

## “ DOMES ANALYSIS WITH VARIABLE THICKNESS”

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### ABSTRACT

Domes are curved structures-they have no angles and no corners and they enclose an enormous amount of space without the help of a single column. They are called “*King of Roofs*”. The study of history reveals that the domes are built usually with constant thickness. The Gol Gumbaz is a hemispherical dome which is built with variable thickness. Here in this project an attempt has been made to analyze domes with variable thickness. Two different domes (Hemispherical and Parabolic) have been analyzed theoretically using membrane theory. Further the same have been analyzed using finite element software Ansys. The values obtained from the finite element analysis are compared with theoretical membrane analysis. The variable thickness helps to built economical structure and optimum structure with optimum cost can be built.

**Key Words:** Dome, Analysis, Hemispherical and Parabolic

### INTRODUCTION

Domes are curved structures-they have no angles and no corners and they enclose an enormous amount of space without the help of a single column. Despite of their thinness, domes are some of the strongest and stiffest structures in existence today. A dome is a shell generated by the revolution of a regular geometrical curve about one axis. A dome can be split into two different directions; vertical sections separated by longitudinal arch lines also called as meridians and horizontal sections separated by hoops or parallels. The dome structures have always been a fascinating area of research. Their unpredictable behavior and difficulties in their mathematical as well as numerical

modeling make these structures a challenge for researchers and engineers. Since domes abound in nature, it is not surprising that they have been widely used as efficient load carrying members in many engineering structures. Domes or shells can sustain large amount of loads with little amount of material. These provide structurally efficient solution to the problem of carrying roof loads over long spans. These three dimensional forms owe their efficiency to translation of applied loads into tensile and compressive stress, as well as shear stress in their plane of surface. These are termed as membrane stresses. Shell may be singly curved in one direction in the form of a cylinder or in the doubly curved to form a dome or to form a saddle shaped surface.

***Classification of Shell Structures:***

**Ruled surface:** A ruled surface is defined as a surface formed by the motion of a straight line which is known as generator or ruling. A surface is said to be singly ruled if, at every point on the surface, a single straight line can be ruled and doubly ruled if at every point two straight lines can be ruled. Ruled surfaces have a practical advantage in that they may be cast on straight forms. Examples for single ruled surfaces are conical shells, conoids and cylinders and doubly ruled surfaces are hyperbolic paraboloid of revolution.

**Surface of translation** is generated by the motion of a plane curve parallel to itself over another curve, the planes containing the two curves being at right angles to each other. Examples are elliptic paraboloid and hyperbolic paraboloid.

A surface of revolution is obtained by rotating a plane curve called the meridian about an axis, lying in the plane of curve. This plane is known as meridian plane.

**Spherical domes:** Revolution of a circular curve about the vertical diameter.

**Elliptical domes:** Revolution of an elliptical curve about one of its axis

**Conical Domes:** Revolution of the hypotenuse of a right angle triangle about one of its sides

**Parabolic domes:** Revolution of a parabolic curve about its axis

Shell surface may be classified as singly curved and doubly curved. Singly curved surfaces are developable and the Gauss curvature is zero for such shell surface. The doubly curved shells may have either positive or negative Gaussian curvature. Shells with negative curvature are called as anticlastic shells. A developable surface can be both a translational and ruled surface such as cylindrical shells.

Similarly an anticlastic shell can be a shell of revolution and ruled surface as in the case of hyperbolic paraboloid of one sheet. There are shell surfaces like corrugated shells, funicular shells which cannot be classified under any one single category. They are normally partly synclastic and partly anticlastic. Some of common type of shells surfaces that are used in practice.

### **OBJECTIVE OF THE STUDY**

In practice, it is common to find analysis and design of domes with constant thickness. It is evident from literature that Gol Gumbaz is a dome of varying thickness. Here the thickness is varied from crown to the base of dome. In view of this the main aim of the present work is to study the effect of variable thickness on the stress distribution in the domes of different shapes.

The Hemispherical Dome and Parabolic domes are analyzed for variable thickness. From this analysis we can understand how the thickness variation affects the nature of Meridional and Circumferential Stresses. The commercially available finite elements software is used for the analysis purpose.

In the present study following work has been carried out

1. Hemispherical dome with variable thickness
  - ✓ Theoretical Membrane Analysis- an equation is derived
  - ✓ Ansys- value is compared
2. Parabolic dome with variable thickness
  - ✓ Theoretical Membrane Analysis - an equation is derived
  - ✓ Ansys- value is compared

The analysis is done using the Membrane theory through which an attempt is made to prove the results obtained are same

## **FINITE ELEMENT TECHNIQUE**

Finite Element Analysis (FEA) was first developed in 1943 by R. Courant, who utilized the Ritz method of numerical analysis and minimization of variation calculus to obtain approximate solutions to vibration systems.

The finite element analysis is a numerical technique. Here the complexities of the problem like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. Because of its diversity and flexibility as an analysis tool. It is receiving much attention in engineering. The fast improvements in computer hardware technology and slashing of cost of computers have boosted this method, since the computer is the basic need for the application of this method.

