Optimization of Plant Under Influence of Perturbed Conditions using PIDA Controller

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Abstract- In present paper, the current investigation was made to stabilize the system in presence of uncertainties using PIDA (Proportional, Integral, Derivative, Accelerator) controller. The research has been done on arbitrary order perturbed plant using frequency domain analysis with graphical techniques. However, a nominal plant under the perturbed conditions, has an additive uncertainty weight sensitivity to cover the entire uncertainty set. In particular, it has been shown that all PIDA gains that robustly stabilize a given uncertain (SISO) (LTI) linear time invariant system by keeping the robust stability criteria of sensitivity $S_\infty = \lvert S_x \rvert << 1$ .The conversion of all problems into the segments of Qusipolynomial equations has been done for achieving the robust stability for plant under the effect of uncertainties,. The results on S_∞ PID designing or tuning are then used into a programming design techniques for obtaining the values of required PIDA controller (proportional, integral , derivative and accelerator) gains (k_p,k_i and k_d, k_a) to achieve robust stability constraint. A new term accelerator (A) controller is added with PID controller that make us enable to bring more stability in system by reducing more overshoot in plant. The main advantage of this technique is that it only depends upon the frequency response of the system and does not require the plant transfer function coefficients.

Keywords- **PIDA Controller, Structure uncertainty, Robust Stability, Sensitivity, Unstructured uncertainty.**

I. INTRODUCTION

The control system emphasized primarily on gathering the comprehensive information for analysis and for designing the object-oriented system .So, the mechanization of object-oriented policies has increased into a hierarchy of object-oriented control systems. Modern control theory is related with systems that have self-organizing, robust and optimum qualities. This interest has increased even greater stimulation among control engineers. An industrial process's control (such as production, manufacturing etc by automatic process rather than manual technique is frequently called automation. An optimum control for robust stabilization and performance and of a plant offers us a great excitation to discuss on some uncertainties or perturbations. The introduction of closed loop feedback system under effect of uncertain conditions gives us motivation to think on present research for achievement of required stability of plant or system. The broad classification of uncertainties in any LTI plant can be done into two ways, Structured and Unstructured uncertainties [1]. The parametric perturbations are included in structured class of uncertainty. As a result, parametric uncertainty or perturbations is an actual deviation in physical parameter of any plant in closed loop system. An unstructured uncertainty is observed as norm-bounded perturbations and uncertainty in system linked with dynamics in system, linearization effects and nonlinearity's, etc [2].

Under effects of structured and unstructured uncertainties, robust stability analysis is included for external results in [2]. Plants with parametric perturbations [3] showed that the robust stability analysis criteria could be determined under structured and unstructured perturbations. The PID stabilization [5] on given results in connection with the Generalized Kharitonov Theorem [4], [5] and [6] give us a platform for stabilizing a given interval plant family using PID. First, PID tuning for robust stability for system with mixed uncertainties are converted into all the problems into the segments of Qusi-polynomial equations [7,8,9] . In [10,11,12,13] and [14, 15] the authors of paper developed techniques for finding all achievable PID controllers that simultaneously stabilized the closed loop system and satisfied an $H\infty$ sensitivity, complementary sensitivity, weighted sensitivity, or robust stability constraint, respectively. The technique used in this paper is applicable to absolute arbitrary order single-input single-output (SISO) systems with time-delay. This technique depends upon the frequency response of the system. If the plant transfer function is determined, we can apply the same method by first computing the frequency response. The goal of the paper is to determine the set of PIDA gains $(k_p, k_i \text{ and } k_d, k_a)$ that satisfies the following S_∞ stability index Criteria $\|S_x\| << 1$ for robust performance under effect of external environmental disturbed conditions. Where, $S_x(s)$ is sensitivity.

II. DESING PROCEDURE

In closed loop system figure (1), $R(s)$ is input and $C(s)$ is output. $G_p(s)$ is transfer function of Plant , $H(s)$ is feedback transfer function.

