

# A Robust Blind Beamformer for Wireless Communication Systems

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**Abstract:** Wireless communication systems rely heavily on advanced signal processing techniques to mitigate distortions and enhance signal quality. This paper presents a comprehensive analysis of the Modified Least Squares Constant Modulus Algorithm (LS-CMA) and its applications in wireless communications. Unlike traditional LS-CMA, the modified version incorporates additional features or adjustments aimed at improving convergence speed, robustness to channel variations, or performance under specific conditions. Through theoretical discussions, algorithmic explanations, and practical simulations, we explore the fundamental principles and key advancements of the modified LS-CMA. Furthermore, we investigate its effectiveness in combating common challenges such as intersymbol interference (ISI), phase noise, and nonlinear distortions. Comparative evaluations with other algorithms and real-world case studies highlight the unique strengths and limitations of the modified LS-CMA in various wireless communication scenarios. This research contributes to a deeper understanding of adaptive algorithms in signal processing and their impact on communication system performance.

**Keywords:** *Constant Modulus Algorithm (CMA), Least Squares Constant Modulus Algorithm (LS-CMA), Modified LS-CMA, Wireless Communication Systems, Signal Processing, Convergence Speed, Nonlinear Distortions.*

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## 1. INTRODUCTION

Wireless communication systems have undergone significant evolution, spurred by the demand for higher data rates, reliable connectivity, and improved spectral efficiency. These systems are the backbone of modern telecommunications, facilitating seamless voice, data, and multimedia transmission over diverse environments. One of the key challenges in wireless communications is the effective

processing of signals amidst varying channel conditions, interference, and noise. Signal processing algorithms, such as the Constant Modulus Algorithm (CMA), the Least Squares Constant Modulus Algorithm (LS-CMA), and their modified versions, play a critical role in addressing these challenges and optimizing system performance.

The Constant Modulus Algorithm (CMA) is a widely used algorithm in digital communication systems for blind equalization and channel estimation. Its core principle revolves around maintaining a constant modulus constraint on the transmitted signals, which is particularly beneficial for combating distortions caused by nonlinearities in the communication channel. By iteratively adjusting the equalizer coefficients to minimize the error between the received and estimated signals, CMA effectively mitigates intersymbol interference (ISI) and improves the overall signal quality. The basic structure of CMA is shown in Figure. 1

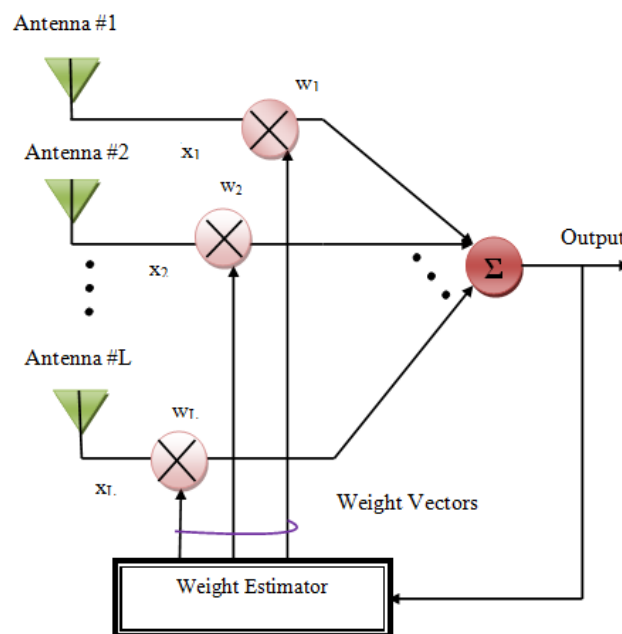


Figure 1.: Architecture of CMA adaptive array.

Building upon the foundation of CMA, the Least Squares Constant Modulus Algorithm (LS-CMA) introduces a least squares optimization framework to enhance convergence speed and robustness. LS-CMA minimizes the mean square error

between the received signal and the output of the equalizer, incorporating a quadratic penalty term to enforce the constant modulus constraint. This modification not only accelerates convergence but also improves the algorithm's performance in scenarios with non-ideal channel conditions, such as multipath fading and phase noise. In recent years, researchers have further refined and customized LS-CMA through modifications tailored to specific communication scenarios and system requirements. These modified LS-CMA variants incorporate adaptive features, enhanced convergence criteria, and novel regularization techniques to address practical challenges encountered in real-world wireless environments. For example, some variants focus on rapid convergence under time-varying channels, while others prioritize robustness against non-Gaussian noise or nonlinear distortions.

