Deadbeat Performance without Parameter Restriction via SCT in LTI Systems

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Abstract:

A technique to realize dead-beat second order and higher order time invariant linear control system without any restriction of system parameters is investigated. There are certain systems, such as biological control systems, where it may not be feasible to incorporate a controller within the system or to process the system's input directly. In such cases, it would be useful to compensate for the system's behaviour by applying a suitable electrical signal that can supplement the normal input. This method of introducing an additional signal to achieve dead-beat transient performance in a control system is referred to as the 'Signal Correction Technique (SCT).' The specific nature of the additional signal varies depending on the type of input to the system, such as a step input, ramp input, parabolic input, or polynomial input. In this work the proposed SCT is developed for the linear continuous time invariant system with the reference input as step input. Here the additional signal is a pulse of appropriate duration and applied at a suitable time and having a specified amplitude with the same rise time as non-compensated system. The parameters of the added pulse to the uncompensated system i.e the start time (t_1), end time (t_2) and the required amplitude (b) are determined without any restriction on system parameters.

Keywords: Dead-beat control, Signal Correction Technique (SCT), Higher-order control systems, Pulse-based compensation, Step input response

1 Introduction

The idea of deadbeat control can easily be visualized by considering the application of a step, ramp or any type of input to a control system. If the response of the system, under this situation, is that it reaches to reference in the minimum time without undergoing any overshoot or undershoots, then the system is defined as a deadbeat one.

In discrete time Control Theory, the deadbeat regulator control problem consists of finding what input signal must be applied to a system in order to bring the output to zero in the smallest number of time steps. For an N-th order linear system it has been shown that this minimum number of steps will be N, provided that the system is null controllable, that is, can be brought to state zero by some input. The solution is to apply feedback such that all poles of the closed-loop transfer function are at the origin of the z-plane. By extension, a closed loop transfer function which has all poles of the transfer function at the origin is sometimes called a deadbeat transfer function. For the discrete tracking problem, the same idea as explained in the above paragraph can be extended. It should be noted that the concept of minimum time is implicitly associated with the dead beat concept. Though in many deadbeat control implementations, the minimum time optimization requirement is not introduced explicitly.

Deadbeat controllers for linear systems have long been investigated, and successful applications have been reported in many literatures [1-7]. Various techniques have been adopted in the past for realizing (compensating) deadbeat transient response of linear control systems [8-16].

Firstly, a suitable deadbeat controller can be designed and put in the forward path of the control loop. The basic idea behind the technique is to cancel some (sometime all) of the plant or process poles by incorporating corresponding zeros and to introduce some desired poles. The difficulties associated with this technique are as follows. The pole-zero cancellations must be perfect, otherwise, not only some zeros will be introduced but also the order of the system will increase giving rise to the problem of system instability. Further, the design of the deadbeat controller is based on the system or plant model. As modeling involves many approximations including linearization, system order reduction by neglecting far poles and zeros of the plant, the deadbeat controller designed on such a model may not produce the desired result when the controller is used with the real plant. It is necessary to study the robustness of the designed deadbeat controller regarding the imperfection of pole-zero cancellation and the imperfection of the model. Such a robust controller design may be turned out to be very tricky. Hopefully, if such a robust controller can be designed, it will alleviate the problem of ageing too. Again, in some applications, such as biological control systems, it may not be possible to incorporate a controller within the system.

Secondly, the input shaping technique is introduced to attain deadbeat control of process with or without dead time. The stability of the closed-loop system need to be guaranteed by the PI controller which may be designed with Nyquist criterion, and the input shaping controller is employed to shape the command input to get deadbeat control. With selecting only two parameters, the sampling period and the proportional gain which can be analytically computed according to the phase margin, the closed-loop system may be designed easily. Unfortunately, the introduction of the PI controller in the system will give rise to all the problems as mentioned in the previous paragraph. Further, in many biological applications, it may not be possible to incorporate the mechanism (input shaping controller) to shape the input command.

Thirdly, there is the signal correction technique (SCT) where a suitable signal is generated by an algorithm using the states of the system and added with the command signal to implement the deadbeat response. The advantage of this system is not that no controller has to be incorporated in the control loop nor is any signal shaping controller required. In some publications, though a general formulation for SCT for deadbeat response of linear systems of any order has been suggested [1,2], but an algorithm for the implementation has been reported only for a second order system with some parameter restriction and it has been developed using some experimental data.

In all the sections of this article, the SCT is applied, where a suitable signal is applied along with reference input to achieve the deadbeat response. In case of deadbeat realization of higher order system, the equivalent second order system is obtained first and then a suitable signal is added to the system without any restriction of system parameters for deadbeat responses. The same signal to be added to original higher order system for the deadbeat output.

2 Discussions of Previous Techniques for Realizing Deadbeat Response

In certain methods for continuous-time linear systems, a dead-beat controller is designed and placed in the forward path of the control loop. The goal of this approach is to cancel out some or all of the system's poles by adding corresponding zeros, and then to introduce the desired poles. However, this technique has significant implications. The pole-zero cancellation must be exact; otherwise, not only can unwanted zeros be introduced, but the system’s order may increase, potentially causing instability. Additionally, the dead-beat controller design relies heavily on the plant model, which involves approximations. These approximations mean that the robustness of the dead-beat controller must be evaluated to account for both imperfections in pole-zero cancellation and inaccuracies in the model. Designing a robust controller with minimal sensitivity can be challenging, but if successfully implemented, it may also help mitigate issues related to system aging. The pole placement problem in both discrete and continuous-time systems has been shown to reduce from an infinite-dimensional to a finite-dimensional problem, even in the presence of feedback delay. Dead-beat control (DB) was first introduced over forty years ago and has since been extensively researched in both continuous and discrete-time control theory [17]. Despite this, it has often been avoided by designers due to concerns with physical reliability and incomplete pole-zero cancellation. However, with the advent of digital signal processing (DSP) systems, many of these issues can now be addressed. In discrete-time systems, dead-beat control ensures zero error at the sampling instants after a finite settling time, regardless of the inter-sample response. Yet, for certain classes of continuous-time systems, dead-beat control can still be problematic due to remaining inter-sample effects, which are generally undesirable.

In certain applications, such as biological control systems, it may not always be feasible to integrate a controller directly into the system. To achieve dead-beat control, even in systems with or without dead time, the input shaping technique is employed. In this approach, a PI controller, designed using the Nyquist criterion, is positioned in the forward path to ensure overall system stability. The input shaping controller is then used to modify the command input in order to produce the desired dead-beat response. While the closed-loop system can be designed relatively easily, introducing the PI controller into the system introduces the same issues mentioned earlier.

3 Proposed Scheme of Deadbeat Realization for Step Input

The signal correction technique (SCT) involves generating an appropriate signal through an algorithm using the system’s states, which is then added to the command signal to achieve a dead-beat response. No controller is required in the control loop, nor is signal shaping necessary. A general formulation for SCT, aimed at achieving a dead-beat response in linear systems of any order, has been proposed in [1,2]. However, the algorithm has only been implemented for second-order systems with certain parameter restrictions. Recent research on RF-DB control systems has led to various proposals for applying modern techniques to control widely used plants [3-8].

In this work, a dead-beat control scheme based on SCT is proposed for second-order time-invariant systems, and an almost dead-beat response for higher-order linear systems, without the need for parameter restrictions or experimental data. The signal for SCT can either represent the state variables within the system or be a pulse. When using an additional pulse input, the pulse's start time is selected to ensure that the rise time remains unchanged, meaning both the compensated and uncompensated systems will have the same rise time (i.e., the time it takes for the output to reach 90% of its steady-state value). In the proposed method, the dead-beat controller design focuses on reducing overshoots to zero by ensuring that the compensated system always reaches the reference input at the pulse’s end time. This novel approach leads to a more effective dead-beat controller design. Theorem 9.1.1 shows that any nth order linear control system can be represented as second-order linear system by minimizing the mean square error of their output responses. Examples are included for obtaining the dead-beat response of higher systems. The simulation results are given in table 9.1..

4 Background and Problem Formulation

In reference [1,2], it was demonstrated that the response of a linear system to a given input can be altered to achieve any desired output by introducing an appropriate signal at the input. However, it was noted that finding such a signal using traditional continuous data control methods would be challenging. The authors proposed a computer simulation approach to address this issue and demonstrated it using a lightly damped linear second-order system. The desired output was chosen so that, after the switch, the system's responses would follow a straight-line trajectory in the phase plane. The additional signal they identified was a single pulse with duration of T, corresponding to the sampling period in digital simulations, applied only at the switching instant. The authors did not provide a method for determining the correct (desired) value of T, and achieving a dead-beat response for a system with arbitrary damping ξ and natural frequency ω_n would require an endless search for the appropriate T. Thus, the proposed method faces practical implementation challenges. Based on experimental results, the authors presented plots of ξ versus T and ξ versus the pulse amplitude. For a given system, by knowing ξ, they were able to determine the values of T and pulse amplitude using these plots. However, they also mentioned that the product of ω_n and T should be 10.425, indicating that the plots and the cited relationship are useful only for implementing the dead-beat response in systems with specific values of ξ and ω_n. Furthermore, the performance of the compensated system concerning T and pulse amplitude had not been analyzed. This limitation motivated the current work, which proposes two new approaches that do not impose any restrictions on system parameters. Additionally, a detailed performance analysis of the compensated system concerning the required pulse parameters has been thoroughly conducted. In the proposed method, it is assumed that the dead-beat response is achieved by adding a signal to the reference input. This additional signal is generated through a proper simulation using the relationship between the state-space variables within the system and the desired output. Another approach to obtaining the dead-beat response is discussed, which involves adding a pulse to the input along with the reference signal when the output reaches 90% of its steady-state value. This approach ensures that the rise time for both the compensated and uncompensated systems remains the same. The duration and amplitude of the pulse are derived in Theorem 6.1.1 using Theorem 5.2, with the rationale for this criterion provided in the next section.

4.1 Deadbeat Representations through Injection of Pulse

The objective of this method is to achieve a dead-beat response by injecting a suitable pulse at an appropriate time in a time-invariant linear continuous system. The required pulse is generated using the system's state variables. The start time t_1 is chosen at the point when the system response reaches 90% of its final value. The choice of start time influences the rise time of the compensated system. If start time is chosen too early, the rise time will be longer than that of the uncompensated system, which is undesirable; hence, the specific selection of t_1. The end time t_2 is determined by the relationship with t_1, as outlined in Theorem 5.2. Once t_1 and t_2 are known, the pulse amplitude can be determined using the values of damping ratio ξ and natural frequency ω_n, as stated in Theorem 6.1.1. The input-output behaviour of a linear second-order system, as shown in Figure 4.1, is described by its transfer function, where R(s) and C(s) are the Laplace transforms of the input and output, respectively.

Let the representation of G(s)= $\left[\frac{k}{s(s+p)}\right]$ and H(s)=1,

$$T(s)=\frac{k}{s^2+p.s+k}=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}, \textit{where } k=\omega_n^2,$$

Consider an input x(t) as the sum of a step of amplitude 'a' applied at t=0 and a pulse of amplitude 'b' applied for the duration from t_1 *to* t_2. Using Inverse Laplace Transform, the output is obtained as

$$c(t)=[a-a\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\{\omega_n t\sqrt{1-\xi^2}+\theta\}] \qquad \textit{for} \quad 0<t<t_1 \qquad (1)$$

$$c(t)=[a-a\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\{\omega_n t\sqrt{1-\xi^2}+\theta\}]-[b-b\frac{e^{-\xi\omega_n(t-t_1)}}{\sqrt{1-\xi^2}}\sin\{\omega_n(t-t_1)\sqrt{1-\xi^2}+\theta\}] \quad t_1<t<t_2 \qquad (2)$$

$$c(t)=[a-a\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\{\omega_n t\sqrt{1-\xi^2}+\theta\}]-[b-b\frac{e^{-\xi\omega_n(t-t_1)}}{\sqrt{1-\xi^2}}\sin\{\omega_n(t-t_1)\sqrt{1-\xi^2}+\theta\}]+$$
$$[b-b\frac{e^{-\xi\omega_n(t-t_2)}}{\sqrt{1-\xi^2}}\sin\{\omega_n(t-t_2)\sqrt{1-\xi^2}+\theta\}] \quad t_2<t<\infty \qquad (3)$$

To obtain the pulse parameters few theorems to be established that can ensure the actual parameters to achieve the dead-beat response.

