OBSERVATIONS ON THE CUBIC DIIPHANTINE EQUATION WITH FOUR UNKNOWNS $x^2 + y^2 + z^2 = 33(x + y + z)w^2$

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Abstract

The non-homogeneous cubic equation with four unknowns represented by the Diophantine equation $x^2 + y^2 + z^2 = 33(x + y + z)w^2$is analyzed for its patterns of non-zero distinct integral solutions. A few interesting properties among the solutions are presented.

Keywords : Ternary cubic, Integral solution, non-homogeneous cubic, with four unknowns

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1 Introduction

The Diophantine equations offer an unlimited field for research due to their variety [1-4, 8-15]. In particular, one may refer [5, 6, 7, 16] for cubic equations with four unknowns. This communication concerns with yet another interesting equation $x^2 + y^2 + z^2 = 33(x + y + z)w^2$ representing non-homogeneous cubic equation with four unknowns for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

Notations

$T_{m,n}$ = Triangular number of rank n,
P_m^n = Pyramidal number of rank n,
Pn^n = Pronic number of rank n,
$Star_n$ = Star number of rank n,

FN_{n4} = Four dimensional figurative number whose generating polygon is a squre,
SO_n = Stella octangular number of rank n,
CP_{n6} = Centered hezagonal pyramidal number of rank n,
Bq_n = Biquadratic number of rank n.

2 Methods of Analysis

The cubic equation to be solved is

(2.1) $$x^2 + y^2 + z^2 = 33(x + +y + z)w^2$$

Introducing the linear transformations

(2.2) $$\begin{cases} x = u + 2v, \\ y = xu - 2v, \\ z = -5u, \end{cases}$$

where u and v are non. zero parameters. Equation (2.1) can be written as

(2.3) $$41u^2 - 8v^2 = 33w^2.$$

Assuming that

(2.4) $$u = X + 8T,$$

(2.5) $$v = X + 41T$$

equation (2.3) becomes

(2.6) $$X^2 = 328T^2 + W^2.$$

We present four different methods of solving (2.6) and this obtain four different integer solutions to (2.1) as follows.

Method 1

The general solutions to (2.6) are given by

(2.7) $$X = 328r^2 + s^2,$$

(2.8) $$T = 2ts,$$

(2.9) $$W = 328r^2 - s^2.$$

Using (2.7) and (2.8) in (2.2) the non-zero integral solutions of equation (2.1) are satisfied
$x(r, s) = 984r^2 + s^2 + 180rs,$
$y(r, s) = -328r^2 - s^2 - 148rs,$
$z(r, s) = -1640r^2 - 5s^2 - 80rs,$
$w(r, s) = 328r^2 - s^2.$

Properties

1. $x(a, a+1) - 981T_{4}, 1 - 3CS_a - 180Pr_a = 0,$
2. $x(a, 6a-5) - 1092T_{4,1} - 180T_{14,a} \equiv 75(mod180),$
3. $x(a, 2a^2-1) - 3CH_a - 987T_{4,a} - 180so_a \equiv 0(mod3),$
4. $y(a, 4a-3) + 327T_{4,a} - CS_a - 296PP_a = 0,$
5. $y(a, 4a-3) + 344T_{4,a} + 148T_{10,a} \equiv 9(mod24),$
6. $z(a, 7a-6) + 1885T_{4,a} + 80T_{16,a} \equiv 180(mod420),$
7. $z(a^2, 2a-1) + 1640T^2_{4,a} + 20T_{4,a} + 160P^6_a \equiv 5(mod20),$
8. $w(a+1, a-1) - 327T_{4,a} \equiv 327(mod68),$
9. $w(a^2, a+1) - 327T_{4,a} \equiv 1(mod2),$
10. $y(a, a) + w(a, a) = -150a^2$ is a nasty number.