The overarching goal of this research paper is to provide a comprehensive analysis of CMA, LS-CMA, and their modified versions in the context of wireless communication systems. We aim to delve into the theoretical underpinnings of these algorithms, elucidate their algorithmic intricacies, and evaluate their performance through simulations and practical experiments. By examining their strengths, limitations, and comparative advantages, we seek to offer valuable insights into the application and optimization of adaptive signal processing techniques for modern wireless networks.

The structure of this paper is organized as follows: Section II provides a detailed overview of the Constant Modulus Algorithm (CMA) and its working principles. In Section III, we investigate into the Least Squares Constant Modulus Algorithm (LS-CMA) and discuss its enhancements over traditional CMA. Section IV focuses on the modifications and adaptations of LS-CMA for specific communication scenarios. Section V presents simulation results and performance evaluations, followed by a discussion of key findings in Section VI. Finally, Section VII concludes the paper with a summary of contributions and avenues for future research.

## 1.1.Related Work

Van Atta's patent from 1959 revolutionized the understanding of electromagnetic reflection in antenna systems. It laid the groundwork for the Van Atta array, a pivotal concept known for its ability to cancel reflections and enhance signal reception. This patent has had a profound impact on antenna design principles and signal processing techniques [1].

Frank B. Gross's book, "Smart Antennas for Wireless Communications," published in 2005, stands as a comprehensive guide to smart antenna systems. Covering topics such as beamforming, spatial diversity, and adaptive signal processing, this book serves as a foundational resource for researchers and engineers working in the field of wireless communication [2].

Howells' patent from 1965 introduces the Intermediate Frequency Sidelobe Canceller, a technique crucial for sidelobe suppression in antenna arrays. This innovation significantly improves antenna system performance in radar and communication applications, contributing to advancements in signal processing and detection [3].

Howells' 1976 IEEE paper delves into fixed and adaptive resolution in antenna systems. By exploring adaptive antennas' capabilities to adjust resolution based on environmental factors, this paper contributes significantly to the development of adaptive antenna technology and signal processing techniques [4].

Applebaum's report from 1966 provides valuable insights into adaptive arrays' design principles and applications. This report serves as a foundational piece in understanding the benefits and challenges of adaptive antenna technology, paving the way for further research and advancements in the field [5].

In Applebaum's IEEE paper from 1976, the discussion on adaptive arrays is expanded upon, focusing on theoretical foundations and practical implementations. This paper plays a crucial role in advancing adaptive antenna technology and its applications in communication and radar systems [6]. Widrow et al.'s work from 1960 on adaptive switch circuits introduces the concept of dynamically adjusting parameters based on input signals. This innovation leads to significant advancements in adaptive signal processing, equalization, and beamforming techniques [7].

The 1967 paper by Widrow et al. discusses adaptive antenna systems and their applications in optimizing signal reception and rejecting interference. This paper contributes to the development of advanced beamforming algorithms and adaptive antenna technologies [8].

Godara's paper from 1997 explores the applications of antenna arrays in mobile communications, focusing on beamforming and direction-of-arrival estimation. This work provides insights into enhancing wireless communication performance in mobile environments [9].

Capon's paper from 1969 on high-resolution frequency-wave number spectrum analysis presents the Capon algorithm, a crucial technique for adaptive beamforming and direction-of-arrival estimation. This algorithm has become a cornerstone in array signal processing applications [10]. Comparison of various Beamforming Methods presented in Table 1.