The overall transfer function of system in figure (1) given below

$$M(s) = \frac{C(s)}{R(s)} \quad (1)$$

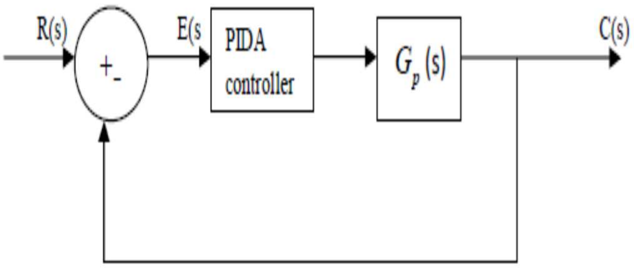

Figure – 1 Closed loop system with feedback and controller [4]

from figure (1) , $K(s)^{PIDA}$ is PIDA controller, the overall transfer function is

$$M(s) = \frac{G_p(s) \times K(s)^{PIDA}}{1 + \{G_p(s) \times K(s)^{PIDA} \times H(s)\}} \quad (2)$$

If $H(s)$ is 1, then

$$M(s) = \frac{G_p(s) \times K(s)^{PIDA}}{1 + \{G_p(s) \times K(s)^{PIDA} \times 1\}} \quad (3)$$

From above formula of $M(s)$ in equation (3), we can take out sensitivity formula given below,

$$S_x = \frac{1}{1 + G_p(s) \times K(s)^{PIDA} \times 1} \quad (4)$$

Any nominal system $G_p(s)$ when working under effect of external noise or disturbed conditions, It must be less sensitive to noise or disturbed conditions. It means mathematically, magnitude of Sensitivity must be less than 1, $\|S_x\| \le 1$. Suppose system is more sensitive to disturbed conditions going to be unstable, and then there is need to reduce sensitivity of system. As sensitivity of system is increasing, It will become a big problem for us to decrease the sensitivity.

But, solution of this problem is to add the uncertainty weight $W_r(s)$ as filter or whose magnitude is to be increased for making system less sensitive to disturbed conditions. It means $|W|_r \propto \left|\frac{1}{S_x}\right|$, uncertainty weight must be inverse proportional to sensitivity to get robust and stable environment in system/plant .

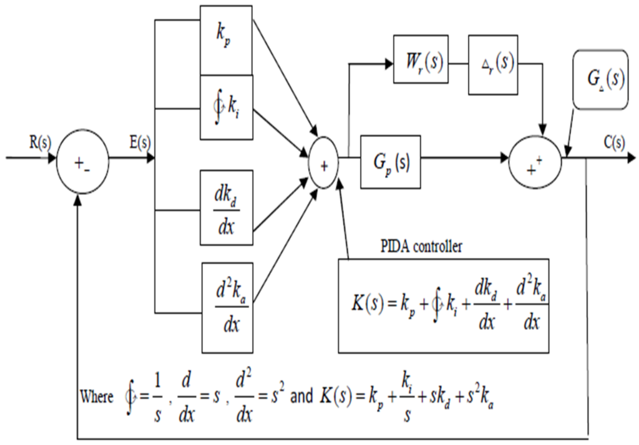

Figure -2 Closed loop system under effect of disturbed conditions with uncertainty weight [4]

In above figure (2), K(s) is PIDA controller with proportional (k_p), integral (k_i) ,derivative (k_d) and accelerator gains (k_a). *R(s)* is input and *C(s)* is output. $\Delta_r(s)$ is stability margin whose value must be less than one to keep system stable. $W_r(s)$ is multiplied with $\Delta_r(s)$ to keep the system in nominal working conditions and we obtain less sensitivity of system to noise environment.