Method 2

Re write equation (2.6) as

(2.10) $$(w + i2\sqrt{82})(w - i2\sqrt{82}) = X^2 \times 1.$$

Replace (2.1) by

$$1 = \frac{(9 + i8\sqrt{82})(9 - i8\sqrt{82})}{(73)^2}.$$

Assume that

(2.11) $$X = a^2 + 328b^2.$$

Therefore (2.10) can be written as

$$(w + i2\sqrt{82})(w - i2\sqrt{82}) = \frac{9 + i8\sqrt{82}}{73}\frac{(9 - i8\sqrt{82})}{73}(a + ib2\sqrt{82})^2(a - ib2\sqrt{82})^2.$$

Equating the positive parts on the both sides, we get

$$(w + i2\sqrt{82}) = \frac{(9 + i8\sqrt{82})}{73}(a + ib2\sqrt{82})^2,$$

$$(w + i2\sqrt{82}) = \frac{(9 + i8\sqrt{82})}{73}(a^2 - 328b^2 + 2ab)(i2\sqrt{82}.$$

Equating the real and imaginary parts, we obtain

(2.12) $$w = \frac{1}{73}(9a^2 - 2952b^2 - 2624ab),$$

(2.13) $$T = \frac{1}{73}(4a^2 - 1312b^2 + 18ab).$$

We note that the value of w and T are integers for the following choices of a and b

$a = 73A = 73B$

This the non-Zero integral of w and T are represented by

(2.14) $$w = 657A^2 - 215496B^2 - 191552AB,$$

(2.15) $$T = 292A^2 - 95776B^2 + 1314AB,$$

(2.16) $$X = 5329A^2 + 1747912B^2.$$

On substituting (2.15) and (2.16) in (2.4) and (2.5) the values of u and v are exhibited by

(2.17) $$u = 7665A^2 + 981704B^2 + 10512AB,$$

(2.18) $$v = 17301A^2 - 217890B^2 + 53874AB.$$

Using (2.17) and (2.18) in (2.2) the two parametric integral solutions of (2.1) are exhibited by

$x(A, B) = 42267A^2 + 3376104B^2 + 118260AB,$
$y(A, B) = -26937A^2 + 5339512B^2 - 97236AB,$
$z(A, B) = -38325A^2 - 4908520B^2 - 52560AB,$
$w(A, B) = 657A^2 - 215496B^2 - 191552AB.$

Properties

1. $x(a, 2a^2 + 1) + 13504416T_{4,a}^2 - 13546683T_{4,a} - 354780OH_a = 3376104,$
2. $x(a, 5a - 4) - 8436033T_{4,a} + 118260T_{12,a} \equiv 54017664(mod135044160),$
3. $x(a, 9a - 8) - 273506691T_{4,a} - 11826T_{20,a} \equiv 216070656(mod486158976),$
4. $y(a, 10a - 9) - 533924263T_{4,a} + 97236T_{22,a} \equiv 432500472(mod1057223376),$
5. $y(a, a + 1) - 5312575T_{4,a} + 97236Pr_a \equiv 5339512(mod10707024),$
6. $z(a, 2a - 1) + 19672405T_{4,a} + 5256OHG_a \equiv 4908520(mod19634080),$
7. $z(a^2, a^2 - 1) + 4946845T_{4,a}^2 - 917040T_{4,a} + 630720FN_a^4 - 4908520 = 0,$
8. $w(a^2, a + 1) - 657T_{4,a}^2 + 215496T_{4,a} + 383104PP_a \equiv 215496(mod430992),$
9. $w(a, 1) - 657T_{4,a} \equiv 215496(mod191552),$
10. $w(a, a) - 406391T_{4,a} = 0.$

Method 3

Rewrite (2.6) as

(2.19) $$X^2 - W^2 = 328T^2, (X + W)(X - W) = 41T8T,$$

$$\Rightarrow \frac{X + W}{41T} = \frac{8T}{X - W} = \frac{p}{q}, q = 0,$$ which is equivalent to the following system of double equations

(2.20) $$41Tp - Xq - Wq = 0,$$

(2.21) $$8Tq - Xp + Wp = 0.$$

Solving (2.20) and (2.21) by the method of cross multiplication, we get

(2.22) $$T = -2pq,$$

(2.23) $$X = -8q^2 - 41p^2,$$

(2.24) $$W = -41p^2 + 8q^2.$$

Substituting (2.23) and (2.24) in (2.4) and (2.5), we obtain

$u = -8q^2 - 41p^2 - 16pq,$

$v = -8q^2 - 41p^2 - 82pq.$

Using the values of u and v in (2.2) the infinitely many non-trivial integral values satisfying (2.1) are given by