A number of popular brand of finite element packages are now available commercially. Some of the packages are now available commercially. Some of popular packages are STAAD-PRO, GT-STRUDEL, NASTRAN, NISA and ANSYS. Using these packages one can analyse several complex structures.

The finite element analysis originated as a method of stress analysis in the design of aircrafts. It started as an extension of matrix method of structural analysis. Today this method is used not only for the analysis in solid mechanics, but even in the analysis fluid flow, heat transfer electric and magnetic field and many others. Civil Engineers used this method extensively for the analysis of beams; space frames plates, shells, folded plates, foundations, rock mechanics and seepage analysis of fluid through porous media. Both static and dynamic problems can be handled by finite element analysis. This method is used extensively for the analysis and design of ships, aircrafts, spacecrafts and electric motors heat engines.

The analysis of domes is done using ANSYS software because of the modeling flexibility available and changes can be made very easily without much loss of time. Another reason for using this software was that it is an integrated package consisting of whole module required for modeling, meshing analysis, and post processing of results.

### ***Finite Element Modeling Of a Dome with Ansys***

The commercially available software Ansys 10 is used to model the dome structure. The finite element analysis involves the continuum by discrete elements. The shape of the element may be triangular, rectangular or quadrilateral etc. The shape of the element is often dictated by the geometric shape of the structure chosen for the analysis. Provided certain conditions are satisfied, the stresses and displacements resulting from the finite element analysis may be expected to converge towards their exact values as we progress from a coarser to the finer subdivision of the structure. Line element is suitable for beam problems and axisymmetric problems. Problems of plane stress and flat plate bending are handled by discretization involving the triangular, rectangular and quadrilateral elements. Thin shells may be idealized by flat faceted, curved, or solid elements in the form of bricks or tetrahedrons

The following procedure is adopted to model a spherical dome using Ansys

#### **ILLUSTRATION PROBLEM**

##### **1.Preferences**

- ✓ Structural

##### **2.Preprocessor**

###### **2.1 Element Type**

- ✓ Shell 61

##### **3.Real Constants**

- ✓ Shell thickness

###### **3.1 Material Properties**

- ✓ Material Models

- ❖ Structural

- ❖ Linear

- ❖ Elastic

- ❖ Isotropic

$$E= 22360 \text{ N/mm}^2$$

$$\text{Poisson's ratio}=0.15$$

$$\text{Density of concrete}=2400 \text{ kg/m}^3$$

#### 4.1 Modeling

Required finite element modeling is done by creating key points and lines in cylindrical co-ordinate system.

#### 5.1 Meshing

Using Mesh tools mesh the line elements.

#### 6.1 Loads

- ✓ Define loads
- ✓ Apply Structural

➤ Displacements

❖ On key points

#### 7.Solution

##### 7.1 Analysis type

- ✓ New Analysis-Static

##### 7.2 Solve

- ✓ Current LS

#### 8. General Post processor

Deformed shape

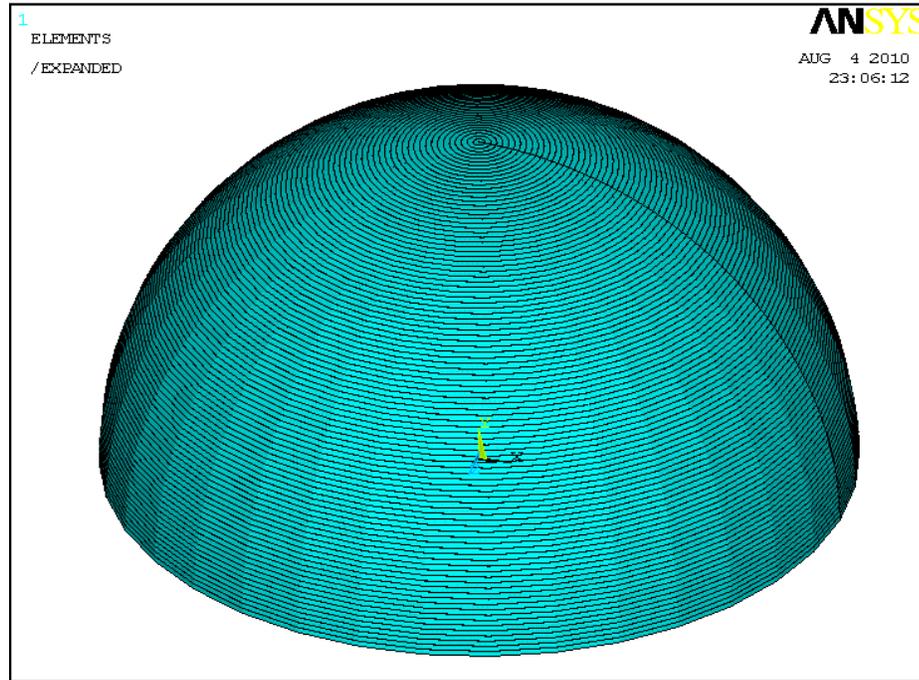
- ✓ Deformed + Un deformed shape

##### 8.1 Element table

- ✓ Define Table

➤ SMIC 6 and SMIC 12 for Bending Moment

➤ LS 7 and LS 19 for  $N_\theta$



➤ LS 5 and LS 17 for  $N_\phi$

### ***The Membrane Theory***

In the mid twentieth century, as architects and structural engineers became distinct and disparate professions, engineers began to prefer analytical method than graphical methods. The membrane theory which is popular today provides a lower bound or safe analysis for axisymmetric thin shell domes through the simple laws of equilibrium. The characteristic thick or thin of a shell are relative terms. The thinness ratio is defined as the ratio of thickness of the shell to Radius of curvature. The thinness ratio is greater than 1/10, the shells are categorized as thick. For thin shells this ratio is often between 1/10 to 1/50. Shells having the ratio less than 1/50 are too thin to be used as efficient load carrying members.