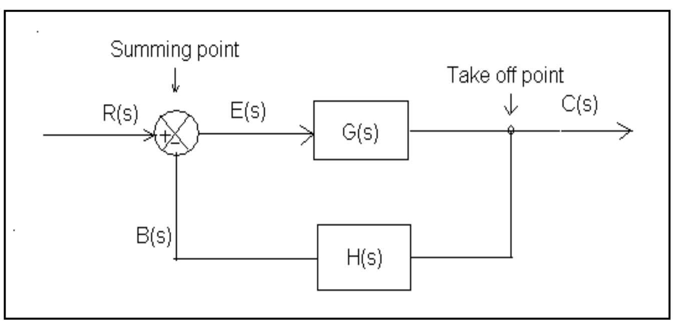

Figure 4.1: The input-output behavior of closed loop linear continuous system

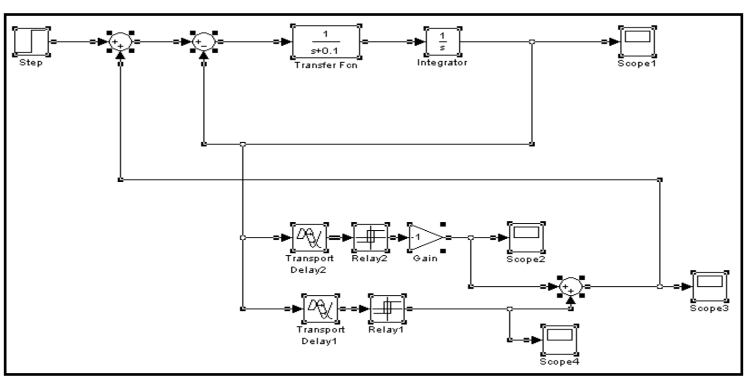

Figure 4.2: Simulation Diagram of Dead Beat System with pulse signal

5 **Theorems Related to Dead Beat Realization**

Theorem 5.1: The time to reach the reference input (t_{ref_input}) of the second order compensated system after the addition of pulse is

$$t_{ref\_input} = \frac{1}{k_3}\left[\Pi - \theta + \tan^{-1}\frac{k_2}{\left(\frac{a}{b} - k_1\right)}\right] \qquad (4)$$

$$k_1 = (e_1 \cos k_3 t_1 - e_2 \cos k_3 t_2) \qquad (5)$$

$$k_2 = (e_2 \sin k_3 t_2 - e_1 \sin k_3 t_1) \qquad (6)$$

$$e_1 = e^{\xi\omega_n t_1}, e_2 = e^{\xi\omega_n t_2} \qquad (7)$$

$$where\ k_3 = \varpi_n\sqrt{1-\xi^2} \qquad (8)$$

Proof:

If t_0 be the time to reach the reference input after the addition of pulse, from (3)

$$c(t) = [a - a\frac{e^{-\xi\omega_n t_0}}{\sqrt{1-\xi^2}}\sin\{\omega_n t_0\sqrt{1-\xi^2} + \theta\}] - [b - b\frac{e^{-\xi\omega_n (t_0 - t_1)}}{\sqrt{1-\xi^2}}\sin\{\omega_n (t_0 - t_1)\sqrt{1-\xi^2} + \theta\}] + [b - b\frac{e^{-\xi\omega_n (t_0 - t_2)}}{\sqrt{1-\xi^2}}\sin\{\omega_n (t_0 - t_2)\sqrt{1-\xi^2} + \theta\}] - a = 0 \qquad (9)$$

$$or\ -\frac{ae^{-\xi\omega_n t_0}}{\sqrt{1-\xi^2}}\sin\left\{\omega_n\sqrt{1-\xi^2}\,t_0 + \theta\right\} + b\frac{e^{-\xi\omega_n (t_0 - t_1)}}{\sqrt{1-\xi^2}}\sin\left\{\omega_n\sqrt{1-\xi^2}(t_0 - t_1) + \theta\right\} - b\frac{e^{-\xi\omega_n (t_0 - t_2)}}{\sqrt{1-\xi^2}}\sin\left\{\omega_n\sqrt{1-\xi^2}(t_0 - t_2) + \theta\right\} = 0 \qquad (10)$$

$$or\ -a\sin\{k_3 t_0 + \theta\} + be^{\xi\omega_n t_1}\sin\{k_3(t_0 - t_1) + \theta\} - be^{\xi\omega_n t_2}\sin\{k_3(t_0 - t_2) + \theta\} = 0 \qquad (11)$$

$$or\ b\begin{bmatrix} e_1\{\sin k_3 t_0 \cos k_3 t_1 - \cos k_3 t_0 \sin k_3 t_1\}\cos\theta + e_1\{\cos k_3 t_0 \cos k_3 t_1 + \sin k_3 t_0 \sin k_3 t_1\}\sin\theta - \\ e_2\{\sin k_3 t_0 \cos k_3 t_2 - \cos k_3 t_0 \sin k_3 t_2\}\cos\theta - e_2\{\cos k_3 t_0 \cos k_3 t_2 + \sin k_3 t_0 \sin k_3 t_2\}\sin\theta \end{bmatrix} = a\sin(k_3 t_0 + \theta) \qquad (12)$$

$$or\ e_1\cos k_3 t_1 \sin(k_3 t_0 + \theta) - e_1 \sin k_3 t_1 \cos(k_3 t_0 + \theta) - e_2 \cos k_3 t_2 \sin(k_3 t_0 + \theta) + e_2 \sin k_3 t_2 \cos(k_3 t_0 + \theta) = a\sin(k_3 t_0 + \theta) \qquad (13)$$

$$or\ k_1 \sin(k_3 t_0 + \theta) + k_2 \cos(k_3 t_0 + \theta) = a \sin(k_3 t_0 + \theta) \quad (14)$$

$$or\ bk_1 \tan(k_3 t_0 + \theta) + bk_2 = a \tan\left(k_3 t_0 + \theta\right) \quad (15)$$

$$or\ \tan(k_3 t_0 + \theta) = \frac{bk_2}{\left(a - bk_1\right)} \quad (16)$$

$$or\ \tan(k_3 t_0 + \theta) = \frac{k_2}{\left(\frac{a}{b} - k_1\right)} \quad (17)$$

$$or\ t_0 = t_{ref\_input} = \frac{1}{k_3}\left[\Pi - \theta + \tan^{-1}\left(\frac{k_2}{\frac{a}{b} - k_1}\right)\right] \quad (18)$$

Hence proved.

Remark 5.1: The result obtained in theorem 5.1 is used in developing theorem 5.2.

Theorem 5.2: The necessary condition of a second order linear system to be a dead-beat system in the SCT scheme is that, the output response of the compensated system must reaches to reference input at the time t_2 (the end time of the applied pulse) under the particular case, k_2=0

where k_2 is defined in theorem 5.1.

Remark 5.2:
It is obvious that other necessary conditions can also be obtained. The following necessary condition is obtained using theorem 5.2.
Proof:

It is assumed that there exist an overshoot after the time t_{ref_input} and occurs at t=$t_{ref\_input} + h$
From (16),

$$bk_2 \cos(k_3 t_{ref\_input} + \theta + h) - (a - bk_1)\sin(k_3 t_{ref\_input} + \theta + h) = c \quad (19)$$

$$or\ \tan(k_3 t_{ref\_input} + \theta + h) = \frac{bk_2}{(a - bk_1)} - \frac{c}{(a - bk_1)}\sec(k_3 t_{ref\_input} + \theta + h) \quad (23)$$

$$or\ (a - bk_1)\sin(k_3 t_{ref\_input} + \theta + h) - bk_2 \cos(k_3 t_{ref\_input} + \theta + h) + c = 0 \quad (20)$$

$$or\ (a - bk_1)\sin(k_3 t_{ref\_input} + \theta + h) = bk_2 \cos(k_3 t_{ref\_input} + \theta + h) - c \quad (21)$$

$$or\ (a - bk_1)\tan(k_3 t_{ref\_input} + \theta + h) = bk_2 - c.\sec(k_3 t_{ref\_input} + \theta + h) \quad (22)$$

$$or\ \tan(k_3 t_{ref\_input} + \theta + h) = \frac{bk_2}{(a - bk_1)} - \frac{c}{(a - bk_1)}\sec(k_3 t_{ref\_input} + \theta + h) \quad (23)$$

$$or\ \tan Y = X - \frac{c}{(a - bk_1)}\sqrt{1 + \tan^2 Y} \qquad (24)$$

$$where\ X = \frac{bk_2}{(a - bk_1)} \qquad (25)$$

$$or\ (\tan Y - X) = -c_1\sqrt{1 + \tan^2 Y} \qquad (26)$$

$$where\ c_1 = \frac{c}{(a - bk_1)} \qquad (27)$$

$$or\ (\tan Y - X)^2 = c_1^2(1 + \tan^2 Y) \qquad (28)$$

$$or\ \tan^2 Y - 2X\tan Y + X^2 = c_1^2(1 + \tan^2 Y) \qquad (29)$$

$$or\ (c_1^2 - 1)\tan^2 Y + 2X\tan Y + (c_1^2 - X^2) = 0 \qquad (30)$$

In particular case by comparing the LHS with RHS it can be said $(c_1^2 - X^2) = 0$ .From the middle term of L.H.S if X=0 is considered, then $c_1 = 0.\ From\ (3.46)\ \tan Y = X = 0$. As $c_1 = 0, it\ implies\ c = 0$ i.e. there should not be any overshoot after t_{ref_input} .By same way it can be proved also that there is no overshoot before t_{ref_input} and after t_2 .That means the output response of the compensated system always reach to reference input at the time t_2 and no overshoot occurs after t_2 . That implies $t_{ref\_input} = t_2$ .under the particular case, X=0, i.e. $k_2 = 0$

Remark 5.3:

The main implication of theorem 5.2 is that the applied pulse parameter t_2 can be selected as time to reach the reference input. In the subsequent development use this value of t_2 can be used.

6 Determination of Pulse Parameters

6.1. Choice of Pulse Parameters in Linear Second Order System:

It is assumed that the start time of the required pulse is the time where original uncompensated output response attains 90% of its reference input. If it is applied before this time the rise time of the output response of the system will be increased and IE (Integral Error) of the dead-beat system will be increased. With this choice of t_1 the other two parameters t_2 and b can be obtained as given in theorem 6.1.1.

Theorem 6.1.1: The pulse parameters i.e. start time (t_1), end time (t_2) and amplitude (b) of the applied additional pulse to get the dead-beat response of the second order linear system, can be obtained as

$$t_1 = \left[\frac{\Pi - \sin^{-1}\left\{0.1\sqrt{1 - \xi^2}\ e^{\xi\omega_n t_1}\right\} - \theta}{\omega_n\sqrt{1 - \xi^2}} \right] \qquad (31)$$

$$t_2 = \frac{1}{k_3}\left[\Pi - \sin^{-1}\{\exp(\log(\sin k_3 t_1)) + \xi\omega_n t_1 - \xi\omega_n t_2\}\right] \qquad (32)$$

$$b = \left(\frac{a}{k_1}\right) \qquad (33)$$

Proof:

As the start time t_1 is selected at the instant when the system response attains 90% of its final value the value of t_1 can be obtained as

$$c(t) = [a - \frac{ae^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\{\omega_n t\sqrt{1-\xi^2} + \theta\}] = 0.9a \;\; at \;\; t = t_1 \qquad (34)$$

$$t_1 = \left[\frac{\Pi - \sin^{-1}\left\{0.1\sqrt{1-\xi^2}\, e^{\xi\omega_n t_1}\right\} - \theta}{\omega_n\sqrt{1-\xi^2}}\right] \qquad (35)$$

The initial value of t_1 can be started with the value zero i.e. from (35) t_1 can be written as

$$t_1 = \left[\frac{\Pi - \sin^{-1}\left(0.1\sqrt{1-\xi^2}\right) - \theta}{\omega_n\sqrt{1-\xi^2}}\right] \qquad (36)$$

Now the iteration will be continued by putting the value of t_1 in (35) until the successive two values of t_1 in (35) will be same, i.e. the value of t_1 converges in (35).

From theorem 5.2 it is already known that $t_{ref\_input} = t_2$.But to obtain the value of t_2, the amplitude of the added pulse (b) must to be known. Hence to get the value of t_2, it is desirable to think in some other way, so that it is dependent on t_1 only.