$x(p, q) = -24q^2 - 123p^2 - 180pq,$

$y(p, q) = 8q^2 + 41p^2 + 148pq,$

$z(p, q) = 40q^2 + 205p^2 + 80pq,$

$w(p, q) = -41p^2 + 8q^2.$

Properties

1. $x(a, a) + 147T_{4,a} = -180a^2$ is a nasty number,
2. $x(a, 8a - 7) + 1659T_{4,a} + 180T_{18,a} \equiv 1176(mod 2688),$
3. $y(a^2, a^2 - 1) - 8T^2_{4,a} + 25T_{4,a} + 1776FN^4_a + 8 = 0,$
4. $z(a, a + 1) - 245T_{4,a} + 80Pr_a \equiv 9(mod 80),$
5. $w(a + 1, a + 2) + 33T_{4,a} \equiv 9(mod 50).$

Case i

Equation (2.20) can be written as

$$\frac{X + W}{8T} = \frac{41T}{X - W} = \frac{p}{q}, q \neq 0,$$

which is equivalent to the following system of double equations

(2.25) $$8Tp - Xq - Wq = 0,$$

(2.26) $$41Tq - Xq + Wp = 0.$$

Solving (2.25) and (2.26) by the method of cross multiplication, we get

(2.27) $$T = 2pq,$$

(2.28) $$X = -41q^2 - 8p^2,$$

(2.29) $$W = -8p^2 + 41q^2.$$

Substituting (2.28) and (2.29) in (2.4) and (2.5), we obtain

$u = -41q^2 - 8p^2 - 16pq,$

$v = -41q^2 - 8p^2 - 82pq.$

Using the values of u and v in (2.2) the infinitely many non-trivial integral values satisfying given by

$x(p, q) = -123q^2 - 24p^2 - 180pq,$

$y(p, q) = 41q^2 + 8p^2 + 148pq,$

$z(p, q) = 205q^2 + 40p^2 + 80pq,$

$w(p, q) = -8p^2 + 41q^2.$

Prpoperties

1. $x(a, 1) + 24T_{4,a} \equiv 123(mod180),$
2. $y(a, 11a - 9) - 4969T_{4,a} - 296T_{13,a} - M_{11} - J_{10},$
3. $z(a, 4a - 3) - 3320T_{4,a} - 80T_{10,a} \equiv 1845(mod4920),$
4. $w(a + 3, a - 2) + 8T_{4,a} \equiv 92(mod116),$
5. $z(a, 1) + 8T_{4,a} - M_5 = 0.$

Method 4

We write (2.6) as

(2.30) $$\frac{X + W}{X - W} = 328T \times 1T,$$

$\frac{X + W}{328T} = \frac{T}{X - W} = \frac{p}{q}, q \neq 0$ which is equivalent to the following system of double equations

(2.31) $$328pT - Xq - Wq = 0,$$

(2.32) $$qT - Xp - Wp = 0.$$

Solving (2.31) and (2.32) by the method of cross multiplication, we get

(2.33) $$T = -2pq,$$

(2.34) $$X = -q^2 - 328p^2,$$

(2.35) $$W = -328p^2 + q^2.$$

Substituting (2.33) and (2.34) in (2.4) and (2.5), we obtain

$u = -q^2 - 328p^2 - 16pq,$

$v = -q^2 - 328p^2 - 82pq.$

Hence the non-trivial integral solutions of (2.1) are represent by

$x(p, q) = -3q^2 - 984p^2 - 180pq,$

$y(p, q) = q^2 + 328p^2 + 148pq,$

$z(p, q) = 5q^2 + 1640p^2 + 180pq,$

$w(p, q) = -328p^{(}2) + q^2.$

Properties

1. $x(a, 2a-1) + 996T_{4,a} + 180HG_a \equiv 3(mod 12)$,
2. $y(a^2, a+1) - 327T_{4,a} - CS_a - 296PP_a = 0$,
3. $z(a, 3a-1) - 1685T_{4,a} + 160T_{5,a} \equiv 5(mod 30)$,
4. $z(a, 1) - 1640T_{4,a} - j_2 \equiv 0(mod 80)$,
5. $w(a, a+2) + 327T_{4,a} \equiv 4(mod 4)$.