### Equation of Equilibrium

The equation of equilibrium can be obtained as discussed by Timoshenko.

Consider the conditions of equilibrium of an element cut from a shell by two adjacent meridian planes and two sections perpendicular to the meridian. It can be concluded from the condition of symmetry that only normal stresses will act on the sides of the element lying in the meridian plane

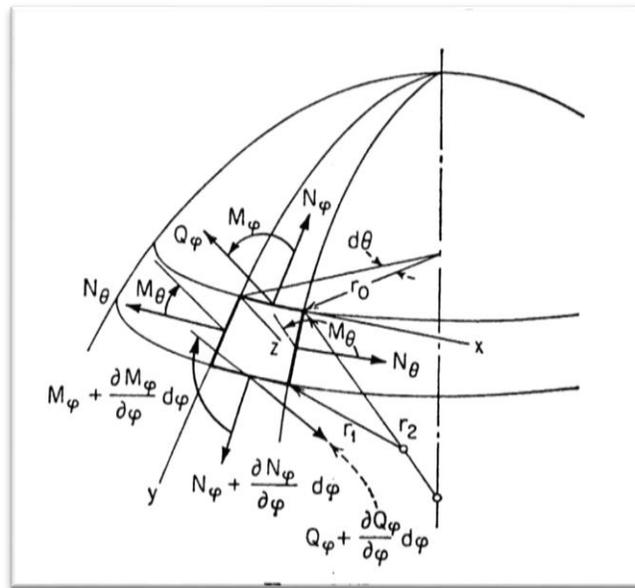


Fig 3: Different forces in a dome

Sum of all forces along  $\phi$ - direction = 0

$$N\phi r_0 d\theta + \left[ \left( N\phi + \frac{\partial N\phi}{\partial \phi} d\phi \right) \left( r_0 + \frac{\partial r_0}{\partial \phi} d\phi \right) d\theta \right] - N\theta d\phi \cos\phi r_1 d\phi - Q\phi r_1 d\phi r_0 d\theta + y r_0 d\theta r_1 d\phi = 0$$

$$\frac{\partial(N\phi r_0)}{\partial \phi} - N\theta r_1 \cos\phi - Q\phi r_0 + y r_0 r_1 = 0$$

Sum of all the forces along Z direction = 0

$$N\theta d\theta \sin\varphi r_1 d\varphi + N\varphi d\varphi r_0 d\theta - Q\varphi r_0 d\theta + \left[ \left( Q\varphi + \frac{\partial Q\varphi}{\partial \varphi} d\varphi \right) (r_0 d\theta) \right] + Zr_0 d\theta r_1 d\varphi = 0$$

$$N\varphi r_0 + N\theta r_1 \sin\varphi + \frac{\partial(Q\varphi r_0)}{\partial \varphi} + Zr_0 r_1 = 0$$

Sum of Moments along the  $\theta$  direction = 0

$$-M\varphi r_0 d\theta + \left[ \left( M\varphi + \frac{\partial M\varphi}{\partial \varphi} d\varphi \right) \left( r_0 + \frac{\partial r_0}{\partial \varphi} d\varphi \right) d\theta \right] - M\theta r_1 d\varphi \cos\varphi d\theta - Q\varphi r_1 d\varphi r_0 d\theta + \left[ \left( Q\varphi + \frac{\partial Q\varphi}{\partial \varphi} d\varphi \right) \left( r_0 + \frac{\partial r_0}{\partial \varphi} d\varphi \right) d\theta r_1 d\varphi \right] = 0$$

$$\frac{\partial(M\varphi r_0)}{\partial \varphi} - M\theta r_1 \cos\varphi - Q\varphi r_0 r_1 = 0$$

Thus the equations of equilibrium are

$$\frac{\partial(N\varphi r_0)}{\partial \varphi} - N\theta r_1 \cos\varphi - Q\varphi r_0 + Zr_0 r_1 = 0$$

$$N\varphi r_0 + N\theta r_1 \sin\varphi + \frac{\partial(Q\varphi r_0)}{\partial \varphi} + Zr_0 r_1 = 0$$

### ***Parametric Study for Hemispherical Dome with Variable Thickness***

Different cases have been solved by varying the thickness from crown to the bottom for the hemispherical dome.