From theorem 5.2, $k_2 = 0$ *i.e.* $e_2 \sin k_3 t_2 - e_1 \sin k_3 t_1 = 0$

$$\therefore \quad \xi\omega_n t_2 + \log(\sin(k_3 t_2)) = \xi\omega_n t_1 + \log(\sin(k_3 t_1)) \qquad (37)$$

$$or \quad \log(\sin(k_3 t_2)) = \xi\omega_n t_1 + \log(\sin(k_3 t_1)) - \xi\omega_n t_2 \qquad (38)$$

$$or \quad t_2 = \frac{1}{k_3}\left[\Pi - \sin^{-1}\{\exp(\log(\sin(k_3 t_1)) + \xi\omega_n t_1 - \xi\omega_n t_2\}\right] \qquad (39)$$

As $t_2 > t_1$, It is assumed that the initial solution of t_2 is t_1. The first iterated value of t_2 from (39) can be considered as $t_2 = \frac{1}{k_3}\left[\Pi - \sin^{-1}(\sin k_3 t_1)\right]$ (40)

Now the iteration will be continued by putting the value of t_2 (40) in (39) until the successive two values of t_2 in (39) will be same, i.e. the value of t_2 converges in (39). From the theorem 5.1 and theorem 5.2, it is understood that to make the second order linear System into dead-beat system through the injection of additional pulse input it can be made

$k_2 = 0$ that implies $(a - bk_1)\sin(k_3 t_r + \theta) = 0$ theorem 5.2. (41)

$$or\ b = \frac{a}{k_1} \qquad (42)$$

Hence proved.

Remark 6.1.1: The theorem 6.1.1 is the most important theorem that gives the necessary pulse parameters (t_1, t_2, b) for the dead-beat realization of second order linear system. The transient behaviour of the dead-beat system is very important. It is preferable that the transient response should be non-oscillating. This is guaranteed for the proposed dead-beat system as given in theorem 6.2.1.

6.2 Verification of Dead beat Response with Addition of Pulse Signal:

Theorem 6.2.1: The output response $c(t)$ of the second order dead-beat system must follow the following conditions

1. $c(t)$ is monotonically increasing for $0 \le t \le t_1$ *and* $t_1 \le t \le t_2$
2. $c(t) = u(t)$ *for* $t \ge t_2$, where $u(t)$ is the step input.

Remark 6.2.1:
It can be proved that the output response of the compensated dead-beat system from (1), (2) and (3) are non decreasing in time domain using the help of theorem 5.1 and theorem 5.2. The function defined in (1) is always non decreasing function as the start time t_1 is selected at the instant when the system response attains 90% of its final value. The function defined in (3) is also non-decreasing as it is proved in theorem 5.2. It is necessary to prove the transient response represented in (2) is a non decreasing function of t.

Proof:

The first order derivative of function defined in (2) can be written as

$$c'(t) = a\frac{\omega_n}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t\right] - b\frac{\omega_n}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,(t-t_1)\right] \quad for\ t_1 < t < t_2 \qquad (43)$$

$$we\ have\ to\ prove\ c'(t) \ge 0$$

$$i.e\ to\ prove\ a.\sin(k_3 t) - be^{\xi\omega_n t_1}\sin[k_3(t-t_1)] \ge 0 \qquad (44)$$

$$The\ above\ expression\ is \ge 0$$

$$\frac{\sin[k_3 t]}{\sin[k_3(t-t_1)]} \geq \frac{e_1}{e_1 \cos k_3 t_1 - e_2 \cos k_3 t_2} \qquad (45)$$

$$or\ \ if\ \ e_1 \sin k_3 t \cos k_3 t_1 - e_2 \sin k_3 t \cos k_3 t_2 \geq e_1 (\sin k_3 t \cos k_3 t_1 - \cos k_3 t \sin k_3 t_1) \quad (46)$$

$$or\ \ if\ \ -e_2 \sin k_3 t \cos k_3 t_2 \geq -e_1 \cos k_3 t \sin k_3 t_1 \qquad (47)$$

$$or\ \ if\ \ e_1 \cos k_3 t \sin k_3 t_1 \geq e_2 \sin k_3 t \cos k_3 t_2 \qquad (48)$$

$$if\ \ a \sin(k_3 t) \geq b e_1 \sin[k_3(t-t_1)] \qquad (49)$$

$$or\ \ if\ \ \frac{\sin[k_3 t]}{\sin[k(t-t_1)]} \geq \frac{b e_1}{a} \qquad (50)$$

$$or\ \ if\ \ \frac{\sin[k_3 t]}{\sin[k_3(t-t_1)]} \geq \frac{e_1}{k_1} \quad \left(\sin ce\ b = \frac{a}{k_1}\ from\ theorem\ 3\right) \qquad (51)$$

$$or\ \ if\ \ \frac{\sin[k_3 t]}{\sin[k_3(t-t_1)]} \geq \frac{e_1}{e_1 \cos k_3 t_1 - e_2 \cos k_3 t_2} \qquad (52)$$

$$or\ \ if\ \ e_1 \sin k_3 t \cos k_3 t_1 - e_2 \sin k_3 t \cos k_3 t_2 \geq e_1 (\sin k_3 t \cos k_3 t_1 - \cos k_3 t \sin k_3 t_1) \quad (53)$$

$$or\ \ if\ \ -e_2 \sin k_3 t \cos k_3 t_2 \geq -e_1 \cos k_3 t \sin k_3 t_1 \qquad (54)$$

$$or\ \ if\ \ e_1 \cos k_3 t \sin k_3 t_1 \geq e_2 \sin k_3 t \cos k_3 t_2 \qquad (55)$$

$$or\ \ if\ \ \frac{e_1}{e_2} \geq \frac{\sin k_3 t \cos k_3 t_2}{\cos k_3 t \sin k_3 t_1} \qquad (56)$$

$$or\ \ if\ \ \frac{e_1}{e_2} \geq \frac{\sin k_3 t \cos k_3 t_2}{\cos k_3 t \left(\frac{e_2}{e_1}\right) \sin k_3 t_2} \quad \text{(from theorem 5.1 and theorem 5.2)} \qquad (57)$$

$$or\ \ if\ \ \tan k_3 t_2 \geq \tan k_3 t \qquad (58)$$

$$or\ \ if\ \ \frac{\sin k_3 t_2}{\cos k_3 t_2} - \frac{\sin k_3 t}{\cos k_3 t} \geq 0 \qquad (59)$$

$$or\ \ if\ \ \sin k_3 (t_2 - t) \geq 0 \qquad (60)$$

$$or\ \ if\ \ \Pi \geq k_3 (t_2 - t) \geq 0 \qquad (61)$$

$$or\ \ if\ \ t \leq t_2 \leq \frac{\Pi}{k_3} \qquad (62)$$

Which is always true from (2).
Hence it is proved.

7 Simulation Results

The simulation diagram of dead-beat system is shown in figure 4.2. In the figure 4.2, it is shown that how the additional pulse has been obtained in the feedback loop of the uncompensated system to make the resultant output with dead-beat response. The parameters of the additional pulse, i.e. the start time (t_1), end time (t_2) and amplitude (b) are derived using theorem 6.1.1. The table 9.1 shows that using theorem 6.1.1, the pulse parameters are obtained for different systems. The resultant output responses of dead-beat systems are shown in figure 7.2 to figure 7.7 respectively for different systems of table 9.1.

Transient Responses of Different Second Order Systems with deadbeat realization

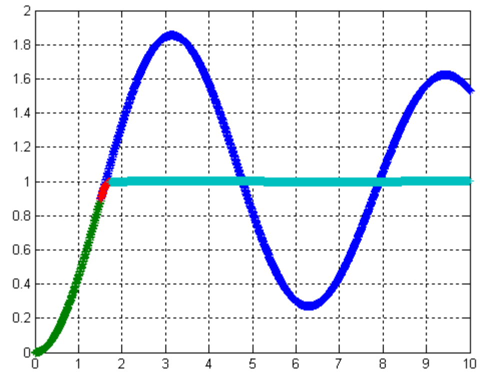

Figure 7.2. k=1. p=0.1, ζ=0.05

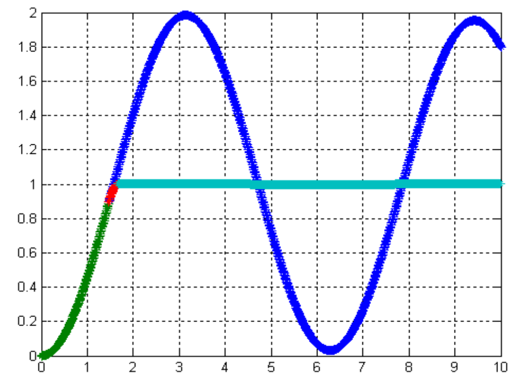

Figure 7.3. k=1. p=0.01, ζ=0.005

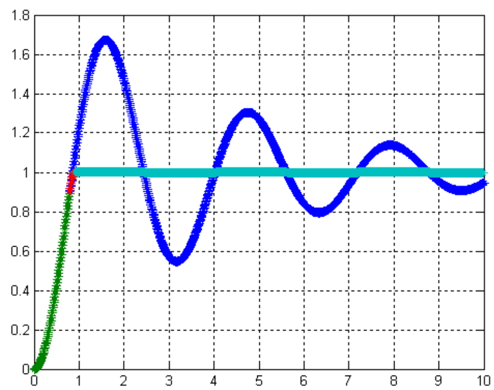

Figure 7.4. k=4. p=0.5,ζ=0.1250

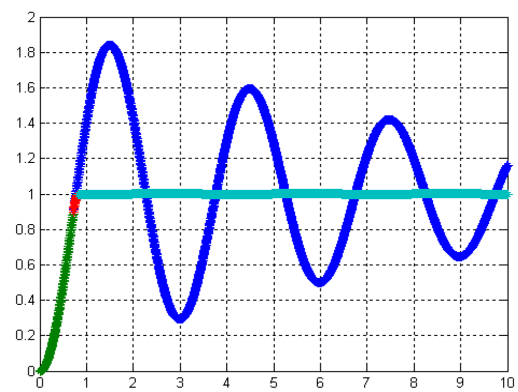

Figure 7.5. k=9. p=1.8, ζ=0.30

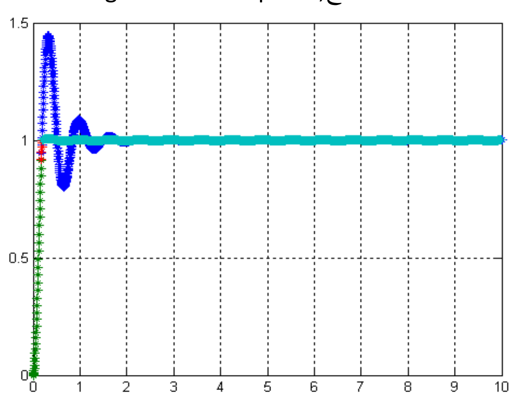

Figure 7.6. k=100. p=5, ζ=0.25

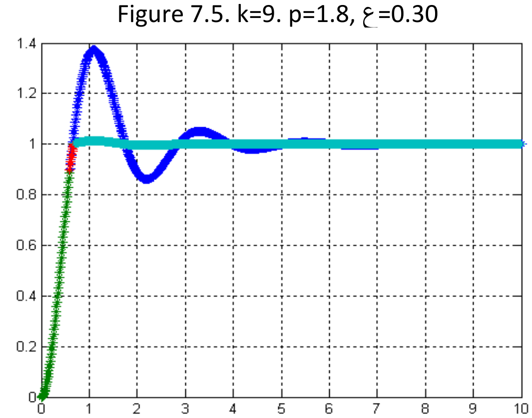

Figure 7.7. k=4.4324. p=0.2316, ζ=0.0550

8 Dead-beat Control Scheme for Higher Order Control System

This above technique of dead-beat realization can be applied to the higher order linear control system by converting the higher order system to the equivalent second order system and obtaining the pulse parameter values (t_1, t_2, b). This pulse, then, can be applied to the original higher order system (nth order system) for the almost dead-beat implementation. In this work the nth order linear control system are represented as second order linear system through section 9, by minimizing their square of error response with respect to two unknown residues of second order system (real or imaginary) by unchanging their corresponding poles.

8.1 Representation of Higher order Linear Continuous System in terms of Poles and Residues.

Theorem 8.1.1: Any higher order linear control system can be represented as

$$c(t) = -\left(\sum_{i=1}^{n} \alpha_i\right) + \sum_{i=1}^{n} \alpha_i e^{A_i t} \qquad (63)$$

$$or \quad c(t) = -\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i t} + 2\sum_{j=1}^{p} e^{C_j t}\left(\beta_j \cos D_j t - \gamma_j \sin D_j t\right) \qquad (64)$$

$$where \ n = m + 2p$$

$$or \quad c(t) = -2\sum_{j=1}^{p} \beta_j + 2\sum_{j=1}^{p} e^{C_j t}\left(\beta_j \cos D_j t - \gamma_j \sin D_j t\right), \ where \ n = 2p \qquad (65)$$

where

A_i (i = 1,2.....m)are the distinct real poles, α_i be the residue of the poles at $s = A_i$

c_j be the real part of complex poles, D_j be the imaginary part of complex poles.

