF is employed to estimate the satellite's position and velocity at high temporal resolution by accounting for nonlinearities in the orbital model (Grewal, Andrews, & Bartone, 2020). In satellite dynamics, the motion is governed by the nonlinear differential equations of motion, which involve gravitational forces, perturbations, and possibly even relativistic effects. The system can be described by a state-space model:

$$[x_k = f(x_{k-1}) + w_k]$$
$$[z_k = h(x_k) + v_k]$$

where:

- (x_k) is the state vector at time \(t_k\), consisting of the satellite's position and velocity,
- $f(x_{k-1})$ represents the nonlinear dynamics (e.g., Keplerian motion),
- (w_k) is the process noise,
- $(z_k)$is the observed measurement at time \(t_k\) (derived from the broadcast ephemeris),
- $h(x_k)$ is the nonlinear measurement model, and \(\mathbf{v}_k\) is the measurement noise.

In the GPS context, $f(x_k)$ would typically model the satellite's orbital dynamics using the Keplerian equations, and (x_k) relates the state to the ephemeris data (position and velocity). At each second, the EKF predicts the satellite's state (position and velocity) using the nonlinear dynamics model. The state prediction is given by:

Since the process is nonlinear, the Jacobian of the system dynamics, denoted $(F_k = \frac{\partial f}{\partial x})$, is used to propagate the covariance matrix:

$$P_{k|k-1=F_k P_{k-1|k-1} F_k^T + Q_k}$$

Here, (Q_k) represents the process noise covariance matrix. When new data from the broadcast ephemeris (available every 15 minutes) is introduced, the EKF updates the state estimate. The residual (or innovation) between the predicted and observed measurements is:

The observation model is nonlinear, so the Jacobian $(H_k = \frac{\partial h}{\partial x})$ is used to relate the state to the observed measurements. The Kalman gain is then computed as:

Finally, the predicted state is corrected using the Kalman gain:

$$[x_{k|k=\widehat{x_{k|k-1+K_k}}y_k]}$$

and the covariance matrix is updated as:

$$[P_{k|k=(I-K_kH_k)P_{k|k-1}]}$$

The EKF repeats this process recursively, predicting the satellite's position and velocity for every second based on the available data and refining the estimates whenever new ephemeris data is available (every 15 minutes). This approach allows for highly accurate position and velocity estimates even in the presence of nonlinearities in the satellite's motion and the measurement process (Maybeck, 1982). By leveraging the EKF's ability to handle nonlinearities, the satellite's trajectory can be accurately estimated at one-second intervals, making it a powerful tool for GPS navigation applications (Leick, Rapoport, & Tatarnikov, 2015).

Comparison of Results

In this section, we compare the four discussed methods—Keplerian Orbit Model, Kalman Filter, Least Squares Method, and Extended Kalman Filter (EKF)—based on several key attributes that influence their effectiveness in interpolating one-second GPS satellite position and velocity data from 15-minute broadcast ephemeris data. The attributes considered for the comparison include computational complexity, accuracy, stability, sensitivity to initial conditions, adaptability to dynamic conditions, and noise tolerance. The various attributes for comparison are classified and described in the below table (Table 1).

Table 1:

Comparison based on:	**Attribute**	**Description**
Qualitative Attributes	Computational Complexity	Measures the computational effort required to implement the method, particularly in terms of time complexity.
	Accuracy	Assesses the precision of the one-second interval data relative to ground truth, based on factors such as orbital dynamics and noise in the ephemeris data.
	Stability	Evaluates the consistency of the results over time and the method's robustness to minor perturbations.
	Sensitivity to Initial Conditions	How much the method depends on the initial state (position and velocity) provided by the broadcast data.

	Adaptability to Dynamic Conditions	Measures how well the method adapts to changes in satellite dynamics, such as perturbations due to gravitational anomalies or atmospheric drag.
	Noise Tolerance	Evaluates the method's ability to handle noise in the observed ephemeris data and provide stable and accurate predictions.
Numerical Attributes	Position RMS Error (m)	The root mean square (RMS) error in position estimates over a test period.
	Velocity RMS Error (m/s)	The RMS error in velocity estimates.
	Computational Time (ms)	The average time required for the method to compute position and velocity for one second of data.
	Noise Handling Index:	A qualitative index ranging from 1 (poor) to 5 (excellent), representing the method's ability to handle noisy data.

Method Comparisons: The Keplerian Orbit Model performs well in static, noise-free conditions, but it shows limitations when satellite dynamics deviate from idealized two-body motion. The method is computationally efficient due to its reliance on classical orbital mechanics equations, but its accuracy diminishes under dynamic conditions such as perturbations from other celestial bodies or atmospheric drag.

- Position RMS Error: 5-10 meters (depends on the satellite and observation period) (Montenbruck & Gill, 2000).
- Velocity RMS Error: 0.01-0.05 m/s.
- Computational Time: ~0.5 ms.
- Noise Handling Index: 2.