- I. Hemispherical dome with 40m diameter and thickness varying from 0.30 m to 0.60 m (n=2)

- II. Hemispherical dome with 40m diameter and thickness varying from 0.30 m to 0.90 m (n=3)
- III. Hemispherical dome with 40m diameter and thickness varying from 0.30 m to 1.20 m (n=4)

The other data used are,

Young’s modulus of elasticity (E) – 1000e6 N/mm<sup>2</sup>

Density of the material – 1900 kg/m<sup>3</sup>

The same data is used for finite element analysis will be made using Ansys.

**Case I: Spherical Dome with variable thickness 0.3 m to 0.6 m**

Span of the dome = 40 m

Radius of dome = 20 m

Density of the material (masonry) = 1900 kg/m<sup>3</sup>

Thickness variation (n=2)  $t_0 = 0.30$  m at the crown and  $2t_0 = 0.60$  m at the base

**Table 3: Values of  $\sigma_\phi$  and  $\sigma_\theta$  theoretically when n=2 at the base**

| $\phi$ | Constant thickness of 0.6 m        |                                      | Variable thickness from 0.3 m to 0.6 |                                      |
|--------|------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
|        | $\sigma_\phi$ (N/mm <sup>2</sup> ) | $\sigma_\theta$ (N/mm <sup>2</sup> ) | $\sigma_\phi$ (N/mm <sup>2</sup> )   | $\sigma_\theta$ (N/mm <sup>2</sup> ) |
| 1      | 0.190                              | 0.190                                | 0.189                                | 0.191                                |
| 5      | 0.190                              | 0.188                                | 0.187                                | 0.192                                |
| 10     | 0.191                              | 0.183                                | 0.185                                | 0.189                                |
| 15     | 0.193                              | 0.174                                | 0.184                                | 0.183                                |
| 20     | 0.196                              | 0.161                                | 0.184                                | 0.173                                |
| 25     | 0.199                              | 0.145                                | 0.185                                | 0.160                                |
| 30     | 0.204                              | 0.125                                | 0.187                                | 0.143                                |
| 35     | 0.209                              | 0.102                                | 0.189                                | 0.122                                |
| 40     | 0.215                              | 0.076                                | 0.193                                | 0.098                                |

|    |       |        |       |        |
|----|-------|--------|-------|--------|
| 45 | 0.223 | 0.046  | 0.197 | 0.071  |
| 50 | 0.231 | 0.013  | 0.203 | 0.041  |
| 55 | 0.241 | -0.024 | 0.210 | 0.008  |
| 60 | 0.253 | -0.063 | 0.218 | -0.028 |
| 65 | 0.267 | -0.107 | 0.228 | -0.068 |
| 70 | 0.283 | -0.153 | 0.240 | -0.110 |
| 75 | 0.302 | -0.204 | 0.253 | -0.155 |
| 80 | 0.324 | -0.258 | 0.269 | -0.203 |
| 85 | 0.350 | -0.316 | 0.288 | -0.255 |
| 90 | 0.380 | -0.380 | 0.311 | -0.311 |

**Case II: Spherical Dome with variable thickness 0.3 m to 0.9 m**

Span of the dome = 40 m

Radius of dome = 20 m

Density of the material (masonry) = 1900 kg/m<sup>3</sup>

Thickness variation (n=3)  $t_0 = 0.30$  m at the crown and  $3t_0 = 0.90$  m at the base

**Table 4: Values of  $\sigma_\phi$  and  $\sigma_\theta$  theoretically when n=3 at the base**

| $\phi$ | Constant thickness of 0.9 m        |                                      | Variable thickness from 0.3 m to 0.9 m |                                      |
|--------|------------------------------------|--------------------------------------|--|--------------------------------------|
|        | $\sigma_\phi$ (N/mm <sup>2</sup> ) | $\sigma_\theta$ (N/mm <sup>2</sup> ) | $\sigma_\phi$ (N/mm <sup>2</sup> )     | $\sigma_\theta$ (N/mm <sup>2</sup> ) |
| 1      | 0.285                              | 0.285                                | -0.191                                 | 0.193                                |
| 5      | 0.286                              | 0.282                                | -0.194                                 | 0.205                                |
| 10     | 0.287                              | 0.274                                | -0.198                                 | 0.214                                |
| 15     | 0.290                              | 0.261                                | -0.202                                 | 0.217                                |
| 20     | 0.294                              | 0.242                                | -0.208                                 | 0.214                                |
| 25     | 0.299                              | 0.218                                | -0.214                                 | 0.206                                |
| 30     | 0.305                              | 0.188                                | -0.220                                 | 0.191                                |
| 35     | 0.313                              | 0.154                                | -0.228                                 | 0.171                                |
| 40     | 0.323                              | 0.114                                | -0.237                                 | 0.144                                |
| 45     | 0.334                              | 0.069                                | -0.246                                 | 0.112                                |

|    |       |        |        |        |
|----|-------|--------|--------|--------|
| 50 | 0.347 | 0.019  | -0.257 | 0.074  |
| 55 | 0.362 | -0.035 | -0.270 | 0.031  |
| 60 | 0.380 | -0.095 | -0.285 | -0.019 |
| 65 | 0.401 | -0.160 | -0.301 | -0.073 |
| 70 | 0.425 | -0.230 | -0.320 | -0.133 |
| 75 | 0.453 | -0.305 | -0.342 | -0.199 |
| 80 | 0.486 | -0.387 | -0.367 | -0.270 |
| 85 | 0.524 | -0.475 | -0.397 | -0.348 |
| 90 | 0.570 | 0.570  | -0.432 | -0.432 |