β_j and γ_j be the real and imaginary residues of the complex poles.

Proof:

1. Both the poles are real poles
2. Both the poles are complex poles.

Case 1 : The output signal of second order linear system can be written as

$$\frac{\alpha_1}{s-A_1} + \frac{\alpha_2}{s-A_2} - \frac{(\alpha_1+\alpha_2)}{s} \qquad (66)$$

Taking the inverse Laplace of (66) it can be written as

$$L^{-1}\frac{\alpha_1}{s-A_1} + L^{-1}\frac{\alpha_2}{s-A_2} - L^{-1}\frac{(\alpha_1+\alpha_2)}{s} \qquad (67)$$

$$= \alpha_1 e^{A_1 t} + \alpha_2 e^{A_2 t} - (\alpha_1 + \alpha_2) \qquad (68)$$

$$= -\sum_{i=1}^{2} \alpha_i + \sum_{i=1}^{2} \alpha_i e^{A_i t} \qquad (69)$$

Case 2: The output signal of second order linear system can be written as

$$\frac{\beta_1 + i\gamma_1}{s-(C_1+iD_1)} + \frac{\beta_1 - i\gamma_1}{s-(C_1-iD_1)} - \frac{2\beta_1}{s} \qquad (70)$$

Taking inverse Laplace of (3.90)

$$L^{-1}\frac{\beta_1 + i\gamma_1}{s-(C_1+iD_1)} + L^{-1}\frac{\beta_1 - i\gamma_1}{s-(C_1-iD_1)} - L^{-1}\frac{2\beta_1}{s} \qquad (71)$$

$$= L^{-1}\left[\frac{2(s\beta_1 - C_1\beta_1 - \gamma_1 D_1)}{(s-c_1)^2 + D_1{}^2}\right] - L^{-1}\left[\frac{2\beta_1}{s}\right] \qquad (72)$$

$$= 2L^{-1}\left[\frac{s\beta_1}{(s-c_1)^2 + D_1{}^2}\right] - 2L^{-1}\left[\frac{C_1\beta_1 + \gamma_1 D_1}{(s-c_1)^2 + D_1{}^2}\right] - L^{-1}\left[\frac{2\beta_1}{s}\right] \qquad (73)$$

$$= 2\beta_1\left[e^{C_1 t}\cos D_1 t + \frac{C_1}{D_1}e^{C_1 t}\right] - 2(C_1\beta_1 + \gamma_1 D_1)\left[\frac{1}{D_1}e^{C_1 t}\sin D_1 t\right] - 2\beta_1 \qquad (74)$$

$$= -2\beta_1 + 2\beta_1 e^{C_1 t}\cos D_1 t + 2\frac{\beta_1 C_1}{D_1}e^{C_1 t}\sin D_1 t - 2\frac{\beta_1 C_1}{D_1}e^{C_1 t}\sin D_1 t - 2\gamma_1 e^{C_1 t}\sin D_1 t \qquad (75)$$

$$= -2\beta_1 + 2e^{C_1 t}(\beta_1\cos D_1 t - \gamma_1 \sin D_1 t) \qquad (76)$$

Similarly for the 3rd order system of all the poles of the system are real, the output c(t) of the system is defined as

$$= -\sum_{i=1}^{3}\alpha_i + \sum_{i=1}^{3}\alpha_i e^{A_i t} \qquad (77)$$

For one real pole and two complex poles the output signal of 3rd order linear system can be defined as

$$\frac{\alpha_1}{s-A_1} + \frac{\beta_1 + i\gamma_1}{s-(C_1+iD_1)} + \frac{\beta_1 - i\gamma_1}{s-(C_1-iD_1)} - \frac{(\alpha_1 + 2\beta_1)}{s} \qquad (78)$$

Taking the inverse Laplace of (78) we can get

$$L^{-1}\frac{\alpha_1}{s-A_1} + L^{-1}\frac{\beta_1 + i\gamma_1}{s-(C_1+iD_1)} + L^{-1}\frac{\beta_1 - i\gamma_1}{s-(C_1-iD_1)} - L^{-1}\frac{(\alpha_1 + 2\beta_1)}{s} \qquad (79)$$

From (3.89) *and* (3.96)

$$c(t) = -(\alpha_1 + 2\beta_1) + \alpha_1 e^{A_1 t} + 2e^{C_1 t}(\beta_1\cos D_1 t - \gamma_1 \sin D_1 t) \qquad (80)$$

Hence for nth order system

$$c(t) = -\left(\sum_{i=1}^{n} \alpha_i\right) + \sum_{i=1}^{n} \alpha_i e^{A_i t} \qquad (81)$$

Where all poles are real

or

$$c(t) = -\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i t} + 2\sum_{j=1}^{p} e^{C_j t}\left(\beta_j \cos D_j t - \gamma_j \sin D_j t\right) \qquad (82)$$

where m number of real poles and 2p number of complex poles and $(m + 2p)$ is equal to n.

or

$$c(t) = -2\sum_{j=1}^{p} \beta_j + 2\sum_{j=1}^{P} e^{C_j t}\left(\beta_j \cos D_j t - \gamma_j \sin D_j t\right) \qquad (83)$$

where all poles are complex and $2p = n$

Hence proved.

Therefore, from (62) the nth order compensated linear control system can be represented as

$$c(t) = -\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i t} + 2\sum_{j=1}^{p} e^{C_j t}\left(\beta_j \cos D_j t - \gamma_j \sin D_j t\right) \qquad (84)$$

$$for\ 0 < t < t_1$$

$$c(t) = -\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i t} + 2\sum_{j=1}^{p} e^{C_j t}\left(\beta_j \cos D_j t - \gamma_j \sin D_j t\right)$$
$$-b\left[-\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i (t-t_1)} + 2\sum_{j=1}^{p} e^{C_j (t-t_1)}\left(\beta_j \cos D_j (t-t_1) - \gamma_j \sin D_j (t-t_1)\right)\right] \qquad (85)$$

$$for\ t_1 < t < t_2$$

$$c(t) = -\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i t} + 2\sum_{j=1}^{p} e^{C_j t}\left(\beta_j \cos D_j t - \gamma_j \sin D_j t\right)$$
$$-b\left[-\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i (t-t_1)} + 2\sum_{j=1}^{p} e^{C_j (t-t_1)}\left(\beta_j \cos D_j (t-t_1) - \gamma_j \sin D_j (t-t_1)\right)\right]$$
$$+b\left[-\left(\sum_{i=1}^{m} \alpha_i + 2\sum_{j=1}^{p} \beta_j\right) + \sum_{i=1}^{m} \alpha_i e^{A_i (t-t_2)} + 2\sum_{j=1}^{p} e^{C_j (t-t_2)}\left(\beta_j \cos D_j (t-t_2) - \gamma_j \sin D_j (t-t_2)\right)\right]$$

$$for\ t_2 < t < \infty \qquad (86)$$

8.2 Representation of Theorem 5.1, 5.2 and 6.1.1 in terms of Poles for Linear Second Order System.

The time to reach the reference input (t_{ref_input}) of the second order compensated system (after the addition of pulse) stated in theorem 5.1 can be represented in terms of poles as follows.

$$t_{ref\_input} = \frac{\left[\pi - \theta + \tan^{-1}\left\{\dfrac{1-b\left(e_1 \cos D_1 t_1 - e_2 \cos D_1 t_2\right)}{b\left(e_1 \sin D_1 t_1 - e_2 \sin D_1 t_2\right)}\right\}\right]}{D_1} \qquad (87)$$

$$where\ e_1 = e^{-C_1 t_1},\ where\ e_2 = e^{-C_1 t_2} \qquad (88)$$

Proof:

If t_0 be the time to reach the reference input after the addition of pulse, from (76)

$$-2\beta_1 + 2e^{C_1 t}\left(\beta_1 \cos D_1 t_0 - \gamma_1 \sin D_1 t_0\right) - b\left\{-2\beta_1 + 2e^{C_1 t_0}\left(\beta_1 \cos D_1 t_0 - \gamma_1 \sin D_1 t_0\right)\right\}$$

$$+ b\left\{-2\beta_1 + 2e^{C_1 t_0}\left(\beta_1 \cos D_1 t_0 - \gamma_1 \sin D_1 t_0\right)\right\} - 2\beta_1 = 0 \qquad (89)$$

$$or\ 2e^{C_1 t_0}\left(\beta_1 \cos D_1 t_0 - \gamma_1 \sin D_1 t_0\right) - 2be^{C_1\left(t_0 - t_1\right)}\left\{\beta_1 \cos D_1\left(t_0 - t_1\right) - \gamma_1 \sin D_1\left(t_0 - t_1\right)\right\}$$

$$+ 2be^{C_1\left(t - t_2\right)}\left\{\beta_1 \cos D_1\left(t_0 - t_2\right) - \gamma_1 \sin D_1\left(t_0 - t_2\right)\right\} = 0 \qquad (90)$$

$$or\ e^{C_1 t}\left(\beta_1 \cos D_1 t_0 - \gamma_1 \sin D_1 t_0\right) - be^{C_1\left(t_0 - t_1\right)}\left\{\beta_1 \cos D_1\left(t_0 - t_1\right) - \gamma_1 \sin D_1\left(t_0 - t_1\right)\right\}$$

$$+ be^{C_1\left(t_0 - t_2\right)}\left\{\beta_1 \cos D_1\left(t_0 - t_2\right) - \gamma_1 \sin D_1\left(t_0 - t_2\right)\right\} = 0 \qquad (91)$$

or

$$e^{C_1 t} r\left(\cos\theta \cos D_1 t_0 - r \sin\theta \sin D_1 t_0\right) - be^{C_1\left(t_0 - t_1\right)} r\left\{\cos\theta \cos D_1\left(t_0 - t_1\right) - r \sin\theta \sin D_1\left(t_0 - t_1\right)\right\}$$

$$+ be^{C_1\left(t_0 - t_2\right)} r\left\{\cos\theta \cos D_1\left(t_0 - t_2\right) - r \sin\theta \sin D_1\left(t_0 - t_2\right)\right\} = 0 \qquad (92)$$

$$or\ e^{C_1 t} \cos\left(D_1 t_0 + \theta\right) - be^{C_1\left(t_0 - t_1\right)} \cos\left\{D_1\left(t_0 - t_1\right) + \theta\right\} + be^{C_1\left(t_0 - t_2\right)} \cos\left\{D_1\left(t_0 - t_2\right) + \theta\right\} = 0 \qquad (93)$$

$$or\ \cos\left(D_1 t_0 + \theta\right) = be^{-C_1 t_1} \cos\left\{D_1\left(t_0 - t_1\right) + \theta\right\} - be^{-C_1 t_2} \cos\left\{D_1\left(t_0 - t_2\right) + \theta\right\} \qquad (94)$$

$$or\ \cos\left(D_1 t_0 + \theta\right) = b\left[e_1 \cos\left\{D_1\left(t_0 - t_1\right) + \theta\right\} - e_2 \cos\left\{D_1\left(t_0 - t_2\right) + \theta\right\}\right] \qquad (95)$$

$$or\ \cos(D_1t_0+\theta)=b\begin{bmatrix}e_1\{\cos D_1(t_0-t_1)\cos\theta-\sin D_1(t_0-t_1)\sin\theta\}\\ -e_2\{\cos D_1(t_0-t_2)\cos\theta-\sin D_1(t_0-t_2)\sin\theta\}\end{bmatrix} \quad (96)$$

$$or\ \cos(D_1t_0+\theta)=be_1\{\cos D_1t_0\cos D_1t_1+\sin D_1t_0\sin D_1t_1\}\cos\theta$$

$$\begin{aligned}&-be_1\{\sin D_1t_0\cos D_1t_1-\cos D_1t_0\sin D_1t_1\}\sin\theta\\&-be_2\{\cos D_1t_0\cos D_1t_2+\sin D_1t_0\sin D_1t_2\}\cos\theta\\&+be_2\{\sin D_1t_0\cos D_1t_2-\cos D_1t_0\sin D_1t_2\}\sin\theta \quad (97)\end{aligned}$$

$$\begin{aligned}&or\ \cos(D_1t_0+\theta)=b(e_1\cos D_1t_1-e_2\cos D_1t_2)\cos(D_1t_0+\theta)\\&+b(e_1\cos D_1t_1-e_2\cos D_1t_2)\sin(D_1t_0+\theta) \quad (98)\end{aligned}$$

$$or\ b(e_1\sin D_1t_1-e_2\sin D_1t_2)\sin(D_1t_0+\theta)=\{1-b(e_1\cos D_1t_1-e_2\cos D_1t_2)\}\cos(D_1t_0+\theta) \quad (99)$$

$$or\ \tan(D_1t_0+\theta)=\frac{\{1-b(e_1\cos D_1t_1-e_2\cos D_1t_2)\}}{b(e_1\sin D_1t_1-e_2\sin D_1t_2)} \quad (100)$$

$$t_0=\frac{\left[\pi-\theta+\tan^{-1}\left\{\frac{1-b(e_1\cos D_1t_1-e_2\cos D_1t_2)}{b(e_1\sin D_1t_1-e_2\sin D_1t_2)}\right\}\right]}{D_1} \quad (101)$$

Hence proved.