From above figure (3) , we can calculate disturbed plant as given below.

$$G_{\Delta}(s) = G_p(s) + W_r(s) \times \Delta_r(s) \qquad (5)$$

Where

$G_{\Delta}(s)$ is disturbed plant, $G_p(s)$ is nominal plant without effect of external noise or disturbance . $W_r(s)$ is uncertainty weight

III. Steps to calculate uncertainty weight

To calculate uncertainity weight, $\Delta_r(s)$ is stability margin whose value must be less than one to keep system stable

$$\Delta_r(s) << 1 \quad approximate....\Delta_r(s) \cong 1 \qquad (6)$$

From above equation (5), it can be rewritten given below

$$\Delta_r(s) = \frac{G_{\Delta}(s) - G_p(s)}{W_r(s)} \xrightarrow{\text{Rearrange}} \frac{G_{\Delta}(s) - G_p(s)}{W_r(s)} = \Delta_r(s)$$

$$\frac{G_{\Delta}(s) - G_p(s)}{W_r(s)} \cong 1 \xrightarrow{\text{Rearrange}} W_r(s) \cong G_{\Delta}(s) - G_p(s) \qquad (7)$$

Where

$$W_{\Delta/p}(s) = G_{\Delta}(s) - G_p(s) \ , \quad W_r(s) > W_{\Delta/p}(s) \qquad (8)$$

From above equation (7), $W_{\square/p}(s)$ must be equal to difference of disturbed palnt and nominal plant, Equation (8) $W_r(s)$ reveals that Its magnitude must be greater than $W_{\square/p}(s)$, In this way, we can find out the $W_r(s)$ magnitude .

IV. Steps to design sensitivity constraint

Now find out overall transfer function of figure (2) given below

$$M(s) = \frac{C(s)}{R(s)} = \frac{G_{\square}(s) \times K(s)^{PIDA}}{1 + \{G_{\square}(s) \times K(s)^{PIDA} \times 1\}} \tag{9}$$

Now, System sensitivity will be given below.

$$S_x = \frac{1}{1 + \{G_{\square}(s) \times K(s)^{PIDA} \times 1\}} \tag{10}$$

We always know that magnitude of sensitivity that must be less than 1 to achieve more robust stability of nominal plant as shown in equation (11) [4].

Robust sensitivity stability constraint is

$$\|S_x\| \le 1 \tag{11}$$

Put value of S_x from equation (10) into robust stability constraint (11), we get following equation.

$$\left\| \frac{1}{1 + \{G_{\square}(s) \times K(s)^{PIDA} \times 1\}} \right\| \le 1 \tag{12}$$

Where

$$K(s) = k_p(s) + \frac{k_i}{s} + s \times k_d + s^2 \times k_a \qquad \textbf{\textit{PIDA controller}} \tag{13}$$

An equation (14) as robust stability constraint can be achieved by putting value of $G_{\square}(s)$ from equation (5) and $K(s)$ from equation (13) into above equation (12)..

$$\left\| \frac{1}{1 + \left\| \begin{array}{l} \left\langle G_p(s) + \{W_r(s) \times \Delta_r(s)\} \right\rangle \times \\ \left\langle k_p + \frac{k_i}{s} + s \times k_d + s^2 \times k_a \right\rangle \end{array} \right\|} \right\| \le 1 \tag{14}$$

The polar form [15] of any equation can be $PF = \|PF\| e^{j\theta}$, apply this form to equation (14), we get equation (15) as given below

If

$$PF = \frac{1}{1+\left\| \begin{matrix} \left\langle G_p(s)+\{W_r(s)\times\Delta_r(s)\}\right\rangle \times \\ \left\langle k_p+\frac{k_i}{s}+s\times k_d+s^2\times k_a \right\rangle \end{matrix} \right\|} \quad (15)$$

As per polar form and equation (15), re-write above equation (14) given below,

$$\left\| \frac{1}{1+\left\| \begin{matrix} \left\langle \begin{matrix} G_p(s)+ \\ \{W_r(s)\times\Delta_r(s)\} \end{matrix} \right\rangle \times \\ \left\langle k_p+\frac{k_i}{s}+s\times k_d+s^2\times k_a \right\rangle \end{matrix} \right\|} \right\| = \frac{1}{1+\left\| \begin{matrix} \left\langle \begin{matrix} G_p(s)+ \\ \{W_r(s)\times\Delta_r(s)\} \end{matrix} \right\rangle \times \\ \left\langle k_p+\frac{k_i}{s}+s\times k_d+s^2\times k_a \right\rangle \end{matrix} \right\|} \times e^{j\angle-\theta}$$