**Case III: Spherical Dome with variable thickness 0.3 m to 0.6 m**

Span of the dome

Radius of dome = 20 m

Density of the material (masonry) = 1900 kg/m<sup>3</sup>

Thickness variation (n=4)  $t_0 = 0.30$  m at the crown and  $4t_0 = 1.20$  m at the base

**Table 5: Values of  $\sigma_\phi$  and  $\sigma_\theta$  theoretically when n=4 at the base**

| $\phi$ | Constant thickness of 1.20 m       |                                      | Variable thickness from 0.3 m to 1.20 m |                                      |
|--------|------------------------------------|--------------------------------------|---|--------------------------------------|
|        | $\sigma_\phi$ (N/mm <sup>2</sup> ) | $\sigma_\theta$ (N/mm <sup>2</sup> ) | $\sigma_\phi$ (N/mm <sup>2</sup> )      | $\sigma_\theta$ (N/mm <sup>2</sup> ) |
| 1      | 0.380                              | 0.380                                | 0.192                                   | 0.196                                |
| 5      | 0.381                              | 0.376                                | 0.200                                   | 0.218                                |
| 10     | 0.383                              | 0.366                                | 0.211                                   | 0.238                                |
| 15     | 0.387                              | 0.348                                | 0.221                                   | 0.251                                |
| 20     | 0.392                              | 0.322                                | 0.231                                   | 0.256                                |
| 25     | 0.399                              | 0.290                                | 0.242                                   | 0.252                                |
| 30     | 0.407                              | 0.251                                | 0.254                                   | 0.240                                |
| 35     | 0.418                              | 0.205                                | 0.267                                   | 0.219                                |

|    |       |        |       |        |
|----|-------|--------|-------|--------|
| 40 | 0.430 | 0.152  | 0.280 | 0.190  |
| 45 | 0.445 | 0.092  | 0.295 | 0.153  |
| 50 | 0.463 | 0.026  | 0.312 | 0.107  |
| 55 | 0.483 | -0.047 | 0.330 | 0.053  |
| 60 | 0.507 | -0.127 | 0.351 | -0.009 |
| 65 | 0.534 | -0.213 | 0.374 | -0.079 |
| 70 | 0.566 | -0.306 | 0.401 | -0.157 |
| 75 | 0.604 | -0.407 | 0.431 | -0.243 |
| 80 | 0.648 | -0.516 | 0.466 | -0.337 |
| 85 | 0.699 | -0.633 | 0.506 | 0.441  |
| 90 | 0.760 | -0.760 | 0.553 | -0.553 |

***Observation for Spherical Dome***

From the above table the inference is tabulated as below

**Table 6: Comparison of forces when the dome is constant thick and variable thick**

| Force at base | n | Variable thick    | Constant thickness | angle change from compression to tension (variable thickness) |
|---------------|---|-------------------|--------------------|---|
| $N_{\theta}$  | 2 | $1.63 \gamma a t$ | $2 \gamma a$       | 56.2  |
| $N_{\theta}$  | 3 | $2.27 \gamma a t$ | $3 \gamma a$       | 58.2  |
| $N_{\theta}$  | 4 | $2.91 \gamma a t$ | $4 \gamma a$       | 59.4  |

It is seen that for a constant thickness that the change of angle from compression is  $51.2^{\circ}$ . But for a variable thickness it changes as tabulated in the table.

The stress  $N_{\theta}$  and  $N_{\phi}$  also reduces in the case of variable thickness than in case of constant thickness.

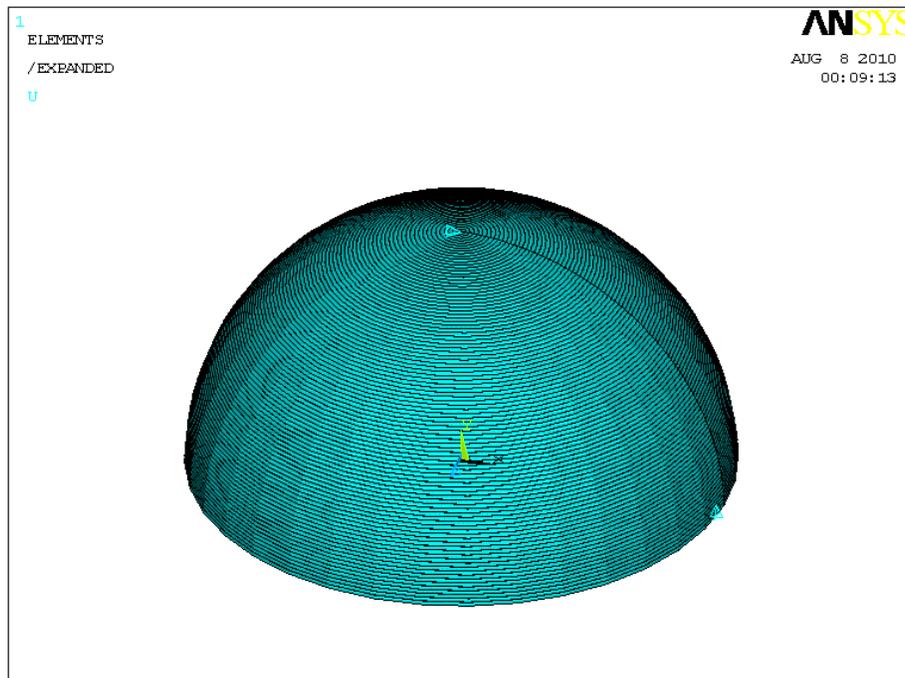
***The Finite Element Analysis for Hemispherical Dome with Variable Thickness***