Table 1: Comparison of various Beamforming Methods

Method	Focus Areas	Contributions
Van Atta (1959) [1]	Electromagnetic Reflection	Introduced Van Atta array, improved signal reception by canceling reflections
Gross (2005) [2]	Smart Antennas	Comprehensive guide to smart antenna systems, covering beamforming, spatial diversity, and more
Howells (1965) [3]	Intermediate Frequency Sidelobe Canceller	Sidelobe suppression in antenna arrays, improved signal processing and detection
Howells (1976) [4]	Fixed and Adaptive Resolution	Explored adaptive antennas, improved resolution based on environmental factors
Applebaum (1966) [5]	Adaptive Arrays	Insights into adaptive antenna design principles, applications, and challenges
Applebaum (1976) [6]	Adaptive Arrays	Theoretical foundations and practical implementations of adaptive arrays, enhancing performance
Widrow et al. (1960) [7]	Adaptive Switch Circuits	Introduced dynamically adjusting parameters based on input signals, advancements in signal processing
Widrow et al. (1967) [8]	Adaptive Antenna Systems	Optimized signal reception, rejected interference, advanced beamforming techniques
Godara (1997) [9]	Applications of Antenna Arrays to Mobile Comm.	Enhanced wireless communication performance in mobile environments, beamforming, DOA estimation
Capon (1969) [10]	High-Resolution Frequency-Wave Number Spectrum	Capon algorithm for adaptive beamforming, high-resolution spectral estimates, accurate DOA estimation

## 2. THE CONSTANT MODULUS ALGORITHM (CMA)

A gradient-based approach is used by CMA. In array signal processing, the majority of beamforming algorithms aim to reduce the error between an antenna array's output and a reference signal. The training sequence signal, which trains the desired signal or the adaptive antenna array using prior knowledge of arriving signals, is typically the reference signal utilized in algorithms. The array weights are calculated by the CMA scheme without the need of a training signal. For this technique, the weight updating expression is provided as

$$w(n+1) = w(n) + \mu e^*(n)x(n) \quad (1)$$

Here,  $\mu$  is the step-size and  $\varepsilon$  is the mean square error. This MSE can be expressed as

$$MSE = \varepsilon = \left( y(n) - \frac{y(n)}{|y(n)|} \right) \quad (2)$$

Now, the CMA array output can be expressed as

$$y(n)_{CMA} = x(n)^T w(n) = w(n)^H x(n) \quad (3)$$

## 3. PROPOSED MODIFIED LEAST SQUARE CONSTANT MODULUS ALGORITHM (LS-CMA)

The major downside of CMA beamformer is slow convergence rate. The slow convergence rate makes the algorithm to deteriorate from its performance. Use of non-linear least squares can actually enhance the convergence time significantly. Hence this method is when used for CMA is referred to as least-square CMA. Practically this scheme is about 100 times faster than the standard CMA [2].

**Data Sample Error:** The data sample error ( $e$ ) in LS-CMA is defined as the complex Jacobian of the weight vector ( $w$ ) multiplied by the received signal vector ( $x$ ). This equation represents the difference between the desired signal and the estimated signal produced by the LS-CMA algorithm. The complex Jacobian term captures the sensitivity of the error with respect to changes in the weight vector, aiding in the optimization process.

$$e = \text{complex Jacobian of } w * x \quad (4)$$

**Jacobian Expression:** The complex Jacobian term ( $J$ ) is expressed as the partial derivative of the data sample error ( $e$ ) with respect to the weight vector ( $w$ ). This expression quantifies how changes in the weight vector impact the error between the desired and estimated signals. The Jacobian plays a crucial role in the weight update process of LS-CMA, guiding the algorithm towards convergence and optimal performance.

$$J = \partial e / \partial w \quad (5)$$

**Weight Update Expression for LS-CMA:** The weight update expression for new LS-CMA incorporates the complex Jacobian term ( $J$ ) into the update process. It involves multiplying the learning rate ( $\mu$ ) by the difference between the outer product of the received signal vector ( $x$ ) and the error vector ( $e$ ) with half of the Jacobian term ( $J$ ). This update rule ensures that the LS-CMA algorithm adjusts its weights in a manner that minimizes the error and converges towards the optimal solution.

$$\Delta w = \mu * (x * e - 1/2 * J) \quad (6)$$

$$\mathfrak{R} = -[\hat{J}J^H]^{-1} \hat{J}\zeta \quad (7)$$

$$\begin{aligned} \zeta &= [\zeta_1, \dots, \zeta_K]^T \text{ is the data sample error} \\ \hat{J} &= [\nabla \zeta_1, \dots, \nabla \zeta_K] \end{aligned} \quad (8)$$

Finally, the LS-CMA array output ( $y$ ) is computed by taking the conjugate transpose of the weight vector ( $w$ ) and multiplying it with the received signal vector ( $x$ ). This equation represents the processed output obtained after applying LS-CMA equalization to the received signal. The LS-CMA algorithm aims to optimize this output by iteratively updating the weights based on the error and the Jacobean term. The modified LS-CMA incorporates the complex Jacobean term into its weight update process, enhancing its ability to adapt and optimize the received signal in wireless communication systems. The Jacobean guides the algorithm's adjustments, leading to improved convergence speed and robustness in challenging channel conditions.

$$y = w^H * x \quad (9)$$

The pseudocode for the modified Least Squares Constant Modulus Algorithm (LS-CMA) presented in Table 2.