Where

$$\theta_x = \angle-\theta = \angle \frac{-1}{1+\left\| \left\langle G_p(s)+\{W_r(s)\times\Delta_r(s)\}\right\rangle \times \left\langle k_p+\frac{k_i}{s}+s\times k_d+s^2\times k_a \right\rangle \right\|}$$

Using above equation (14) and its polar form, an equation (16) can be achieved given below

$$\frac{e^{j\theta_x}}{1+\left\| \begin{matrix} \left\langle G_p(s)+\{W_r(s)\times\Delta_r(s)\}\right\rangle \times \\ \left\langle k_p+\frac{k_i}{s}+s\times k_d+s^2\times k_a \right\rangle \end{matrix} \right\|} \le 1 \quad (16)$$

Put into equation (16) Approximate 0.99<=1 $\Rightarrow$ 0.99 $\approx$<=1

We get following equation (17).

$$\frac{e^{j\theta_x}}{1+\left\| \begin{matrix} \left\langle G_p(s)+\{W_r(s)\times\Delta_r(s)\}\right\rangle \times \\ \left\langle k_p+\frac{k_i}{s}+s\times k_d+s^2\times k_a \right\rangle \end{matrix} \right\|} = 0.99 \quad (17)$$

Rearranging above equation (17), equation (18) can be rewritten given below.

$$\left(1+\left\|\begin{matrix}\left\langle G_p(s)+\{W_r(s)\times\Delta_r(s)\}\right\rangle\times\\ \left\langle k_p+\frac{k_i}{s}+s\times k_d+s^2\times k_a\right\rangle\end{matrix}\right\|\right)-\frac{e^{i\theta_x}}{0.99}=0 \tag{18}$$

Where

$$G_p(s)=\partial_p(s)+j\lambda_p(s) \tag{19}$$

$\partial_p(s)$ and $\lambda_p(s)$ is real and imaginary part of plant

$$W_r(s)=\Psi_r(s)+j\Phi_r(s) \tag{20}$$

$\Psi_r(s)$ and $\Phi_r(s)$ is real and imaginary part of uncertainty weight.

$$\Delta_r(s)<<1 \quad \text{approximate} \quad \Delta_r(s)\approx 1 \tag{21}$$

As mathematically, It is known from [4] written given below in form of equation (22).

$$e^{j\theta_x}=\cos\theta_x+j\sin\theta_x \;, \qquad s=j\ \omega \tag{22}$$

Where [4]

$j^2=-1$ and $j=\sqrt{-1}$

Put all value of equations (19), (20). (21), (22) into equation (18), we get equation as follows,

$$\left(1+\left\|\begin{matrix}\left\langle\begin{matrix}\{\partial_p(j\omega)+j\lambda_p(j\omega)\}+\\ \{\{\Psi_r(j\omega)+j\Phi_r(j\omega)\}\times 0.99\}\end{matrix}\right\rangle\times\\ \left\langle k_p-j\frac{k_i}{\omega}+j\times\omega\times k_d-\omega^2\times k_a\right\rangle\end{matrix}\right\|\right)-\frac{\cos\theta_x+j\sin\theta_x}{0.99}=0 \tag{23}$$

Break above equation (23) into real and imaginary part in form of equation (24),(25) and (26).

$$\begin{bmatrix}A11\times k_p+\left(\frac{B11}{\omega}\right)\times k_i-\left(B11\times\omega\times k_d\right)-(A11\times\omega^2\times k_a)\\ =\frac{\cos\ \theta_x}{0.99}-1\end{bmatrix}^{REAL\_PART} \tag{24}$$

$$\begin{bmatrix} B11\times k_p-\left(\dfrac{A11}{\omega}\right)\times k_i+\left(A11\times\omega\times k_d\right)-\left(B11\times\omega^2\times k_a\right) \\ =\dfrac{\sin\theta_x}{0.99} \end{bmatrix}^{IAMGINARY\_PART} \tag{25}$$

Where

$$\begin{matrix} A11=\left(\partial_p(j\omega)+\{0.99\times\Psi_r(j\omega)\}\right) \\ B11=\left(\lambda_p(j\omega)+\{0.99\times\Phi_r(j\omega)\}\right) \end{matrix} \tag{26}$$