The modeling in Ansys is carried using Shell 61 element. The same properties considered for the theoretical analysis is considered in finite element analysis. The membrane analysis is carried out for following parameters

- I. Hemispherical dome with 40m diameter and thickness varying from 0.30 m to 0.60 m (n=2)
- II. Hemispherical dome with 40m diameter and thickness varying from 0.30 m to 0.90 m (n=3)
- III. Hemispherical dome with 40m diameter and thickness varying from 0.30 m to 1.20 m (n=4)

The analysis is carried out for the two boundary condition and bending moment are obtained for same set of parameters.

- 1. Simply support condition
- 2. Fixed support condition



**Fig 4.25: Hemispherical dome expanded view with variable thickness 0.3 m at crown and 1.2 m at base**

**Observation for Hemispherical Dome with constant & Variable Thickness**

The following observations are made

**Table 7: Comparison of stress Constant and variable thickness hemispherical dome**

| Diameter of dome 20 m   | $\sigma_\phi$<br>N/m <sup>2</sup> | $\sigma_\theta$<br>N/mm <sup>2</sup> | %<br>reduction   | $\phi$ at<br>$\sigma_\theta = 0$ | max<br>$M_\phi$<br>Nm  |
|---|-----------------------------------|--------------------------------------|------------------|----------------------------------|------------------------|
| Hemispherical dome with constant thickness 0.6 m at base          | 0.380 (C)                         | 0.380 (T)                            | <b>18.2</b><br>% | 51.2°                            |                        |
| Hemispherical dome with variable thickness 0.3 m to 0.6 m at base | 0.311 (C)                         | 0.311 (T)                            |                  | 56.3°                            | 0.047 x10 <sup>6</sup> |
| Hemispherical dome with constant thickness 0.9 m at base          | 0.572 (C)                         | 0.572 (T)                            | <b>24.2</b><br>% | 51.2°                            |                        |
| Hemispherical dome with variable thickness 0.3 m to 0.9 m at base | 0.432 (C)                         | 0.432 (T)                            |                  | 57.9°                            | 0.132 x10 <sup>6</sup> |
| Hemispherical dome with constant thickness 1.2 m at base          | 0.760 (C)                         | 0.760 (T)                            | <b>27.2</b><br>% | 51.2°                            |                        |
| Hemispherical dome with variable thickness 0.3 m to 1.2 m at base | 0.553 (C)                         | 0.553 (T)                            |                  | 58.8°                            | 0.279 x10 <sup>6</sup> |

- ✓ The above table infers that the *stresses reduce* when the dome is of variable thickness.
- ✓ The angle  $\phi$  at  $\sigma_\theta = 0$  also *increases* in case of variable thickness.
- ✓ The modeling with Ansys helps us to get the stress values for any variable thickness hemispherical domes and constant thickness dome.
- ✓ The stress values of finite element analysis come with close agreement with theoretical membrane analysis value.
- ✓ The change from compression to tension in the  $N_\theta$  also comes in agreement.
- ✓ In case of simply supported condition the Bending moment is maximum where the angle changes from tension to compression.
- ✓ In case of simply supported condition the Bending moment is zero at the supports and increases as the thickness at the base increases.

- ✓ In case of Fixed support condition the Bending moment takes the maximum value at the support and gets damped to zero.
- ✓ In case of fixed support condition the Bending moment increases as the thickness at the support increases.

### ***Parametric Study for Parabolic Dome with Variable Thickness***

Different cases have been solved by varying the thickness from crown to the bottom for the parabolic dome.

- I. Parabolic dome with 40 m diameter, 10 m rise thickness varying from 0.30 m to 0.60 m (n=2)
- II. Parabolic dome with 40 m diameter, 10 m rise thickness varying from 0.30 m to 0.90 m (n=3)

The other data used are,

Young's modulus of elasticity (E) – 1000e6 N/mm<sup>2</sup>

Density of the material – 1900 kg/m<sup>3</sup>

The same data will be used and finite element analysis will be made using Ansys

#### **Case I: Parabolic Dome with variable thickness 0.3 m to 0.6 m**

Span of dome = 40 m

Radius of dome = 20 m

Rise of dome = 10 m

Density of the material (masonry) = 1900 kg/m<sup>3</sup>

Thickness variation (n=2)  $t_0 = 0.30$  m at the crown and  $2t_0 = 0.60$  m at the base