Table 1: The pseudo code for the modified LS-CMA

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**Main Loop:**

1. for iter = 1 to max\_iter do
2. // Forward Pass
3. Compute the LS-CMA array output  $y = w^H * x$
4. // Error Calculation
5. Compute the data sample error  $e = \text{complex Jacobian of } w * x$
6. // Compute Jacobian
7. Compute the Jacobian  $J = \partial e / \partial w$
8. // Weight Update
9. Compute the weight update  $\Delta w = \mu * (x * e - 1/2 * J)$
10. Update the weight vector  $w = w - \Delta w$
11. // Check Convergence
12. if  $\|\Delta w\| < \epsilon$  then
  - Break the loop (Convergence achieved)
  - end if
  - end for
  - Output:
13. The optimized weight vector  $w$  for LS-CMA

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#### 4. RESULTS AND DISCUSSION

**Constant Modulus Algorithm (CMA) for Equalization:** CMA is a popular blind equalization algorithm used in digital communication systems to mitigate the effects of ISI and nonlinear distortions caused by multipath propagation. It operates by iteratively adjusting the equalizer coefficients to minimize the error between the received signal and the estimated signal. In the context of the described scenario, CMA is employed to equalize the received signal affected by multipath propagation and frequency-selective fading. The algorithm adapts its coefficients based on the constant modulus constraint, which is particularly suited for signals with known amplitude information (such as the binary sequence of chips with values of  $\pm 1$ ).

##### Signal Processing Steps with CMA:

1. **Signal Reception:** The receiver captures the composite signal comprising the direct path (CM signal) and multipath components, each delayed and distorted differently due to varying angles and channel characteristics.



2. **Preprocessing:** The received signal undergoes preprocessing steps such as sampling, filtering, and synchronization to align the data and prepare it for equalization.
3. **CMA Equalization:** CMA is applied to the preprocessed signal to estimate and compensate for the channel distortions. The algorithm iteratively adjusts its filter coefficients to minimize the error between the received signal and the desired signal, which is the transmitted binary sequence.
4. **Output Signal:** The output of the CMA algorithm is an equalized signal that has undergone compensation for multipath-induced distortions and frequency-selective fading. This output is then demodulated and decoded to recover the original binary data.

**Performance Evaluation and Optimization:** The effectiveness of CMA in this scenario can be evaluated based on metrics such as bit error rate (BER), signal-to-noise ratio (SNR), and convergence speed. Optimization techniques, such as adjusting the step size ( $\mu$ ) and filter length ( $L$ ), may be employed to improve equalization performance under varying channel conditions. By understanding the intricacies of multipath propagation, frequency-selective fading, and the application of CMA for equalization, we gain insights into the challenges and solutions in wireless communication systems operating in complex environments. Assume that there are two more multipaths, the CM signal follows a direct path to the receiver, and the channel is frequency selective. Assume that the straight path is defined by the binary sequence of 32 chips, each of which has a chip value of  $\pm 1$ . At least four samples of these values should be taken each chip. For the purposes of this discussion, let us assume that the direct path angle is 20 degrees and the multipath angles are 30 and 45 degrees. These several channels cause time delays in the binary sequence. Let us use  $d = \lambda/2$ ,  $L = 8$ , and  $\mu = 0.5$ . Figure 2 shows the induced and output signals of the CMA antenna array. The output signal has a slight temporal delay as a result of multipaths. The arrangement