The necessary condition stated in theorem 5.2 for $t_{ref\_input}=t_2$ can be drawn as

$$1-b(e_1\cos D_1t_1-e_2\cos D_1t_2)=0 \quad (102)$$

where e_1 *and* e_2 *are given in* (88)

Proof:

It is assumed that there exist an overshoot after the t_{ref_input} time and occurs at t=$t_{ref\_input}+h$

From (100), it can be written as

$$\begin{aligned}&b(e_1\sin D_1t_1-e_2\sin D_1t_2)\sin(D_1t+\theta+h)-\cos(D_1t+\theta+h)\\&+b(e_1\cos D_1t_1-e_2\cos D_1t_2)\cos(D_1t+\theta+h)=c \quad (103)\\&or\ b(e_1\sin D_1t_1-e_2\sin D_1t_2)\sin(D_1t+\theta+h)-\cos(D_1t+\theta+h)\\&+b(e_1\cos D_1t_1-e_2\cos D_1t_2)\cos(D_1t+\theta+h)-c=0 \quad (104)\\&or\ b(e_1\sin D_1t_1-e_2\sin D_1t_2)\tan Y=1+c.\sec Y-b(e_1\cos D_1t_1-e_2\cos D_1t_2) \quad (105)\end{aligned}$$

$$\begin{aligned}&or\ \tan Y=\left\{\frac{1}{b(e_1\sin D_1t_1-e_2\sin D_1t_2)}-\frac{(e_1\cos D_1t_1-e_2\cos D_1t_2)}{(e_1\sin D_1t_1-e_2\sin D_1t_2)}\right\}\\&+\left\{\frac{c.\sec Y}{b(e_1\sin D_1t_1-e_2\sin D_1t_2)}\right\} \quad (106)\end{aligned}$$

$$or \quad \tan Y = \left\{ \frac{1 - b(e_1 \cos D_1 t_1 - e_2 \cos D_1 t_2)}{b(e_1 \sin D_1 t_1 - e_2 \sin D_1 t_2)} \right\} + \left\{ \frac{c.\sec Y}{b(e_1 \sin D_1 t_1 - e_2 \sin D_1 t_2)} \right\} \qquad (107)$$

$$or \ \tan Y = X + c_1 \sqrt{1 + \tan^2 Y} \qquad (108)$$

$$or \ (\tan Y - X) = c_1 \sqrt{1 + \tan^2 Y} \qquad (109)$$

$$or \ \tan^2 Y - 2X \tan Y + Y^2 = c_1^2 (1 + \tan^2 y) \quad (110)$$

$$(c_1^2 - 1)\tan^2 y + 2X \tan Y + (c_1^2 - X^2) = 0 \qquad (111)$$

In particular case by comparing the LHS with RHS it can be said $(c_1^2 - X^2) = 0$ .From the middle term of L.H.S if X=0 is considered, then $c_1 = 0$. From (109) $\tan Y = X = 0$. As $c_1 = 0, it\ implies\ c = 0$ i.e. there should not be any overshoot after t_{ref_input}.By same way it can be proved also that there is no overshoot before t_{ref_input} and after t_2.That means the output response of the compensated system always reach to reference input at the time t_2 and no overshoot occurs after t_2. That implies $t_{ref\_input} = t_2$ .under the particular case, X=0, i.e.

$$1 - b(e_1 \cos D_1 t_1 - e_2 \cos D_1 t_2) = 0$$

The pulse parameters i.e. start time (t_1), end time (t_2) and amplitude (b) of the applied additional pulse to get the dead-beat response of the second order linear system stated in theorem 6.1.1 can be represented in terms of poles as

$$t_1 = \frac{\{\cos^{-1}(0.1\cos\theta\, e^{-C_1 t}) - \theta\}}{D_1} \qquad (112)$$

$$t_2 = \frac{\pi - \sin^{-1}[\exp\{c_1 t_2 - c_1 t_1 + \log(\sin D_1 t_1)\}]}{D_1} \qquad (113)$$

$$b = \frac{1}{e_1 \cos D_1 t_1 - e_2 \cos D_1 t_2} \qquad (114)$$

$$\text{From theorem } 5.2,\ k_2 = 0\ i.e.\ e_2 \sin k_3 t_2 - e_1 \sin k_3 t_1 = 0 \qquad (115)$$

$$or \ (e_1 \sin D_1 t_1 - e_2 \sin D_1 t_2) = 0 \qquad (116)$$

$$or \ e_1 \sin D_1 t_1 = e_2 \sin D_1 t_2 \qquad (117)$$

$$or \ \log e_1 + \log(\sin D_1 t_1) = \log e_2 + \log(\sin D_1 t_2) - c_1 t_1 + \log(\sin D_1 t_1)$$

$$= -c_1 t_2 + \log(\sin D_1 t_2) \quad (118)$$

$$or \ \log(\sin D_1 t_2) = c_1 t_2 - c_1 t_1 + \log(\sin D_1 t_1) \qquad (119)$$

$$or \ \sin D_1 t_2 = \exp\{c_1 t_2 - c_1 t_1 + \log(\sin D_1 t_1)\} \qquad (120)$$

$$or \ t_2 = \frac{\pi - \sin^{-1}[\exp\{c_1 t_2 - c_1 t_1 + \log(\sin D_1 t_1)\}]}{D_1} \qquad (121)$$

$$if\ the\ initial\ solution\ of\ t_2\ is\ t_1$$

$$then\ t_2 = \frac{\pi - \sin^{-1}(\sin D_1 t_1)}{D_1} \qquad (122)$$

Now the iteration will be continued by putting the value of (122) in (121) until the successive two values of in (121) will be same, i.e. the value of converges in (121). From the theorem 5.1 and theorem 5.2, it is understood that to make the second order linear system into dead-beat system through the injection of additional pulse signal, the following equation can be obtained.

$$\frac{1}{b(e_1 \sin D_1 t_1 - e_2 \sin D_1 t_2)} - \frac{(e_1 \cos D_1 t_1 - e_2 \cos D_1 t_2)}{(e_1 \sin D_1 t_1 - e_2 \sin D_1 t_2)} = 0 \qquad (123)$$

$$or\ b = \frac{1}{e_1 \cos D_1 t_1 - e_2 \cos D_1 t_2} \qquad (124)$$

Here a following lemma can be proposed, which will be used in theorem 9.1.1.

Lemma 8.2.1: For any output response of any second order linear system in the form of with the $Y(s) = \left(\frac{k}{1 + 2 * \mathrm{Zeta} * \mathrm{Tw} * \mathrm{s} + \mathrm{Tw}^2 * \mathrm{s}^2}\right)\frac{1}{s}$ step input the following conditions hold.

$$(i)\ \beta_1 C_1 = \gamma_1 D_1 \qquad (125)$$

$$or\ (ii)\ \alpha_1 A_1 = -\alpha_2 A_2 \qquad (126)$$

Proof (i): It is assumed that both the poles are complex.

$$T(s) = \frac{Y(s)}{u(s)} = \left(\frac{k}{1 + 2 * \mathrm{Zeta} * \mathrm{Tw} * \mathrm{s} + (\mathrm{Tw} * \mathrm{s})\text{^}2}\right) \qquad (127)$$

$$Y(s) = \left(\frac{k}{1 + 2 * \mathrm{Zeta} * \mathrm{Tw} * \mathrm{s} + (\mathrm{Tw} * \mathrm{s})\text{^}2}\right) \qquad (128)$$

$$Y(s) = \frac{\beta_1 + i\gamma_1}{s - (C_1 + iD)} + \frac{\beta_1 - i\gamma_1}{s - (C_1 - iD_1)} - \frac{2\beta_1}{s} \qquad (129)$$

$$or\ Y(s) = \frac{\beta_1 s - \beta_1 C_1 + i\beta_1 D_1 + i\gamma_1 s - i\gamma_1 C_1 - D_1\gamma_1 + \beta_1 s - \beta_1 C_1 - i\beta_1 D_1 - i\gamma_1 s + i\gamma_1 C_1 - D_1\gamma_1}{s^2 - 2sC_1 + (C_1^2 + D_1^2)} - \frac{2\beta_1}{s} \qquad (130)$$

$$or\ Y(s) = \frac{2\beta_1 s - 2\beta_1 C_1 - 2D_1\gamma_1}{s^2 - 2sC_1 + (C_1^2 + D_1^2)} - \frac{2\beta_1}{s} \qquad (131)$$

$$or\ Y(s) = \frac{2\beta_1 s^2 - 2\beta_1 C_1 s - 2D_1\gamma_1 s - 2\beta_1 s^2 + 4\beta_1 C_1 s - 2\beta_1 (C_1^2 + D_1^2)}{s\{s^2 - 2sC_1 + (C_1^2 + D_1^2)\}} \qquad (132)$$

$$or\ Y(s) = \frac{2\beta_1 C_1 s - 2D_1\gamma_1 s - 2\beta_1 (C_1^2 + D_1^2)}{s\{s^2 - 2sC_1 + (C_1^2 + D_1^2)\}} \qquad (133)$$

$$or\ Y(s)=\frac{2\left(\beta_1 C_1 - D_1\gamma_1\right)s - 2\beta_1\left(C_1{}^2 + D_1{}^2\right)}{s\left\{s^2 - 2sC_1 + \left(C_1{}^2 + D_1{}^2\right)\right\}} \qquad (134)$$

It is implied from (128) and (134)

$$\left(\beta_1 C_1 - \gamma_1 D_1\right) = 0$$
$$or\ \ \beta_1 C_1 = \gamma_1 D_1\ \ proved$$

Again to prove condition (ii) let us consider both the poles are real.

$$Y(s)=\frac{\alpha_1}{s-A_1}+\frac{\alpha_2}{s-A_2}-\frac{\alpha_1+\alpha_2}{s} \qquad (135)$$

$$Y(s)=\frac{\alpha_1 s-\alpha_1 A_2+\alpha_2 s-\alpha_2 A_1}{(s-A_1)(s-A_2)}-\frac{\left(\alpha_1+\alpha_2\right)}{s} \qquad (136)$$

$$or\ Y(s)=\frac{\alpha_1 s^2-\alpha_1 A_2 s+\alpha_2 s^2-\alpha_2 A_1 s-\left(s^2-A_1 s-A_2 s+A_1 A_2\right)\left(\alpha_1+\alpha_2\right)}{\left(s-A_1\right)\left(s-A_2\right)s} \qquad (137)$$

$$or\ Y(s)=\frac{\alpha_1 A_1 s+\alpha_2 A_2 s-A_1 A_2 \alpha_1 - A_1 A_2\alpha_2}{\left(s-A_1\right)\left(s-A_2\right)s} \qquad (138)$$

$$\mathrm{or}\ \mathrm{Y(s)}=\frac{\left(\alpha_1 A_1+\alpha_2 A_2\right)s-A_1 A_2\alpha_1-A_1 A_2\alpha_2}{\left(s-A_1\right)\left(s-A_2\right)s}\ \text{from} \qquad (139)$$

$$\text{from (89) and(100)}\ \ \left(\alpha_1 A_1+\alpha_2 A_2\right)=0 \qquad (140)$$

$$or\ \ \alpha_1 A_1=-\alpha_2 A_2\ \ proved \qquad (141)$$

9 Conversion of nth Order System to Equivalent 2nd Order System

Every physical system can be translated into mathematical model. The mathematical procedure of system modeling often leads to comprehensive description of a process in the form of high-order differential equations which are difficult to use either for analysis or controller synthesis. It is, therefore, useful, and sometimes necessary, to find the possibility of finding some equations of the same type but of lower order that may be considered to adequately reflect the dominant characteristics of the system under consideration. Some of the reasons for using reduced-order models of high order linear systems could be as follows:

- To have a better understanding of the system,
- To reduce computational complexity,
- To reduce hardware complexity,
- To make feasible controller design.

Many control system applications, such as satellite altitude control, fighter aircraft control, model-based predictive control, control of fuel injectors, automobile spark timer, possess a mathematical model of the process with higher order, due to which the system defined becomes complex. These higher order models are cumbersome to handle. As a result, lower order system modeling can be performed, which helps in alleviating computational complexity and implementation difficulties involved in the design of controllers and compensators for higher order systems. Further, the development and usage of micro controllers and microprocessors in the design and implementation of control system components has increased the importance of lower order system modeling.