**Table 8: Values of  $\sigma_\phi$  and  $\sigma_\theta$  theoretically when n=2 at the base**

| $\phi$ | Constant thickness of 0.6 m          |  | Variable thickness from 0.3 m to 0.6 m |  |
|--------|--------------------------------------|--|--|--|
|        | $\sigma_{\phi}$ (N/mm <sup>2</sup> ) | $\sigma_{\theta}$ (N/mm <sup>2</sup> ) | $\sigma_{\phi}$ (N/mm <sup>2</sup> )   | $\sigma_{\theta}$ (N/mm <sup>2</sup> ) |
| 1      | 0.190                                | 0.209                                  | 0.188                                  | 0.193                                  |
| 5      | 0.191                                | 0.209                                  | 0.184                                  | 0.201                                  |
| 10     | 0.194                                | 0.211                                  | 0.182                                  | 0.210                                  |
| 15     | 0.200                                | 0.213                                  | 0.182                                  | 0.216                                  |
| 20     | 0.209                                | 0.215                                  | 0.187                                  | 0.221                                  |
| 25     | 0.221                                | 0.219                                  | 0.194                                  | 0.225                                  |
| 30     | 0.237                                | 0.223                                  | 0.206                                  | 0.227                                  |
| 35     | 0.258                                | 0.227                                  | 0.224                                  | 0.229                                  |
| 40     | 0.288                                | 0.232                                  | 0.249                                  | 0.230                                  |

**Case II: Parabolic Dome with variable thickness 0.3 m to 0.9 m**

Span of dome = 40 m

Radius of dome = 20 m

Rise of dome = 10 m

Density of the material (masonry) = 1900 kg/m<sup>3</sup>

Thickness variation (n=3)  $t_0 = 0.30$  m at the crown and  $3t_0 = 0.90$  m at the base

**Table 9: Values of  $\sigma_{\phi}$  and  $\sigma_{\theta}$  theoretically when n=3 at the base**

| $\phi$ | Constant thickness of 0.9 m          |  | Variable thickness from 0.3m to 0.9 m |  |
|--------|--------------------------------------|--|---------------------------------------|--|
|        | $\sigma_{\phi}$ (N/mm <sup>2</sup> ) | $\sigma_{\theta}$ (N/mm <sup>2</sup> ) | $\sigma_{\phi}$ (N/mm <sup>2</sup> )  | $\sigma_{\theta}$ (N/mm <sup>2</sup> ) |
| 1      | 0.285                                | 0.285                                  | 0.187                                 | 0.191                                  |
| 5      | 0.287                                | 0.286                                  | 0.178                                 | 0.194                                  |
| 10     | 0.292                                | 0.287                                  | 0.173                                 | 0.196                                  |
| 15     | 0.300                                | 0.290                                  | 0.172                                 | 0.197                                  |

|    |       |       |       |       |
|----|-------|-------|-------|-------|
| 20 | 0.313 | 0.294 | 0.176 | 0.197 |
| 25 | 0.331 | 0.298 | 0.183 | 0.197 |
| 30 | 0.355 | 0.304 | 0.194 | 0.198 |
| 35 | 0.388 | 0.310 | 0.211 | 0.199 |
| 40 | 0.431 | 0.317 | 0.236 | 0.206 |

***Observations for Parabolic Dome with variable thickness 0.3 m to 0.9 m***

From the above table following inference is obtained

1. Both  $N_\phi$  and  $N_\theta$  are compression through out
2. The values are less when compared with constant thickness

***1.1 The Finite Element Analysis for Parabolic Dome with Variable Thickness***

The modeling with Ansys is carried using Shell 61 element. The same properties used for the theoretical analysis is considered in the finite element analysis. The membrane analysis is carried out for following parameters

- I. Parabolic dome with 10 m diameter, 5 m rise and thickness varying from 0.30 m to 0.60 m (n=2)
- II. Parabolic dome with 10 m diameter, 5 m rise and thickness varying from 0.30 m to 0.90 m (n=3)

The analysis is carried out for the two boundary condition and bending moment are obtained for same set of parameters.