V. STEPS TO CALCULATE (k_p, k_i) KEEPING VALUE OF $k_d = k_{dt}$ AND $k_a = k_{at}$ CONSTANT

As per Conversion of equations (24), (25) into Matrix form given below with help of [15].

$$\begin{pmatrix} A11 & \left(\dfrac{B11}{\omega}\right) \\ B11 & \left(\dfrac{-A11}{\omega}\right) \end{pmatrix}\begin{pmatrix} k_p \\ k_i \end{pmatrix} = \begin{pmatrix} \left\{ \begin{matrix} \left(B11\times\omega\times k_{dt}\right)+ \\ \left(A11\times\omega^2\times k_{at}\right)+ \\ \dfrac{\cos\ \theta_x}{0.99}-1 \end{matrix} \right\} \\ \left\{ \begin{matrix} \dfrac{\sin\theta_x}{0.99}- \\ \left(A11\times\omega\times k_{dt}\right)+ \\ \left(B11\times\omega^2\times k_{at}\right) \end{matrix} \right\} \end{pmatrix}_{2\times 2\_matrix} \tag{27}$$

Above equation (27) can be written in form of equation (28)

$$M_1 \times \mathrm{X} = \mathrm{B} \tag{28}$$

Where

$$M_1 = \begin{pmatrix} A11 & \left(\dfrac{B11}{\omega}\right) \\ B11 & \left(\dfrac{-A11}{\omega}\right) \end{pmatrix}, \mathrm{X} = \begin{pmatrix} k_p \\ k_i \end{pmatrix},$$

$$B = \begin{pmatrix} \left\{ \begin{matrix} \left(B11\times\omega\times k_{dt}\right)+ \\ \left(A11\times\omega^2\times k_{at}\right)+\dfrac{\cos\ \theta_x}{0.99}-1 \end{matrix} \right\} \\ \left\{ \begin{matrix} \dfrac{\sin\theta_x}{0.99}- \\ \left(A11\times\omega\times k_{dt}\right)+\left(B11\times\omega^2\times k_{at}\right) \end{matrix} \right\} \end{pmatrix} \tag{29}$$

As from equation (28) ,

$$X = M_1^{-1} \times B \tag{30}$$

In this way , PIDA controller (proportional, integral gains k_p, k_i) can be obtained by using above methodology.

The Value of Proportional controller (k_p,), Integral controller(k_i) given below.

$$\begin{pmatrix} k_p \\ k_i \end{pmatrix} = M_1^{-1} \times \begin{pmatrix} \left\{ \begin{array}{l} (B11 \times \omega \times k_{dt}) + (A11 \times \omega^2 \times k_{at}) + \\ \frac{\cos\ \theta_x}{0.99} - 1 \end{array} \right\} \\ \left\{ \begin{array}{l} \frac{\sin \theta_x}{0.99} - (A11 \times \omega \times k_{dt}) + \\ (B11 \times \omega^2 \times k_{at}) \end{array} \right\} \end{pmatrix} \tag{31}$$

Find out M_1^{-1} and put into equation (31) with help of [16].

$$M_1^{-1} = \frac{Adj(M\ 1)}{|M\ 1|} \text{ and } Adj(M_1) = \begin{pmatrix} \left(\frac{-A11}{\omega}\right) & \left(\frac{-B11}{\omega}\right) \\ -B11 & A11 \end{pmatrix} \tag{32}$$

$$M\ 1^{-1} = \frac{\begin{pmatrix} \left(\frac{-A11}{\omega}\right) & \left(\frac{-B11}{\omega}\right) \\ -B11 & A11 \end{pmatrix}}{|M\ 1|} = \begin{pmatrix} \frac{\left(\frac{-A11}{\omega}\right)}{|M\ 1|} & \frac{\left(\frac{-B11}{\omega}\right)}{|M\ 1|} \\ \frac{-B11}{|M\ 1|} & \frac{A11}{|M\ 1|} \end{pmatrix} \tag{33}$$