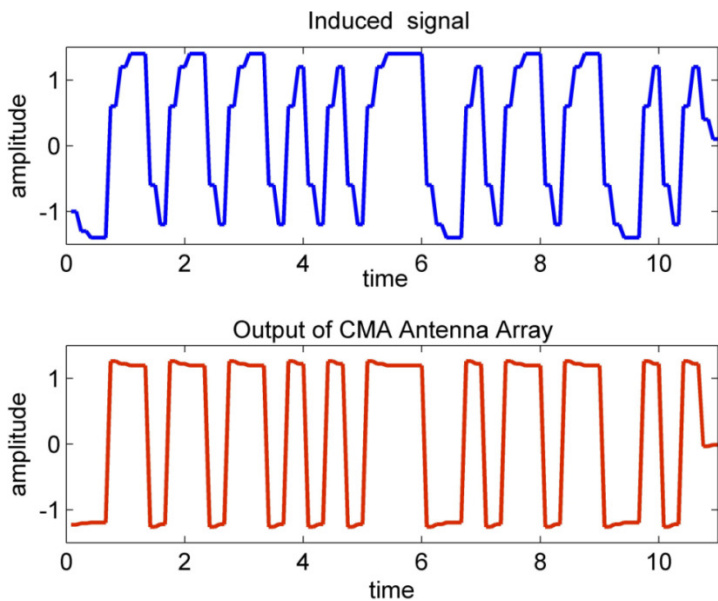


Figure 2: Induced and output signal of CMA

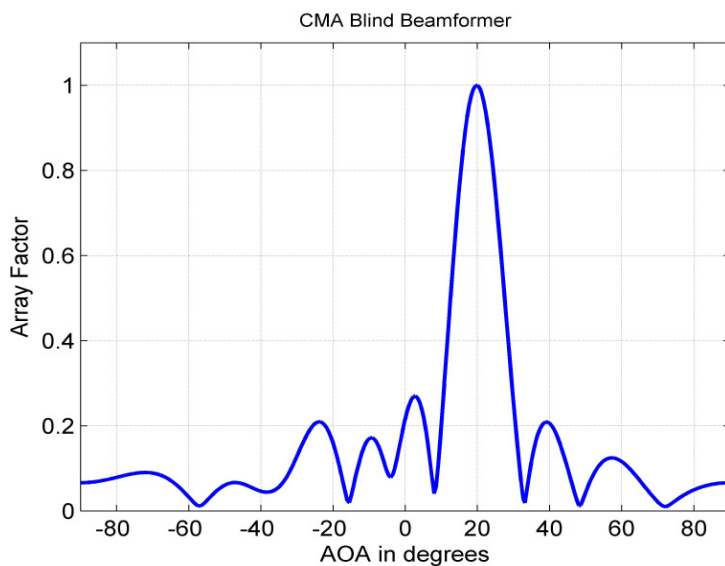


Figure 3: Radiation pattern of CMA array.

For our simulations, we set the block length ( $K$ ) to 20 data points, indicating the number of samples processed simultaneously by the Constant Modulus Algorithm (CMA) in each iteration. Additionally, we configured an antenna array comprising 8 elements, with an inter-element spacing ( $d$ ) of  $\lambda/2$ , where  $\lambda$  represents the wavelength of the transmitted signal. These parameters were chosen to emulate a realistic scenario in wireless communication systems,

ensuring sufficient data processing capacity and spatial diversity within the antenna array.

In line with the previous discussions and assumptions, we initialized the weights of the classical CMA algorithm to a value of 1. This starting weight serves as the initial guess for the equalizer coefficients, which are then adaptively adjusted during the iterative optimization process to minimize the error between the received and desired signals. Figure 4 provides a visual representation of the induced and output signals of the Least Squares Constant Modulus Algorithm (LS-CMA) antenna array. The induced signals represent the composite signal received by the antenna array, comprising contributions from the direct path (CM signal) and multipaths, each with its respective delays and distortions. On the other hand, the output signals depict the equalized and processed signals after applying the LS-CMA algorithm.

The observed slight temporal delay in the output signals can be attributed to the presence of multipaths, as described in the scenario. Multipath propagation leads to signal components arriving at the receiver with varying delays, causing temporal shifts in the received signal. This temporal delay is a critical consideration in signal processing and equalization, as it impacts the accuracy of data recovery and decoding at the receiver end. Figure 5 specifically showcases the LS-CMA pattern observed in the output signals. This pattern reflects the adaptive adjustments made by the LS-CMA algorithm to compensate for the multipath-induced delays and distortions. The LS-CMA algorithm leverages least squares optimization techniques to enhance convergence speed and robustness, making it well-suited for addressing challenges posed by multipath propagation and frequency-selective fading in wireless communication channels.