1. Simply support condition
2. Fixed support condition

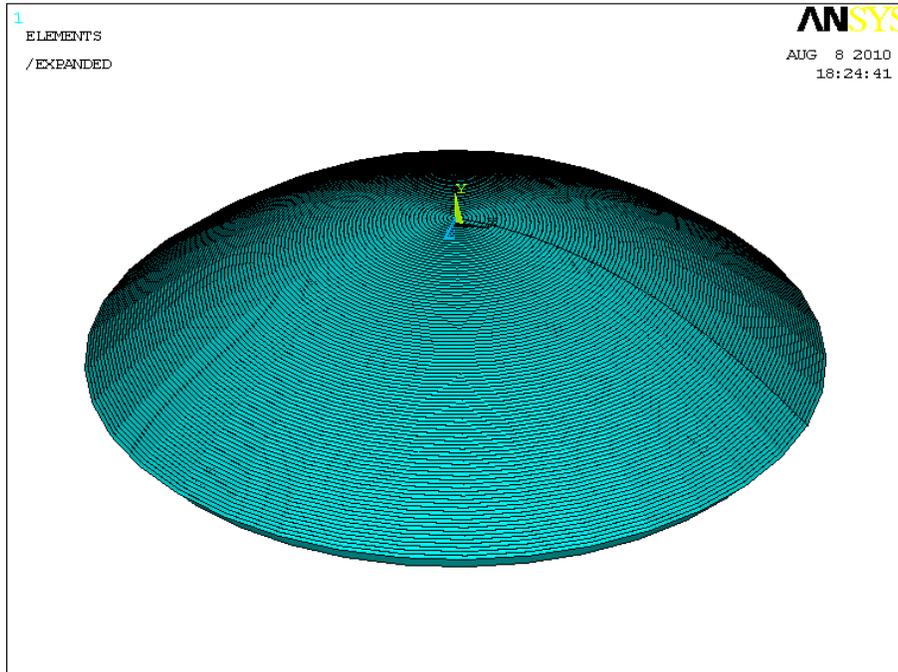


Fig 4.37: Expanded view of the above dome with variable thickness 0.3 m at crown and 0.6 m at base

**Observation for Parabolic Dome with Variable Thickness:**

The following observations are made

**Table 10: Comparison of stresses variable and constant thickness of parabolic dome**

| Diameter of dome 40 m   | $\sigma_\phi$ | $\sigma_\theta$ | Reduction % $\sigma_\phi$ | Reduction % $\sigma_\theta$ | $\phi$ at $\sigma_\theta = 0$ | max $M_\phi$       |
|---|---------------|-----------------|---------------------------|-----------------------------|-------------------------------|--------------------|
| Parabolic dome with constant thickness 0.6 m at base          | 0.288 (C)     | 0.232 (C)       | 13.5 %                    | 0.86 %                      | -                             |                    |
| Parabolic dome with variable thickness 0.3 m to 0.6 m at base | 0.249 (C)     | 0.230 (C)       |                           |                             | -                             | $0.64 \times 10^6$ |
| Parabolic dome with constant thickness 0.9 m at base          | 0.431 (C)     | 0.317 (C)       | 45.2 %                    | 35%                         | -                             |                    |
| Parabolic dome with variable thickness 0.3 m to 0.9 m at base | 0.236 (C)     | 0.206 (C)       |                           |                             | -                             | $0.14 \times 10^7$ |

1. The above table infers that the **stresses reduce** when the dome is of variable thickness.

2. Stresses **will not be zero** both in case of constant and variable thickness.

3. The modeling with Ansys helps us to get the stress values for any variable thickness parabolic domes.
4. The finite element analysis comes with close agreement with theoretical value.
5. The values of  $N_\phi$  and  $N_\theta$  are in compression as in case of parabolic dome
6. In case of simply supported condition the Bending moment is zero becomes at the support and attains a maximum value and dampens to zero.
7. In case of simply supported condition the Bending moment is zero at the supports and increases as the thickness at the base increases
8. In case of Fixed support condition the Bending moment takes the maximum value at the support and gets damped to zero
9. In case of fixed support condition and simply support condition the Bending mom

## RESULTS AND DISCUSSION

The above analysis of the shells with variable thickness following results were obtained

**Table 11: Nature of stresses in domes of variable thickness**

| Thickness          | Hemispherical dome |                        | Parabolic domes |                 |
|--------------------|--------------------|------------------------|-----------------|-----------------|
|                    | $\sigma_\phi$      | $\sigma_\theta$        | $\sigma_\phi$   | $\sigma_\theta$ |
| Constant thickness | Compression        | Compression to tension | Compression     | Compression     |
| Variable thickness | Compression        | Compression to tension | Compression     | Compression     |

1.The results obtained from theoretical analysis and Finite element analysis from Ansys is comparable.

2.The hemispherical dome which is analyzed for variable thickness shows that the  $N_\theta$  value changes from compression to tension is increased to 56 degrees when  $n=2$ , 58 degrees when  $n=3$  and 59 degrees when  $n =4$ . This result helps us to construct the spherical dome with variable thickness up to 56 to 60 degrees.

3.The analysis of hemispherical dome also reveals that the stress values get reduced when the dome has variable thickness.

4. The bending moment is also maximum where the  $N_{\theta}$  changes from compression to tension when it is simply supported and when it is fixed it is maximum at the support. The bending moment also decreases as compared to having constant thickness. This helps to have lesser tension reinforcement.

5. The analysis of parabolic dome shows that both the  $N_{\phi}$  and  $N_{\theta}$  value are in compression

6. The bending moment in case of parabola has zero value at base and has a maximum value then completely dampens out. In case of fixed support it has a maximum value at support and dampens out completely.

7. The thickness variation analysis helps in saving lot of material in the construction process. Hence an optimal structure which satisfies the condition can be built. This improves the economy of the project. Therefore an optimal structure with optimum cost can be built.

### ***Scope for further Studies***

- Bending analysis using Geckler's approximation can be done for the domes with variable thickness.
- Analysis with different loading condition can be studied.
- Dynamic analysis of the domes with variable thickness

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