In recent decades, much effort has been made in the field of model order reduction for linear dynamic systems and several methods like: Aggregation method [18], Pade approximation [19], Routh approximation [20], Moment matching technique [21], Mihailov stability criterion [22], and optimization technique [23], have been proposed. Among them Routh approximation and Pade technique has been recognized as the powerful method. But the serious disadvantage of Pade approximation is that sometimes it leads to an unstable reduced order system for a stable original system. Further, numerous methods of order reduction are also available in the literature [24-31], which are based on minimization of the ISE criterion. However, a familiar aspect in the methods explained in [24-30] is that the denominator coefficients values of the low order system (LOS) are selected arbitrarily by some stability preserving methods such as dominant pole, Routh approximation methods, etc. and then the numerator coefficients of the LOS are determined by minimization of the ISE. In [31], Howitt and Luss recommended a procedure, in which both the numerator and denominator coefficients are considered to be free parameters and are chosen to minimize the ISE in impulse or step responses.

9.1 Reduction to Second Order Linear System without Changing the Poles

Any nth order linear control system can be represented as second order linear system by minimizing their mean square error of output responses with respect to two unknown residues of second order system (real or imaginary) without changing their corresponding poles. In this method, a complex conjugate pole, of the original higher order system, nearest to the origin is considered. The residues of corresponding poles (β, γ) are found by minimizing the mean square error between the transient responses of the original higher system and the reduced second order system. This method is discussed with the numerical examples along with the results using theorem 9.1.1.

Theorem 9.1.1: Any nth order linear control system can be represented as second order linear system by minimizing the mean square error of their output responses with respect to two unknown residues of second order system (real or imaginary) without changing their corresponding poles.

Proof:

Case I: The equivalent second order system with two imaginary poles.

$$MSE=\int_0^\infty\left[\begin{array}{l}\left\{-\left(\sum_{i=1}^{m}\alpha_i+2\sum_{j=1}^{p}\beta_j\right)+\sum_{i=1}^{m}\alpha_i e^{A_1 t}+2\sum_{j=1}^{p}e^{C_j t}\left(\beta_j\cos D_j t-\gamma_j\sin D_j t\right)\right\}\\-\left\{-2\beta_j'+2e^{C_j t}\left(\beta_j'\cos D_j t-\gamma_j'\sin D_j t\right)\right\}\end{array}\right]^2 dt \qquad (142)$$

$$=\int_0^\infty\left[\begin{array}{l}\sum_{i=1}^{m}\alpha_i e^{A_1 t}+2e^{C_j t}\left(\beta_j-\beta_j'\right)\cos D_j t+2e^{C_j t}\left(\gamma_j'-\gamma_j\right)\sin D_j t\\+2\sum_{\substack{k=1\\k\neq j}}^{p}\left(\beta_k\cos D_k t-\gamma_k\sin D_k t\right)\end{array}\right]^2 dt \qquad (143)$$

$$=\int_0^\infty\left(\sum_{i=1}^{m}\alpha_i e^{A_1 t}\right)^2 dt+4\int_0^\infty e^{2C_j t}\left(\beta_j-\beta_j'\right)^2\cos^2 D_j t dt+4\int_0^\infty e^{2C_j t}\left(\gamma_j'-\gamma_j\right)^2\sin^2 D_j t dt$$

$$+4\int_0^\infty\sum_{\substack{k=1\\k\neq j}}^{p}\left(\beta_k\cos D_k t-\gamma_k\sin D_k t\right)^2 dt+8\int_0^\infty\sum_{k=1}^{p}\sum_{\substack{l=1\\k,l\neq j,k<l}}^{p}\left(\beta_k\cos D_k t-\gamma_k\sin D_k t\right)\left(\beta_l\cos D_l t-\gamma_l\sin D_l t\right)dt$$

$$+4\int_0^\infty\sum_{i=1}^{m}\alpha_i e^{A_i t}\sum_{\substack{k=1\\k\neq j}}^{p}\left(\beta_k\cos D_k t-\gamma_k\sin D_k t\right)+8\int_0^\infty e^{C_j t}\left(\beta_j-\beta_j'\right)\cos D_j t\sum_{\substack{k=1\\k\neq j}}^{p}\left(\beta_k\cos D_k t-\gamma_k\sin D_k t\right)dt$$

$$+8\int_0^\infty e^{C_j t}\left(\gamma_j'-\gamma_j\right)\sin D_j t\sum_{\substack{k=1\\k\neq j}}^{p}\left(\beta_k\cos D_k t-\gamma_k\sin D_k t\right)dt$$

$$+4\int_0^\infty\left(\sum_{i=1}^{m}\alpha_i e^{A_1 t}\right)e^{C_j t}\left(\beta_j-\beta_j'\right)\cos D_j t dt$$

$$+4\int_0^\infty\left(\sum_{i=1}^{m}\alpha_i e^{A_1 t}\right)e^{C_j t}\left(\gamma_j-\gamma_j'\right)\sin D_j t dt$$

$$+8\int_0^\infty e^{2C_j t}\left(\beta_j-\beta_j'\right)\left(\gamma_j'-\gamma_j\right)\cos D_j t\sin D_j t dt \qquad (144)$$

$$=2\left(\beta_j-\beta_j'\right)^2\int_0^\infty\left(1+\cos 2D_j t\right)e^{2C_j t}dt+2\left(\gamma_j'-\gamma_j\right)^2\int_0^\infty\left(1-\cos 2D_j t\right)e^{2C_j t}dt$$

$$= 2\left(\beta_j - \beta_j^{'}\right)^2 \int_0^{\infty} e^{2C_j t} dt - 2\left(\beta_j - \beta_j^{'}\right)^2 \int_0^{\infty} e^{2C_j t} \cos 2D_j t dt + 2\left(\gamma_j^{'} - \gamma_j\right)^2 \int_0^{\infty} e^{2C_j t} dt$$

$$- 2\left(\gamma_j^{'} - \gamma_j\right)^2 \int_0^{\infty} e^{2C_j t} \cos 2D_j t dt + 4\left(\beta_j - \beta_j^{'}\right) \int_0^{\infty} e^{C_j t} \left[\sum_{\substack{k=1 \\ k \neq j}}^{p} \beta_k \left\{ \cos\left(D_k + D_j\right)t + \cos\left(D_k - D_j\right)t \right\} \right] dt$$

$$+ 4\left(\gamma_j^{'} - \gamma_j\right) \int_0^{\infty} e^{C_j t} \left[\sum_{\substack{k=1 \\ k \neq j}}^{p} \beta_k \left\{ \sin\left(D_k + D_j\right)t + \sin\left(D_j - D_k\right)t \right\} \right] dt$$

$$+ 8\left(\beta_j - \beta_j^{'}\right) \int_0^{\infty} e^{C_j t} \sum_{\substack{k=1 \\ k \neq j}}^{p} \beta_k \cos D_k t \cos D_j t dt + 8\left(\gamma_j^{'} - \gamma_j\right) \int_0^{\infty} e^{C_j t} \sum_{\substack{k=1 \\ k \neq j}}^{p} \beta_k \cos D_k t \sin D_j t dt$$

$$- 8\left(\beta_j - \beta_j^{'}\right) \int_0^{\infty} e^{C_j t} \sum_{\substack{k=1 \\ k \neq j}}^{p} \gamma_k \sin D_k t \cos D_j t dt - 8\left(\gamma_j^{'} - \gamma_j\right) \int_0^{\infty} e^{C_j t} \sum_{\substack{k=1 \\ k \neq j}}^{p} \gamma_k \sin D_k t \sin D_j t dt$$

$$+ 4\left(\beta_j - \beta_j^{'}\right) \int_0^{\infty} \sum_{i=1}^{m} \alpha_i e^{\left(A_i + C_j\right)t} \cos D_j t dt + 4\left(\gamma_j^{'} - \gamma_j\right) \int_0^{\infty} \sum_{i=1}^{m} \alpha_i e^{\left(A_i + C_j\right)t} \sin D_j t dt$$

$$+ 8e^{2C_j t} \left(\beta_j - \beta_j^{'}\right)\left(\gamma_j^{'} - \gamma_j\right) \int_0^{\infty} \cos D_j t \sin D_j t dt + \int_0^{\infty} K_1(t) dt \tag{145}$$

where $K_1(t)$ is a function, independent of $\beta^{'}$ and $\gamma^{'}$.

$$+ 4\left(\gamma_j^{'} - \gamma_j\right) \int_0^{\infty} e^{C_j t} \left[\sum_{\substack{k=1 \\ k \neq j}}^{p} \beta_k \left\{ \sin\left(D_k + D_j\right)t + \sin\left(D_j - D_k\right)t \right\} \right] dt$$

$$- 4\left(\beta_j - \beta_j^{'}\right) \int_0^{\infty} e^{C_j t} \left[\sum_{\substack{k=1 \\ k \neq j}}^{p} \gamma_k \left\{ \sin\left(D_k + D_j\right)t + \sin\left(D_k - D_j\right)t \right\} \right] dt$$

$$- 4\left(\gamma_j^{'} - \gamma_j\right) \int_0^{\infty} e^{C_j t} \left[\sum_{\substack{k=1 \\ k \neq j}}^{p} \beta_k \left\{ \cos\left(D_k - D_j\right)t - \cos\left(D_k + D_j\right)t \right\} \right] dt$$

$$+4\left(\beta_j-\beta_j^{'}\right)\int_0^{\infty}\left\{\sum_{i=1}^{m}\alpha_i e^{\left(A_i+C_j\right)t}\cos D_j t\right\}dt+4\left(\gamma_j^{'}-\gamma_j\right)\int_0^{\infty}\left\{\sum_{i=1}^{m}\alpha_i e^{\left(A_i+C_j\right)t}\sin D_j t\right\}dt$$

$$+4e^{2C_j t}\left(\beta_j-\beta_j^{'}\right)\left(\gamma_j^{'}-\gamma_j\right)\int_0^{\infty}\sin 2D_j t dt+\int_0^{\infty}K_1(t)dt \qquad (146)$$

The above expression is integrable. $K_1(t)$ is the function which is independent of $\beta_j^{'}$ *and* $\gamma_j^{'}$ After the integration the two unknown residues $\beta_j^{'}$ *and* $\gamma_j^{'}$ are obtained by minimizing the obtained MSE with respect to $\beta_j^{'}$ (to do the partial derivative and equate to zero) after putting the relation $\beta_j^{'}C_j=\gamma_j^{'}D_j$ from lemma 8.2.1. The additional pulse required for the dead-beat system can be achieved using the theorem 6.1.1 and these pulse parameters are used to implement the dead-beat realization for the original higher order system.