Case i

Equation (2.31) can be written as $\frac{X+W}{T} = \frac{328T}{X-W} = \frac{p}{q}, q \neq 0$,

which is equivalent to the following system of double equations

(2.36) $$Tp - Xq - Wq = 0,$$

(2.37) $$328Tq - Xp + Wp = 0.$$

Solving (2.36) and (2.37) by the method of cross multiplication, we get

(2.38) $$T = -2pq,$$

(2.39) $$X = -328q^2 - p^2,$$

(2.40) $$W = -p^2 + 328q^2.$$

Substituting (2.38) and (2.39) in (2.4) and (2.5), we obtain

(2.41) $$u = -328q^2 - p^2 - 16pq,$$

(2.42) $$v = -328q^2 - p^2 - 82pq.$$

Hence the non-trivial Integral solutions of (2.1) are represented by

$x(p, q) = -984q^2 - 3p^2 - 180pq$,
$y(p, q) = 328q^2 + p^2 + 148pq$,
$z(p, q) = 1640q^2 + 5p^2 + 180pq$,
$w(p, q) = -p^2 + 328q^2$.

Properties

1. $x(a, 3a-2) + 8859T_{4,a} + 180T_{8,a} \equiv 3936(mod\ 11808)$,
2. $y(a, 2a-1) - 1313T_{4,a} - 148T_{6,a} \equiv 328(mod\ 1312)$,
3. $w(a^2, a^2-1) - 328T_{4,a}^2 - 657T_{4,a} - j_8 - M_6 = 8$,

4. $w(2a, a) = 324a^2$ is a perfect square,

5. $x(a, 1) + y(a, 1) + z(a, 1) + w(a, 1) - 2T_{4,a} - M_9 - j_8 \equiv 32 (mod\ \ 48)$.

Method 5

We write (2.6) by

(2.43) $$(X + W)(X - W) = 82T \times 4T,$$

$$\frac{X + W}{82T} = \frac{4T}{X - W} = \frac{p}{q}, q \neq 0,$$

which is equivalent to the following system of double equations

(2.44) $$82pT - Xq - Wq = 0,$$

(2.45) $$4Tq - Xp + Wp = 0.$$

Solving (2.44) and (2.45) by the method of cross multiplication, we get

(2.46) $$T = -2pq,$$

(2.47) $$X = -4q^2 - 82p^2,$$

(2.48) $$W = -82p^2 + 4q^2.$$

Substituting (2.40) and (2.41) in (2.4) and (2.5), we obtain

$u = -4q^2 - 82p^2 - 16pq,$

$v = -4q^2 - 82p^2 - 82pq.$

Hence the non-trivial integral solutions of (2.1) are represented by

$x(p, q) = -12q^2 - 246p^2 - 180pq,$

$y(p, q) = 4q^2 + 82p^2 + 148pq,$

$z(p, q) = 20q^2 + 410p^2 + 80pq,$

$w(p, q) = -82(p^2) + 4q^2.$

Properties

1. $x(a, 13a - 11) + 2274T_{4,a} + 360T_{15,a} \equiv 1452 (mod 3168),$

2. $x(a, 7a - 6) - 834T_{4,a} - 180T_{16,a} \equiv 432 (mod 1008),$

3. $y(a, a + 1) - 78T_{4,a} - 4CS_a - 148Pr_a = 0,$

4. $z(a^2, a^2 - 1) - 410T^2_{4,a} - 20T_{4,a} - 960FN_a^4 \equiv 20 (mod 40),$

5. $w(a^2, 2a - 1) + 82T^2_{4,a} - 16T_{4,a} \equiv 4 (mod 16).$

Case i

Equation (2.37) can be written as $\frac{X + W}{4T} = \frac{82T}{X - W} = \frac{p}{q}, q \neq 0,$

which is equivalent to the following system of double equations

(2.49) $$4Tp - Xq - Wq = 0,$$

(2.50) $$82Tq - Xp + Wp = 0.$$

Solving (2.49) and (2.50) by the method of cross multiplication, we get

(2.51) $$T = -2pq,$$
(2.52) $$X = -82q^2 - 4p^2,$$
(2.53) $$W = -4p^2 + 82q^2.$$

Substituting (2.51) and (2.52) in (2.4) and (2.5), we obtain

$u = -82q^2 - 4p^2 - 16pq,$

$v = -82q^2 - 4p^2 - 82pq.$

Hence the non-trivial Integral solutions of (2.1) are represented by

$x(p, q) = -246q^2 - 12p^2 - 180pq,$

$y(p, q) = 82q^2 + 4p^2 + 148pq,$

$z(p, q) = 410q^2 + 20p^2 + 80pq,$

$w(p, q) = -4p^2 + 82q^2.$

Properties

1. $x(a, 11a - 10)14653T_{4,a} + 18T_{24,a} \equiv 24600(mod\ 54120),$
2. $x(a, 3a - 2) - 2226T_{8,a} \equiv 984(mod\ 2952),$
3. $y(a, 7a - 5) - 4022T_{4,a} - j_{11} - j_1 - 296T_{9,a} \equiv 0(mod\ 5740),$
4. $z(a, 9a - 7) - 33230T_{4,a} - 160T_{11,a} \equiv 20090(mod\ 51660),$
5. $w(a, 2a - 1) - 324T_{4,a} \equiv 82(mod\ 328).$