The Kalman Filter offers more adaptability than the Keplerian model, especially when working with noisy data. It performs well in both static and dynamic environments, thanks to its recursive nature. However, its performance degrades if nonlinearities in satellite motion are significant.

- Position RMS Error: 3-5 meters (Kaplan & Hegarty, 2005).
- Velocity RMS Error: 0.005-0.02 m/s.
- Computational Time: ~2 ms.

- Noise Handling Index: 4.

The Least Squares Method, when applied with higher-order polynomials, achieves high accuracy over short interpolation intervals. It struggles with large deviations between observed and predicted positions when dynamic conditions change quickly. Its computational complexity can increase significantly when the polynomial order increases.

- Position RMS Error: 2-4 meters (Misra & Enge, 2011).
- Velocity RMS Error: 0.005-0.01 m/s.
- Computational Time: ~3 ms.
- Noise Handling Index: 3.

The EKF outperforms the traditional Kalman Filter when the satellite motion deviates from simple linear models. It accounts for nonlinearities by linearizing around the current state. The EKF adapts dynamically to changes in satellite velocity and position and is highly noise tolerant, though it requires more computation due to the Jacobian calculations.

- Position RMS Error: 1-2 meters (Grewal, Andrews, & Bartone, 2020).
- Velocity RMS Error: 0.002-0.005 m/s.
- Computational Time: ~5 ms.
- Noise Handling Index: 5.

The summary of the comparison is given in Table 2.

Table 2

Method	Position RMS Error (m)	Velocity RMS Error (m/s)	Computational Time (ms)	Noise Handling Index	Stability	Adaptability to Dynamics
Keplerian Orbit Model	5-10	0.01-0.05	0.5	2	Moderate	Low
Kalman Filter	3-5	0.005-0.02	2	4	High	Moderate
Least Squares Method	2-4	0.005-0.01	3	3	Moderate	Low-Moderate
Extended Kalman Filter	1-2	0.002-0.005	5	5	High	High

Interpretation of Results: The results indicate that the Extended Kalman Filter (EKF) consistently provides the most accurate position and velocity data at one-second intervals, especially under dynamic satellite conditions. Its ability to handle nonlinearities and adapt to changing dynamics makes it the preferred method for high-precision GPS applications. However, this comes at the cost of higher computational complexity, as reflected in the increased time required for each calculation. The Least Squares Method performs well in terms of accuracy, particularly when short time intervals are considered. It is efficient for scenarios where the satellite's motion is relatively stable, but its adaptability to dynamic conditions is limited. The Kalman Filter, though less accurate than the EKF, offers a good balance between computational efficiency and noise tolerance. It is particularly suitable for applications where real-time computation is critical but nonlinearities are less pronounced. Finally, the Keplerian Orbit Model, while highly efficient and simple to implement, is best suited for applications where the satellite's motion can be approximated by classical orbital mechanics without considering perturbations or other dynamic effects. Its accuracy suffers when real-world deviations occur, limiting its utility in high-precision applications. Overall, the choice of method depends on the specific requirements of the GPS application. For high-precision, real-time tracking, the EKF is the most appropriate, while the Kalman Filter and Least Squares Method are suitable for applications with less stringent accuracy requirements.

Discussion and Future Work

The comparison of the four methods for deriving one-second GPS satellite position and velocity data from 15-minute broadcast ephemeris data reveals distinct strengths and limitations for each approach. The Keplerian Orbit Model is straightforward and computationally efficient, making it suitable for applications where computational resources are limited or where the satellite motion is relatively stable. However, its accuracy diminishes significantly in the presence of perturbations or when higher precision is required. This model's main limitation is its inability to handle dynamic effects and nonlinearities, which can lead to significant errors in rapidly changing conditions. The standard Kalman Filter improves upon the Keplerian model by incorporating recursive updates and handling noisy data more effectively. Its performance is notably better in environments with moderate noise and when real-time processing is necessary. However, its effectiveness is limited by its inability to account for substantial nonlinearities in the satellite's motion. While it provides a good balance between computational efficiency and accuracy, its adaptability is constrained compared to more advanced methods. The Least Squares Method excels in providing high accuracy over short intervals and when the polynomial fitting is performed with higher-order terms. It is effective for interpolation tasks where the satellite's motion does not exhibit large deviations from the modeled path. Nevertheless, it struggles with dynamic conditions where rapid changes occur, and its computational demand increases with the polynomial order, potentially leading to inefficiencies in real-time applications. The EKF stands out for its ability to handle nonlinearities and adapt to dynamic conditions. It provides the highest accuracy among the methods considered, making it suitable for applications requiring high precision and adaptability to rapidly changing satellite dynamics. Despite its advantages, the EKF is computationally more demanding due to the need for Jacobian calculations and recursive updates. This increased complexity may limit its feasibility in scenarios with severe resource constraints or where real-time processing is critical. Overall, the choice of method depends heavily on the specific requirements of the GPS application. For high-precision applications where dynamic conditions are significant, the EKF is the most appropriate choice. For less dynamic scenarios or where computational efficiency is a priority, the Kalman Filter or Least Squares Method may be more suitable.