Case II: The equivalent second order system with two real poles.

$$MSE=\int_0^{\infty}\left[\begin{array}{l}\left\{-\left(\sum_{i=1}^{m}\alpha_i+2\sum_{j=1}^{p}\beta_j\right)+\sum_{i=1}^{m}\alpha_i e^{A_i t}+2\sum_{j=1}^{p}e^{C_j t}\left(\beta_j\cos D_j t-\gamma_j\sin D_j t\right)\right\}\\-\left\{\left(\alpha_j+\alpha_k\right)+\alpha_j e^{A_j t}+\alpha_k e^{A_k t}\right\}\end{array}\right]^2 dt \qquad (147)$$

$$=\int_0^{\infty}\left[\sum_{\substack{i=1\\ i\neq j,k}}^{m}\alpha_i e^{A_i t}+\left(\alpha_j-\alpha_j^{'}\right)e^{A_j t}+\left(\alpha_k-\alpha_k^{'}\right)e^{A_k t}+2\sum_{l=1}^{p}e^{C_l t}\left(\beta_l\cos D_l t-\gamma_l\sin D_l t\right)\right]^2 dt \qquad (148)$$

$$=\int_0^{\infty}\sum_{\substack{i=1\\ i\neq j,k}}^{m}\alpha_i^2 e^{2A_i t}dt+\int_0^{\infty}\left(\alpha_j-\alpha_j^{'}\right)^2 e^{2A_j t}dt+\int_0^{\infty}\left(\alpha_k-\alpha_k^{'}\right)^2 e^{2A_k t}dt+4\int_0^{\infty}\sum_{l=1}^{p}e^{2C_l t}\left(\beta_l\cos D_l t-\gamma_l\sin D_l t\right)^2 dt$$

$$+2\int_0^{\infty}e^{A_j t}\left(\alpha_j-\alpha_j^{'}\right)\left(\sum_{\substack{i=1\\ i\neq j,k}}^{m}\alpha_i e^{A_i t}\right)dt+2\int_0^{\infty}e^{A_k t}\left(\alpha_k-\alpha_k^{'}\right)\left(\sum_{\substack{i=1\\ i\neq j,k}}^{m}\alpha_i e^{A_i t}\right)dt$$

$$+4\int_0^{\infty}\left(\sum_{\substack{i=1\\ i\neq j,k}}^{m}\alpha_i e^{A_i t}\right)\left\{\sum_{l=1}^{p}e^{C_l t}\left(\beta_l\cos D_l t-\gamma_l\sin D_l t\right)\right\}dt+2\int_0^{\infty}\left(\alpha_j-\alpha_j^{'}\right)\left(\alpha_k-\alpha_k^{'}\right)\int_0^{\infty}e^{\left(A_j+A_k\right)t}dt$$

$$+2\int_0^{\infty}\left\{\left(\alpha_j-\alpha_j^{'}\right)\sum_{l=1}^{p}e^{\left(A_j+C_l\right)t}\left(\beta_l\cos D_l t-\gamma_l\sin D_l t\right)\right\}dt+2\int_0^{\infty}\left\{\left(\alpha_k-\alpha_k^{'}\right)\sum_{l=1}^{p}e^{\left(A_k+C_l\right)t}\left(\beta_l\cos D_l t-\gamma_l\sin D_l t\right)\right\}dt \qquad (149)$$

$$
\begin{aligned}
&=\left(\alpha_j-\alpha_j'\right)^2\int_0^\infty e^{2A_jt}dt+\left(\alpha_k-\alpha_k'\right)^2\int_0^\infty e^{2A_kt}dt+2\left(\alpha_j-\alpha_j'\right)\int_0^\infty\left(\sum_{\substack{i=1\\ i\neq j,k}}^{m}\alpha_i e^{\left(A_i+A_j\right)t}\right)dt\\
&+2\int_0^\infty\left(\sum_{\substack{i=1\\ i\neq j,k}}^{m}\alpha_i e^{\left(A_i+A_k\right)t}\right)dt+2\left(\alpha_j-\alpha_j'\right)\left(\alpha_k-\alpha_k'\right)\int_0^\infty e^{\left(A_i+A_k\right)t}dt\\
&+2\left(\alpha_j-\alpha_j'\right)\int_0^\infty\left\{\sum_{l=1}^{p}e^{\left(A_j+C_l\right)t}\left(\beta_l\cos D_lt-\gamma_l\sin D_lt\right)\right\}dt\\
&+2\left(\alpha_k-\alpha_k'\right)\int_0^\infty\left\{\sum_{l=1}^{p}e^{\left(A_k+C_l\right)t}\left(\beta_l\cos D_lt-\gamma_l\sin D_lt\right)\right\}dt+\int_0^\infty K_2(t)dt \qquad (150)
\end{aligned}
$$

The above expression is integrable. $K_2(t)$ is the function which is independent of α_j' *and* α_k'. After the integration the two unknown residues α_j', α_k' are obtained by minimizing the obtained MSE with respect to α_j' (to do the partial derivative and equate to zero) after putting the relation $\alpha_j' A_j = -\alpha_k' A_k$ from lemma 8.2.1. The additional pulse required for the dead-beat system can be achieved using the theorem 6.1.1 and these pulse parameters are used to implement the dead-beat realization for the original higher order system.

Remark 9.1.1:

It is found from theorem 8.1.1 that output response of any nth order linear control system can be written as

$$c(t)=-\left(\sum_{i=1}^{m}\alpha_i+2\sum_{j=1}^{p}\beta_j\right)+\sum_{i=1}^{m}\alpha_i e^{A_it}+2\sum_{j=1}^{p}e^{C_jt}\left(\beta_j\cos D_jt-\gamma_j\sin D_jt\right) \quad (151)$$

where m number of real poles and 2p number of complex poles (m+2p) is equal to n.

It is clear from the above equation as any real pole A_i is moved far away from origin the system is equivalent to its next lower order system (since A_i is negative). Similarly the same is true for the complex conjugate poles $C_j \pm iD_j$, as the real part of complex conjugate poles C_j is large the above equation is close to the next lower order system (since C_j is negative). This is the reason where always anyone must have to remove the distant poles either real or complex to obtain the equivalent lower order system for dead- beat realization. In theorem 9.1.1 it is obtained the equivalent second order system by estimating the residues of the second order system with the two poles keeping unchanged (either two real poles or two complex conjugate poles).The poles which are kept unchanged are the nearest pole of the system.

But in case II of theorem 9.1.1, when the equivalent second order system is obtained for higher order system with two nearest real poles, the value of ξ is always greater than equal to one, i.e. the equivalent second order system as well as original higher order system is over damped in nature and the output response of the system is free of oscillation. In the SCT scheme, there is no need to consider this type of case for dead-beat realization. The case II of theorem 9.1.1 is mentioned here only for the theoretical interest of obtaining equivalent second order system from given higher order system but not for the dead-beat implementation of the system.

Example 9.1.1: The 3rd order linear control system is considered with $G(s)=\dfrac{30}{s(s+1)(s+6)}$. To obtain the dead-beat response of the system with the addition of pulse, it is desirable to find out the equivalent second order system and then to obtain the pulse parameters to implement into original given system.

$$T(s)=\frac{G(s)}{1+G(s)}=\frac{30}{s(s+1)(s+6)+30}=\frac{30}{s^3+7s^2+6s+30} \qquad (152)$$

$$\frac{Y(s)}{u(s)}=\frac{30}{s^3+7s^2+6s+30} \qquad (153)$$

$$or\ Y(s)=\frac{30}{s^3+7s^2+6s+30}u(s) \qquad (154)$$

$$or\ Y(s)=\frac{30}{s^3+7s^2+6s+30}.\frac{1}{s} \qquad (155)$$

$$or\ Y(s)=\frac{30}{s^4+7s^3+6s^2+30s+0.s^0} \qquad (156)$$

Let [R, P, K] = RESIDUE (B, A), where RESIDUE (B, A) is the function which finds the residues, poles and direct term of a partial fraction expansion of the ratio of two polynomials B(s)/A(s). If there are no multiple roots,

$$\frac{B(s)}{A(s)}=\frac{R(1)}{s-P(1)}+\frac{R(2)}{s-P(2)}+\ldots+\frac{R(n)}{s-P(n)}+K(s) \qquad (157)$$

Vectors B and A specify the coefficients of the numerator and denominator polynomials in descending powers of s. The residues are returned in the column vector R, the pole locations in column vector P, and the direct terms in row vector K.

$$Y(s)=\frac{-0.0911}{(s+6.7684)}+\frac{-0.4545+0.1716i}{(s+0.1158-2.1021i)}+\frac{-0.4545-0.1716i}{(s+0.1158+2.1021i)}+\frac{1}{s} \qquad (158)$$

Taking Laplace transformation and using theorem 8.1.1, the output response in time domain

$$c(t) = 1 + (-0.0911)e^{(-6.7684)t} + 2e^{(-0.1158)t}\{(-0.4545)\cos(2.1021)t - (0.1716)\sin(2.1021)t\} \quad (159)$$

The above 3rd order system is having one real pole and two imaginary poles. As the poles to be kept unchanged in this 3rd order system, the equivalent 2nd order system can be obtained with same two imaginary poles and their modified residues. The MSE between the above two systems will be minimized to obtain the new residues of imaginary poles of the equivalent second order system. The MSE can be represented as

$$MSE(t) = f(t) =$$

$$\int_0^\infty \left[\left\{1 + \alpha_1 e^{A_1 t} + 2e^{C_1 t}\left(\beta_1 \cos D_1 t - \gamma_1 \sin D_1 t\right)\right\} - \left\{1 + 2e^{C_1' t}\left(\beta_1' \cos D_1' t - \gamma_1' \sin D_1' t\right)\right\}\right]^2 dt$$

$$\text{where } C_1 = C_1' \text{ and } D_1 = D_1' \quad (160)$$

From lemma 8.2.1 the equivalent second order system can be written as

$$Y(s) = \frac{-2\beta_1'\left(C_1^2 + D_1^2\right)}{s\left\{s^2 - 2sC_1 + \left(C_1^2 + D_1^2\right)\right\}} \quad (161)$$

From theorem 9.1.1 $$Y(s) = \frac{0.9436 \times 4.4322}{s\left\{s^2 + 0.2316s + 4.4322\right\}} \quad (162)$$

The system given in (162) is the equivalent second order system which gives similar output response with the same step input given in the original 3rd order system. To achieve the dead-beat response of the system given in (162) the value of start time (t_1), end time (t_2) and pulse amplitude (b) can be obtained using theorem 6.1.1. It is noticed that t_1 and t_2 do not depend on the reference input to the given system but depends on ξ and ω_n only. The pulse amplitude (b) depends on ξ, ω_n, t_1, t_2 and also the reference step input. Here $t_1 = 0.7216, t_2 = 0.8236$ and $b = 4.27557$. The same pulse parameters (t_1, t_2, b) is applied to the given 3rd order system to obtain the deadbeat.

The figure 9.1(a) and 9.1(b) represents dead-beat realization of the given linear system and its equivalent second order system respectively in example 9.1.1.The figure 9.1(c) represents the mean square error of the output response between higher order system and equivalent second order system in example 9.1.1.

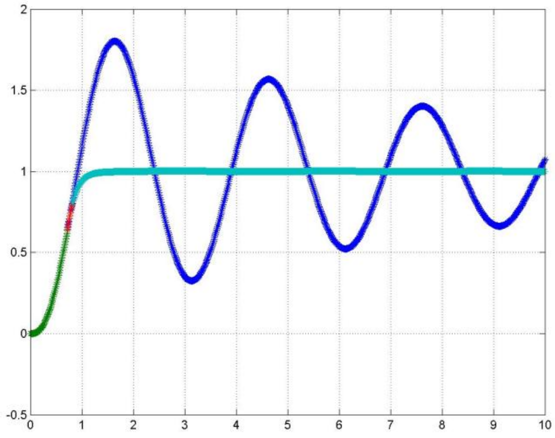

Figure 9.1 (a). Dead beat response of example 9.1.1 using same pulse parameters used in reduced second order system of figure figure 9.1(b)

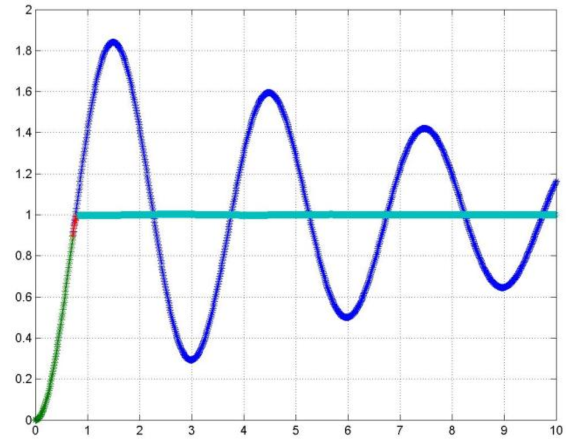

Figure 9.1 (b) Deadbeat response of example 9.1.1 used in reduced second order system of figure 9.1 (a)

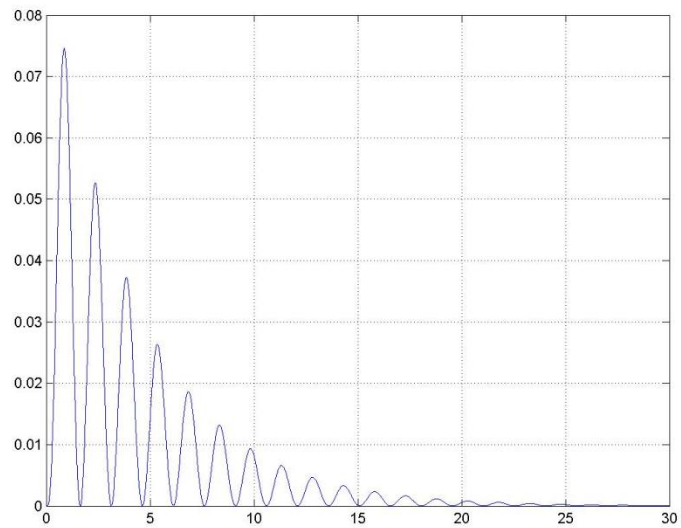

Figure 9.1(c). The mean square error of the output response between higher order system and reduced second order system of example 9.1.1

Example 9.1.2:

The 4th order linear control system is considered with $G(s) = \dfrac{40}{s(s+10)(s+2)(s+1)}$.