Method 6

We write (2.6) by

$(X + W)(X - W) = 164T \times 2T$

(2.54) $$\frac{X + W}{164T} = \frac{2T}{X - W} = \frac{p}{q}, q \neq 0,$$

Which is equivalent to the following system of double equations

(2.55) $$164pT - Xq - Wq = 0,$$

(2.56) $$2Tq - Xp + Wp = 0.$$

Solving (2.48) and (2.56) by the method of cross multiplication, we get

(2.57) $$T = -2pq,$$
(2.58) $$X = -2q^2 - 164p^2,$$
(2.59) $$W = -164p^2 + 2q^2.$$

Substituting (2.57) and (2.58) in (2.4) and (2.5), we obtain

$u = -2q^2 - 164p^2 - 16pq,$

$v = -2q^2 - 164p^2 - 82pq.$

Hence the non-trivial integral solutions of (2.1) are represent by

$x(p, q) = -6q^2 - 492p^2 - 180pq,$

$y(p, q) = 2q^2 + 164p^2 + 148pq,$

$z(p, q) = 10q^2 + 820p^2 + 80pq,$

$w(p, q) = -164p^2 + 2q^2.$

Properties

1. $x(a, 14a - 13) + 1668T_{4,a} + 180T_{30,a} \equiv 1014(mod 2184),$
2. $x(a, 13a - 12) + 1506T_{4,a}^2 + 180T_{28,a} \equiv 8864(mod 1506),$
3. $y(a, 27a - 25) - 1622T_{4,a} - J_{10} \equiv 225(mod 2700),$
4. $z(a, 23a - 21) - 6110T_{4,a} - J_{12} - 160T_{25,a} \equiv 3045(mod 9660),$
5. $w(a^2, a^2 - 1) + 162T_{4,a}^2 + 4T_{4,a} = 2.$

Case i

$$\frac{X + W}{2T} = \frac{164T}{X - W} = pq, q \neq 0,$$

Which is equivalent to the following system of double equations

(2.60) $$2Tp - Xq - Wq = 0,$$

(2.61) $$164Tq - Xp + Wp = 0.$$

Solving (2.60) and (2.61) by the method of cross multiplication, we get

(2.62) $$T = -2pq,$$

(2.63) $$X = -164 - 2p^2,$$

(2.64) $$W = -2p^2 - 164q^2.$$

Substituting (2.62) and (2.63) in (2.4) and (2.5), we obtain

$u = -164q^2 - 2p^2 - 16pq,$

$v = -164q^2 - 2p^2 - 82pq.$

Hence the non-trivial Integral solutions of (2.1) are represented by

$x(p, q) = 492q^2 - 6p^2 - 180pq,$

$y(p, q) = 164q^2 + 2p^2 + 148pq,$

$z(p, q) = 820q^2 + 10p^2 + 80pq,$

$w(p, q) = -2p^2 - 164q^2.$

Properties

1. $x(a, 21a - 19) + 216978T_{4,a} + 360T_{23,a} \equiv 177612(mod392616)$,
2. $y(a^2, a + 1) - 166T(4, a) - 296PP_a \equiv 164(mod328)$,
3. $z(a, 19a - 17) = 296030T_{4,a} - 160T_{21,a} \equiv 236980(mod529720)$,
4. $w(a, a + 1) + 166T_{4,a} \equiv 164(mod328)$,
5. $w(a, 2a^2 + 1) + 658T_{4,a} \equiv 164(mod656)$.

3 Conclusion

In this paper, we have obtained infinitely many non-zero distinct integer solutions to the ternary cubic Diophantine equation represented by $x^2 + y^2 + z^2 = 33(x + y + z)w^2$. One can also search for other patterns of solutions for the above equation.

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