In this way

$$\begin{pmatrix} k_p \\ k_i \end{pmatrix} == \begin{pmatrix} \frac{\left(\frac{-A11}{\omega}\right)}{|M\ 1|} & \frac{\left(\frac{-B11}{\omega}\right)}{|M\ 1|} \\ \frac{-B11}{|M\ 1|} & \frac{A11}{|M\ 1|} \end{pmatrix} \times \begin{pmatrix} \left\{ \begin{array}{l} (B11 \times \omega \times k_{dt}) + \\ (A11 \times \omega^2 \times k_{at}) + \\ \frac{\cos\ \theta_x}{0.99} - 1 \end{array} \right\} \\ \left\{ \begin{array}{l} \frac{\sin \theta_x}{0.99} - \\ (A11 \times \omega \times k_{dt}) + \\ (B11 \times \omega^2 \times k_{at}) \end{array} \right\} \end{pmatrix} \tag{34}$$

$$M1 = \begin{vmatrix} \left(A11 & \left(\frac{B11}{\omega}\right) \right) \\ \left(B11 & \left(\frac{-A11}{\omega}\right) \right) \end{vmatrix} = \begin{Bmatrix} A11 \times \left(\frac{-A11}{\omega}\right) - \\ B11 \times \left(\frac{B11}{\omega}\right) \end{Bmatrix} = \begin{Bmatrix} \left(\partial_p(j\omega) + \{0.99 \times \Psi_r(j\omega)\}\right) \times \\ \left(\frac{-\left(\partial_p(j\omega) + \{0.99 \times \Psi_r(j\omega)\}\right)}{\omega} \right) - \\ \left(\bar{\lambda}_p(j\omega) + \{0.99 \times \Phi_r(j\omega)\}\right) \times \\ \left(\frac{\left(\bar{\lambda}_p(j\omega) + \{0.99 \times \Phi_r(j\omega)\}\right)}{\omega} \right) \end{Bmatrix}$$

$$\begin{aligned} &\textit{where} \\ &A11 = \left(\partial_p(j\omega) + \{0.99 \times \Psi_r(j\omega)\}\right) \\ &B11 = \left(\bar{\lambda}_p(j\omega) + \{0.99 \times \Phi_r(j\omega)\}\right) \end{aligned}$$

Find out gains k_p and k_i keeping value of k_{dt} and k_{at} constant

$$\begin{pmatrix} k_p \\ k_i \end{pmatrix} = \left(\begin{matrix} \left\{ \begin{matrix} \left\langle \frac{\left(\frac{-A11}{\omega}\right)}{|M1|} \right\rangle \times \\ \left\langle \left(B11 \times \omega \times k_{dt}\right) + (A11 \times \omega^2 \times k_{at}) + \frac{\cos\ \theta_x}{0.99} - 1 \right\rangle + \\ \left\langle \frac{\left(\frac{-B11}{\omega}\right)}{|M1|} \right\rangle \times \\ \left\langle \frac{\sin\theta_x}{0.99} - \left(A11 \times \omega \times k_{dt}\right) + (B11 \times \omega^2 \times k_{at}) \right\rangle \end{matrix} \right\} \\ \left\{ \begin{matrix} \left\langle \frac{-B11}{|M1|} \right\rangle \times \\ \left\langle \left(B11 \times \omega \times k_{dt}\right) + (A11 \times \omega^2 \times k_{at}) + \frac{\cos\ \theta_x}{0.99} - 1 \right\rangle + \\ \left\langle \frac{A11}{|M1|} \right\rangle \times \left\langle \frac{\sin\theta_x}{0.99} - \left(A11 \times \omega \times k_{dt}\right) + (B11 \times \omega^2 \times k_{at}) \right\rangle \end{matrix} \right\} \end{matrix} \right) \tag{35}$$