To obtain the dead-beat response of the system with the addition of pulse, it is desirable to find out the equivalent second order system and then to obtain the pulse parameters (t_1, t_2, b) for this equivalent system. The same pulse parameters are applied to the given 4th order system for the deadbeat realization. Here t_1=1.2915, t_2=1.4732 and b=4.46788.

Figure 9.2(a) and 9.2(b) represents dead-beat realization of the given linear system and its equivalent second order system respectively in example 9.1.2. The figure 9.2(c) represents the mean square error of the output response between higher order system and equivalent second order system in example 9.1.2.

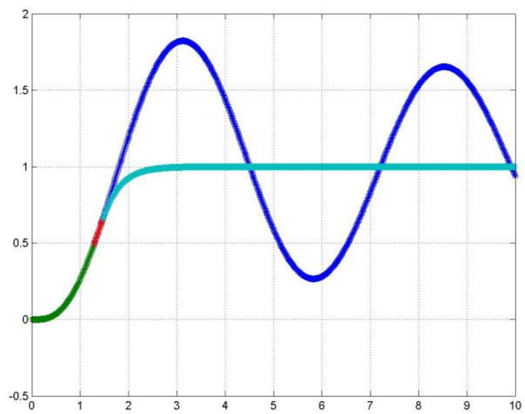

Figure 9.2.(a). Dead beat response of example 9.1.2 using same pulse parameters used in figure 9.2(b)

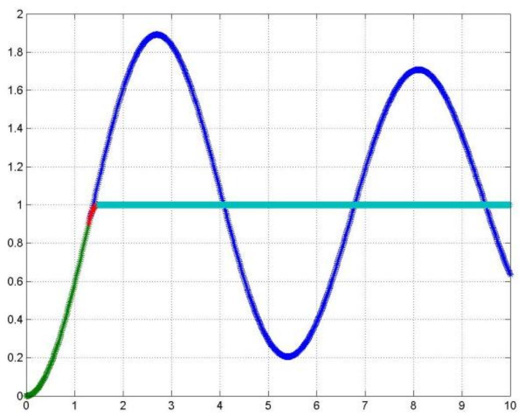

Figure 9.2 (b) Deadbeat response of reduced second order system of figure 9.2 (a)

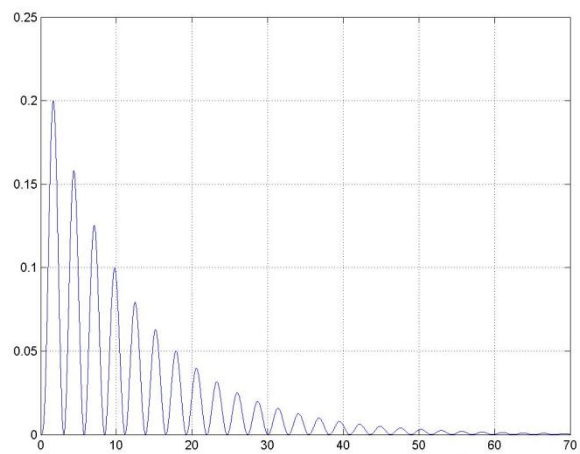

Figure 9.2 (c). The mean square error of the output response between higher order system and equivalent second order system of example 9.1.2

Table 9.1: The pulse parameters for the six different second order linear continuous systems

Sr no	k	p	ξ	t_1	t_2	b
1	1	0.1	0.05	1.5147	1.7306	4.27863
2	1	0.01	0.005	1.4744	1.6772	4.9
3	4	0.5	0.1250	0.7949	0.9140	3.39194
4	9	1.8	0.30	0.5980	0.7108	1.586
5	100	5	0.25	0.1758	0.2006	2.47575
6	4.4324	0.2316	0.0550	0.7216	0.8236	4.27557

10 Conclusions

In this paper the dead-beat control has been achieved by signal correction technique (SCT), which does not require any restriction on the system parameters. In this scheme of dead beat control the additional signal is the pulse of appropriate duration and applied at a suitable time and having a specified amplitude. The application is selected so that the non-dead-beat and the corresponding dead-beat systems have the same rise time (when the system response attains 90% of its final value). The pulse duration is selected so that it ends long before the first overshoot of the uncompensated system. In this work it is decided to make zero overshoot or undershoot after the exact rising time (the system response attains 100% of its final value) to find out the pulse parameters (t_1, t_2, b). This idea is a novel one and results in better deadbeat realization. The proposed method is based on theoretical foundation given in theorem 5.1, theorem 5.2, theorem 6.1.1, and theorem 6.2.1 for second order linear continuous time systems. Theorem 8.1.1 and theorem 9.1.1 extended the method for higher order linear continuous time systems. The parameters of the added pulse to the uncompensated system i.e the start time (t_1), end time (t_2) and the required amplitude (b) are determined without any restriction on system parameters The implementation of the dead-beat system by adding pulse is straight forward and gives good performance as has been seen in many simulations. In this scheme the generation of pulse is quite easy in real time operation and it does not raise the hardware complexity unlike the other scheme done in previous works. Whatever be the order of the system, the system components will not increase in this scheme. But in the previous works of various researchers, the system components will increase as the order of the system being increased, which leads to more hardware and computation complexity. The dead beat control scheme discussed in section 4 to section 6 can be applied to any higher order linear continuous time invariant system with any type of reference input like ramp input, parabolic input or any polynomial type of input. Section 7 represents the simulation result of 2^{nd} order continuous system. Section 8 and section 9 demonstrate how the higher order linear continuous system can undergo deadbeat realization using the same additional pulse signal of its equivalent second order system. In this work deadbeat realization of linear time invariant system is done only for the step input. The other type reference input such as ramp input, sinusoidal input can motivate the authors and other researchers for the future scope of deadbeat realization. Still there are lot of future works left for implementing deadbeat controller over the nonlinear system and time varying system. The Feedback linearization of nonlinear system can introduce locally asymptotically stable system. The thoughts of feedback linearization must to be implemented for global stability and robustness of the non-linear time varying system.

References:

1. Acintya Das, Rajib Bag, And N. G. Nath, “A Modification To Realize Dead-Beat Performance Of Control Systems—Signal Correction Technique”, IEEE Transactions On Instrumentation And Measurement,; Vol. 55, No. 5, 2006, pp: 1546–1550,
2. Rajib Bag, Achintya Das, D. N. Tibarewala,” Signal Correction Technique (SCT) to Realize Dead Beat Performance of Control System”, International Journal of Advanced Engineering & Application, Jan 2011 Issue; pp 47-50.
3. S. Urikura and A. Nagata, “Ripple-free deadbeat control for sampled-data systems”, IEEE Trans. Automat. Control, vol. AC-32, 1987, pp: 474-482,
4. E. Zafiriou and M. Morari, “Digital controllers for SISO systems: a review and a new algorithm”,Int. J. Control, vol. 42, ,1985, pp. 855-876
5. C.A. Karybakas, C.A.Barbargires, “Explicit conditions for Ripple-Free Dead Beat Control”, KYBERNETIKA - Volume 3 2 (1 9 9 6), Number 6 , Pp 601–614.
6. Robert Paz And Hatem Elaydi “Optimal Ripple-Free Deadbeat Controllers”, Int. J. Control, 1998; Vol. 71, No. 6, pp 1087-1104.
7. S. H. Zak and E. E. Blouin, “Ripple-free deadbeat control”, IEEE Control Systems Magazine, 1993; vol.13; issue 4.
8. L. Jetto, “Deadbeat controllers with ripple-free requirement for SISO discrete systems”, IEE Proceedings, Vol. 137, Pt. D, No. 5, September 1990,pp 323-328.
9. Valiollah Ghaffari, “Control Law Design with Dead-beat Property in Continuous-Time Dynamical Systems”, International Journal of Industrial Electronics, Control and Optimization (IECO), Vol. 2, No. 3, pp. 189-196, July (2019)
10. Nina Vunder, Madina Sinetova, A.V. Ushakov, “Dead-Time Pseudo-Compensation for Improvement of Accurate Characteristics of a Continuous Control System with Correction According CP a Smith Predictor”, Optoelectronics Instrumentation and Data Processing 54(1):97-104, January 2018
11. Carlos A. García, Manel Velasco, Cecilio Angulo, Pau Marti, Antonio Camacho, “Revisiting Classical Controller Design and Tuning with Genetic Programming”, Sensors **2023**, 23, 9731, pp: 1-25
12. Rami A. Maher, “Discrete Controller Synthesis Based On Genetic Programming”, WSEAS TRANSACTIONS on SYSTEMS and CONTROL, Volume 10, 2015, pp: 763-775
13. Francisco Vargas, “Optimal ripple-free deadbeat control using an integral of time squared error (ITSE) index”, Automatica, September 2011, 47(9):pp: 2134-2137
14. Eitaku Nobuyama, “Parametrization of all ripple-free deadbeat controllers”, Systems & Control Letters, Elsevier, Volume 21, Issue 3, September 1993, Pages 217-224
15. Abou soufyane Benyouce, Kamel Kara, Chouder Aissa, Santiago Silvestre, “Prediction-based Deadbeat Control for Grid-connected Inverter with L-filter and LCL-filter”, Electric Power Components and Systems, Taylor & Francis Group, 42(12): July 2014, pp:1266–1277
16. Malathi Selvaraj, Dr K Elango, “Implementation of dead beat controller using Particle Swarm Optimization for software defined network”, Computer Communications 155(2), March 2020
17. Dr. S. Janardhanan, “Model Order Reduction and Controller Design Techniques”, Chapter 3,pp 23-36.
18. C.B Vishwakarma* and R. Prasad, “System Reduction using Modified Pole Clustering and Pade Approximation”, XXXII National Systems Conference, NSC 2008, December 17-19, 2008, pp. 592-596.
19. S. Panda, S. K. Tomar, R. Prasad, C. Ardil,” Reduction of Linear Time-Invariant Systems Using Routh-Approximation and PSO”, World Academy of Science, Engineering and Technology 57 2009,pp 937-944.
20. Ismail bin Mohd and Mustafa bin Mamat, “Model Reduction and Moment Matching”, Applied Mathematical Sciences, Vol. 3, 2009, no. 38, pp: 1849 – 1859.
21. Shiv Kumar Tomar* and Rajendra Prasad, “Linear Model Reduction using Mihailov Stability Criterion and Continued Fraction Expansions”, XXXII National Systems Conference, NSC 2008, December 17-19, 2008, pp 603-605.

22. G. Parmar, Life Member SSI, AMIE, S. Mukherjee, FIE, and R. Prasad, "Order Reduction of Linear Dynamic Systems using Stability Equation Method and GA", World Academy of Science, Engineering and Technology 26 2007, pp 372-378.
23. Ramesh, K, "Order reduction by error minimization technique", International Conference on Computing, Communication and Networking, 2008. pp 1-6.
24. S. Mukherjee and R. N. Mishra, "Order reduction of linear systems using an error minimization technique", Journal of Franklin Institute, Vol. 323, No. 1, 1987,pp. 23-32.
25. Devender Kumar Saini, Rajendra Prasad," Mixed Evolutionary Techniques to Reduce Order of Linear Interval Systems using Generalized Routh Array", International Journal of Engineering Science and Technology Vol. 2(10), 2010, pp: 5197-5205.
26. Parmar, S. Mukherjee, R. Prasad, "Reduced Order Modelling of Linear Dynamic Systems using Particle Swarm Optimized Eigen Spectrum Analysis" International Journal of Computational and Mathematical Sciences, 2007, pp: 45-52.
27. G. Parmar, Mukherjee and R.N. Mishra, "Reduced order modeling of linear multivariable systems using particle swarm optimisation technique", International Journal of Innovative Computing and Applications, Volume 1 Issue 2, January 2007 Pages 128-137.
28. S. Panda, J. S. Yadav, N. P. Patidar and C. Ardil," Evolutionary Techniques for Model Order Reduction of Large Scale Linear Systems, International Journal of Engineering and Applied Sciences 5:1 2009,pp 22-28.
29. Swadhin Ku. Mishra, Sidhartha Panda, Simanchala Padhy and C. Ardil," MIMO System Order Reduction Using Real- Coded Genetic Algorithm", International Journal of Information and Mathematical Sciences 6:4 2010, pp 232-236.
30. G.Vasu, J. Nancy Namratha, P.Murari," Order Reduction of Linear Dynamic Systems using Improved Generalize Least-Squares Method and PSO", International Journal of Engineering Research and Applications (IJERA), Vol. 2, Issue 2, Mar-Apr 2012, pp.581-587.
31. G.D. Howitt and R. Luus, "Model reduction by minimization of integral square error performance indices", Journal of Franklin Inst., Vol. 327, 1990, pp. 343-357,