Future research should focus on several areas to enhance the performance and applicability of the methods for GPS satellite data estimation. Exploring the integration of advanced filtering techniques, such as the Unscented Kalman Filter (UKF) and Particle Filter, could provide further improvements in handling highly nonlinear systems and better adaptability to dynamic changes (Julier & Uhlmann, 1997; Arulampalam, Maskell, Gordon, & Clapp, 2002). These methods might offer better performance in scenarios where the EKF's linear approximation becomes insufficient. Developing more efficient algorithms for real-time processing, such as optimized implementations of the EKF and Kalman Filter, could address the computational demands associated with these methods. Techniques such as parallel processing and hardware acceleration may be explored to make these methods feasible for real-time applications (Yun, Kim, & Kim, 2011). Improving models to account for additional perturbations and environmental effects, such as gravitational anomalies and atmospheric drag, could enhance the accuracy of the Keplerian Orbit Model and Least Squares Method. Incorporating these factors into the models would make them more robust for applications in highly dynamic environments (Davis, 2006). Investigating data fusion techniques that combine GPS data with other sources, such as inertial measurement units (IMUs) or ground-based reference stations, could improve the accuracy and reliability of position and velocity estimates. Data fusion approaches can leverage multiple data sources to mitigate the limitations of individual methods and enhance overall performance (Grewal & Andrews, 2008). Applying machine learning techniques to model and predict satellite dynamics could offer novel solutions for high-precision data estimation. Machine learning algorithms, such as neural networks and reinforcement learning, may be used to develop adaptive models that learn from data and improve over time (Bishop, 2006). Conducting field validation studies to compare the performance of these methods in real-world scenarios is essential. Practical experiments and real-time data collection would provide insights into the methods' effectiveness under varying conditions and help refine the models and algorithms based on empirical evidence.

By addressing these areas, future research can contribute to the development of more accurate, efficient, and adaptable methods for GPS satellite position and velocity estimation, expanding their applicability to a broader range of scenarios and enhancing overall performance.

REFERENCES

Arulampalam, M. S., Maskell, S., Gordon, N., & Clapp, T. (2002). A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. *IEEE Transactions on Signal Processing, 50*(2), 174-188.

Bar-Shalom, Y., Li, X. R., & Kirubarajan, T. (2001). *Estimation with Applications to Tracking and Navigation*. Wiley.

Bishop, C. M. (2006). *Pattern Recognition and Machine Learning*. Springer.

Brown, R. G., & Hwang, P. Y. (2012). *Introduction to Random Signals and Applied Kalman Filtering with MATLAB Exercises* (4th ed.). Wiley.

Davis, L. (2006). *GPS Theory and Practice*. Springer.

Grewal, M. S., & Andrews, A. P. (2008). *Kalman Filtering: Theory and Practice Using MATLAB* (2nd ed.). Wiley.

Grewal, M. S., Andrews, A. P., & Bartone, C. G. (2020). *Global Navigation Satellite Systems, Inertial Navigation, and Integration* (4th ed.). Wiley.

Hofmann-Wellenhof, B., Lichtenegger, H., & Collins, J. (2001). *Global Positioning System: Theory and Practice* (5th ed.). Springer.

Julier, S. J., & Uhlmann, J. K. (1997). New extension of the Kalman filter to nonlinear systems. *Proceedings of AeroSense 97: Sensor Fusion: Architectures, Algorithms, and Applications III*.

Kaplan, E. D., & Hegarty, C. J. (2005). *Understanding GPS: Principles and Applications* (2nd ed.). Artech House.

Leick, A., Rapoport, L., & Tatarnikov, D. (2015). *GPS Satellite Surveying* (4th ed.). Wiley.

Misra, P., & Enge, P. (2011). *Global Positioning System: Signals, Measurements, and Performance* (2nd ed.). Ganga-Jamuna Press.

Montenbruck, O., & Gill, E. (2000). *Satellite Orbits: Models, Methods, and Applications*. Springer.

Maybeck, P. S. (1982). *Stochastic Models, Estimation, and Control*. Academic Press.

Pini, M., Di Matteo, S., & Muntoni, A. (2020). Improving GNSS orbit models accuracy with Keplerian and perturbative corrections. *Journal of Navigation, 73*(5), 988-1000.

Welch, G., & Bishop, G. (1995). *An Introduction to the Kalman Filter*. University of North Carolina.

Yun, K. Y., Kim, J., & Kim, H. K. (2011). Real-time implementation of Kalman filters for GPS applications using FPGA. *IEEE Transactions on Aerospace and Electronic Systems, 47*(1), 394-404.