The Value of Proportional controller (k_p,), Integral controller(k_i) has been achieved using Matrix techniques [15]

$$k_p = \left\{ \begin{matrix} \left\langle \frac{\left(\frac{-A11}{\omega}\right)}{|M1|} \right\rangle \times \left\langle \left(B11 \times \omega \times k_{dt}\right) + (A11 \times \omega^2 \times k_{at}) + \frac{\cos\ \theta_x}{0.99} - 1 \right\rangle + \\ \left\langle \frac{\left(\frac{-B11}{\omega}\right)}{|M1|} \right\rangle \times \left\langle \frac{\sin\theta_x}{0.99} - \left(A11 \times \omega \times k_{dt}\right) + (B11 \times \omega^2 \times k_{at}) \right\rangle \end{matrix} \right\} \tag{36}$$

$$k_i = \begin{Bmatrix} \left\langle \frac{-B11}{|M1|} \right\rangle \times \left\langle \left(B11 \times \omega \times k_{dt}\right) + (A11 \times \omega^2 \times k_{at}) + \frac{\cos\ \theta_x}{0.99} - 1 \right\rangle + \\ \left\langle \frac{A11}{|M1|} \right\rangle \times \left\langle \frac{\sin\theta_x}{0.99} - \left(A11 \times \omega \times k_{dt}\right) + (B11 \times \omega^2 \times k_{at}) \right\rangle \end{Bmatrix} \quad (37)$$

VI. RESULTS

As considered nominal plant given below

$$G_p(s) = \frac{64.4}{\left(s \times \left(s + 34.7\right)\right)} \times e^{-t_d^{nominal\_delay} \times s}$$

Where at

$t_d^{nominal\_delay} \in [0.1]$ Plant will remain stable

Nominal plant selected- $G_p(s) = \frac{64.4}{\left(s \times \left(s + 34.7\right)\right)} \times e^{-0.1 \times s}$ (38)

From equation (8) uncertainty weight can be selected given below

$$W_{\square/p}(s) = G_{\square}(s) - G_p(s) = \left\langle \begin{matrix} \left\{ \frac{64.4}{\left(s \times \left(s + 34.7\right)\right)} \times e^{-t_{ud}^{uncertain\_delay} \times s} \right\} - \\ \left\{ \frac{64.4}{\left(s \times \left(s + 34.7\right)\right)} \times e^{-t_d^{nominal\_delay} \times s} \right\} \end{matrix} \right\rangle$$

Where

$t_{ud}^{uncertain\_delay} \subset [0.01, 0.20]$ for disturbed plant

Uncertainty weight selected to cover all uncertain conditions by obtaining given formula as $W_r(s) > W_{\square/p}(s)$

$$W_r(s) = \frac{0.14 \times 20.40 \times 100.02}{\left((s + 20.40) \times \left(s + 100.2\right)\right)} \quad (39)$$

As, we know from above equation (13), PIDA controller selected given below on stable region in figure (4).

$$K(s)^{PIDA} = \begin{bmatrix} K_1(s) \\ K_2(s) \end{bmatrix} = \begin{bmatrix} \frac{10s^3 + 0.7s^2 - 318.5s + 6768}{s} \\ \frac{10s^3 + 0.7s^2 + 834.7s + 3390}{s} \end{bmatrix}_{\text{PIDA}} \quad (40)$$

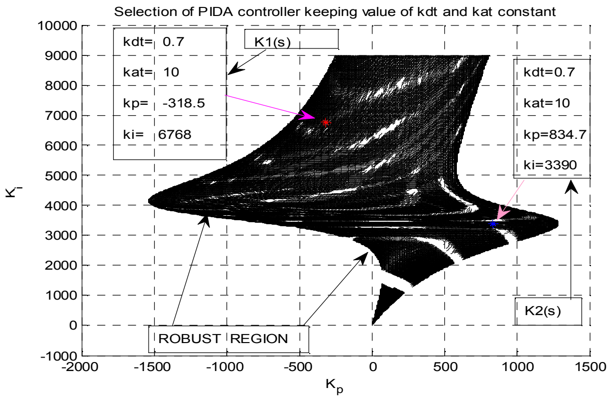

Figure-4 selection of K1(s) and K2(s) PIDA controller with constant value of k_{dt} =0.7and k_{at} =10

Here, sensitivity stability constraint was achieved with help of equations (11), (12), (14) , (38) ,(39). (40), (41) and (42) given below