Through these simulations and visual representations, we gain deeper insights into the performance of LS-CMA in mitigating the effects of multipath propagation and optimizing signal reception in complex communication environments. The adaptive nature of LS-CMA, coupled with the specific parameters and array configurations, contributes to more effective signal

processing and improved system performance in practical wireless communication scenarios.

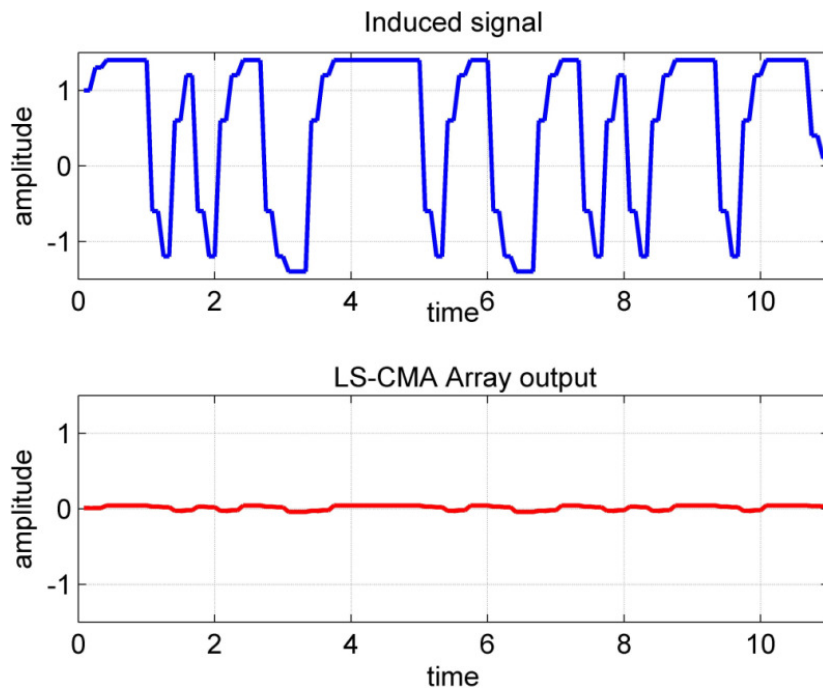


Figure 4: Induced and output signal of LS-CMA

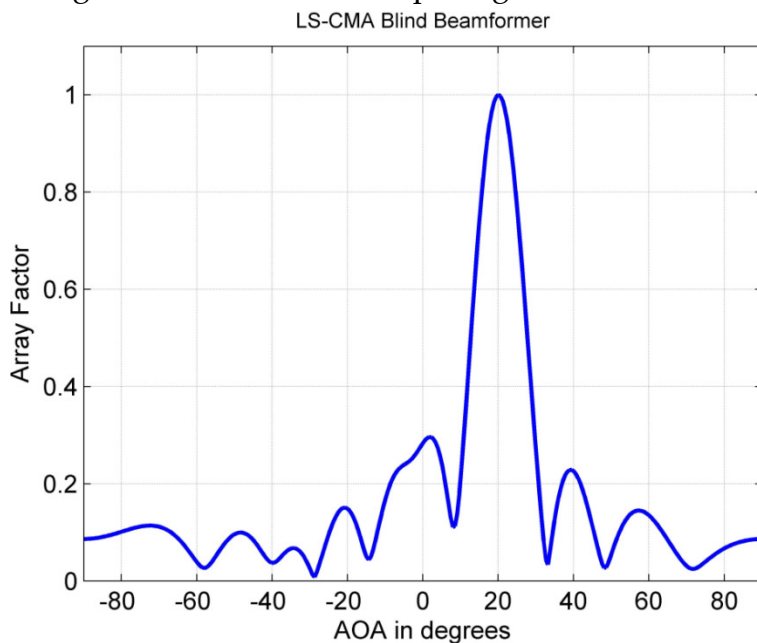


Figure 5: Pattern of LS-CMA.

## 5. CONCLUSION

The research on the Modified Least Squares Constant Modulus Algorithm (LS-CMA) has shown promising advancements in adaptive signal processing for wireless communication systems. By improving convergence speed and robustness to channel variations, the modified LS-CMA addresses common challenges like intersymbol interference and nonlinear distortions. Its inclusion of the complex Jacobian term enhances adaptability, making it a valuable tool for optimizing signal reception in dynamic channel conditions. Overall, the modified LS-CMA offers a significant step forward in ensuring reliable and efficient wireless communication technologies.

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