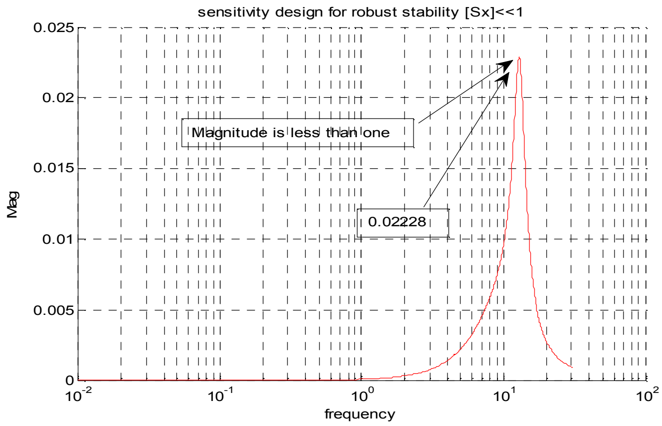

Figure-5 Magnitude of sensitivity

After selection of an uncertainty weight from equation (39), nominal plant from equation (38) and K1(s) PIDA controller from equation (40) with help of figure (4) to Insert all selected values into sensitivity constraint, A required robust stability criteria cn be achieved as shown in equations (41) and figure (5).

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$$S_x = \left\| \frac{1}{1+\left\| \left\langle G_p(s)+\{W_r(s)\times 0.99\} \right\rangle \times K1(s)^{PIDA} \right\|} \right\| = 0.02228 \quad (41)$$

Uncertainty weight from equation (39), nominal plant from equation (38) and K2(s) PIDA controller from equation (40) is selected with help of figure (4) to Insert all selected values into sensitivity constraint, A required robust stability criteria can be achieved as shown in equations (42) and figure (6).

$$S_x = \left\| \frac{1}{1+\left\| \left\langle G_p(s)+\{W_r(s)\times 0.99\} \right\rangle \times K2(s)^{PIDA} \right\|} \right\| = 0.05457 \quad (42)$$

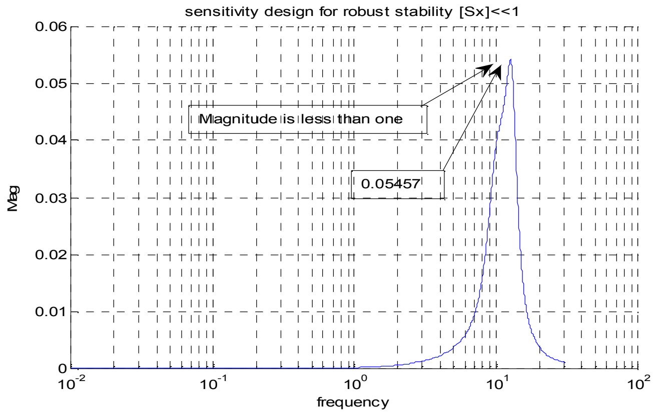

Figure-6 Magnitude of sensitivity

In this way, The sensitivity stability region after plotting (K_p, K_i) region as shown in figure-4 can be achieved. After selection of K1 and K2(s) PIDA controller on stable region in figure (4) , A robust sensitive stability criteria can be achieved as shown in figure (5) and figure (6).

VII. CONCLUSION

In this paper, practical design method was proposed for S_∞ sensitivity design with PIDA controller for general single input single output LTI(linear time invariant) system with time delay. .Note that design problem is shown to the PIDA controller that has been converted into simultaneous stabilization of the complex quasi-polynomial equations. The sensitivity stability region after plotting (K_p, K_i) region as shown in figure-4 is achieved. After selection of K1 PIDA controller on stable region and K2(s) PIDA controller on stable region , A robust sensitive stability criteria was achieved as shown in figure (5) and figure (6). With wide use of PID and PI controller in industrial process, a methodology was created here to design here PIDA controller . PIDA controller has proportional, integral , derivative and accelerator gains that definitely reduce instability in plant . It is expected that the all the result calculated in this paper will contribute to the development of practical control system design.

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