# Learning-Based Reconstruction of Finite rate of innovation Signals

# T. Tirupal<sup>1</sup>, E.Devendra goud<sup>2</sup>, V.Arun kumar<sup>3</sup>, C.Hemanth Kumar<sup>4</sup>

<sup>1</sup> HOD, Dept., of ECE, G. Pullaiah College of Engineering and Technology, A.P, India <sup>2,3,4</sup> Dept., of ECE, G. Pullaiah College of Engineering and Technology, A.P, India

Abstract - Finite innovation rate (FRI) signal reconstruction is a significant area of signal processing that seeks to precisely retrieve negative signals with the use of a few parameters. Reconstructing continuous, band-free signal classes with a limited number of parameters from low-rate discrete samples is made possible by the Finite Rate of Innovation (FRI) sampling theory. This challenge frequently becomes an estimation of the spectrum, which can be done by techniques that estimate the signal subspace that tends to vanish at a certain signal-to-noise ratio (PSNR). We are thinking of another approach that makes advantage of data in tagged files in order to prevent this issue. We put forth two model-based learning techniques: building encoder-decoder deep neural networks to describe the detection process and denoising depth unfolded spectrum estimation. Comparing simulation results for both learning algorithms to traditional subspace-based techniques, a notable improvement in breaking PSNR is observed. Although encoderdecoder networks outperform deep unrolled networks and perform comparable to classical FRI approaches at low noise levels, the latter can reconstruct FRI signals even in the absence of an identified sample kernel. Additionally, we produced more precise predictions and attained competitive results in both positive and negative rate pulse detection from in vivo calcium imaging data. Applications in sectors like communications and pharmaceuticals can benefit from the reconstruction of the FRI signal by mathematical methods, structural design, and negative signal recovery. This allows for data capture and compression while storing crucial information with less resources.

Key words: FRI signals, Sampling theory, PSNR, Signal recovery

# 1. INTRODUCTION

Limited Innovation Rate (FRI) Signal processing is a straightforward problem with numerous applications, including radar, biomedical imaging, and communications. Due to their limited freedom, FRI signals can be represented by unusual objects such as impact location and shape [1]. But because of their short range, FRI signals are frequently ignored, which makes reconstructing them challenging [2]. Recovering the low level measurement's underlying signal is the primary goal of the FRI signal reconstruction process. The procedure entails figuring out the material pulses of the signal's place and form. To do this, a variety of mathematical and signal processing techniques such cluster modeling, smoothing, and optimization are applied [3, 4]. This technique exploits the difference between FRI signals to accurately reconstruct the signal from limited data, ultimately restoring the original signal with high accuracy.

Signals from discrete samples that are invariant with continuous translation are perfectly reproduced by classical sampling theory. The results have been extended to signals that are not limited to being independent at a certain moment in time in recent years with the advent of finite rate of innovation (FRI) sampling theory [5, 6, 7]. Given that the amplitude and position of the K pulses will determine the signal, the most basic FRI signal is a stream of K pulses with a recency rate of 2K. This has prompted the employment of a number of techniques, including electrocardiography (ECG), radar, functional magnetic resonance imaging (fMRI), ultrasound imaging, and calcium monitoring. Current FRI signal reconstruction techniques frequently alter the continuous location estimation problem into a frequency estimation problem,

which can be resolved using spectrum estimation techniques as the matrix pen method and Prony's Cadzow denoiser. Through the use of numerical value decomposition (SVD), the signal subspace is estimated using this technique.

The reconstruction silently followed the plain's Cramer-Rao vineyard. But when the peak signal-to-noise ratio (PSNR) falls below a particular level, it becomes worse [8, 9, 10]. This is expected to be caused by confusion between the signal subspace and the variation subspace, or the orthogonal subspace in the noise. Compressed sensing (CS), in addition to the traditional FRI method, enables the determination of the pulse stream's position on the grating. To replicate the signal, however, requires knowledge of the sampling kernel since both the CS and FRI approaches require determining the sampling kernel's Fourier coefficients at a specific frequency.. The details of the nucleus' structure are unclear for several applications, including calcium imaging in neurology [11, 12]. While atomic or convex relaxation models have been extended to reconstruct continuous lines using CS, they still have issues that are comparable to those with the FRI model, namely the need to recognize the significance of the pulse image. As an alternative, the FRI signal can be reconstructed from the under sample obtained by several cores without being aware of its shape thanks to compressed multi-channel blind deconvolution. In this essay, however, we'll focus on the scenario in which there is just one pulse image. Here, we use a data-driven learning strategy in an attempt to overcome these constraints [13, 14, 15]. Deep neural networks are being used in certain research to do spectrum estimation for tasks like determining the frequency of different sound waves or the direction in which they arrive [16, 17]. In this research, we concentrate on building interpretable networks using the current FRI architecture.

Prior to addressing the frequency conversion issue in the traditional FRI technique, we first suggest the depth unwrapping noise removal technique to address the performance issue. While transferring iterative techniques to network layers with learnable parameters, deep decompression maintains the domain information of the data; in our example, the spectral sample matrix goes to Eplitz and low level [18, 19]. Here, we select the predicted Wirtinger gradient descent (PWGD) algorithm, which enables learning to be excluded from the network architecture. PWGD is a minor variation of Cadzow denoising. Dirac's reconstruction is possible after the unwinding network is connected to the Prony technique.. Our objective is to minimize the expected denoising matrix by utilizing the suggested zero eigenvalue-based loss functions [19, 20]. The filter's orientation with respect to the ground truth is eliminated while projecting into the orthogonal subspace. By doing this, the likelihood of subspace exchange events is decreased, enhancing PSNR burstiness. Alternatively, we propose to avoid the spectral estimation by learning the  $g\psi(\bullet)$ encoder network, which is intended to infer directly from the well-known Dirac position model. This is because the transition from the continuous estimation problem to the exponential frequency problem still requires knowledge of the kernel structure. Next, we carry out the optimization by expanding the  $f\theta(\bullet)$ decoder network, which simulates the signals that the FRI process receives and restructures them based on its forecasts. and wavelength [21, 22, 23]. The FRI Encoder-Decoder Network (FRIED-Net) is formed by them collectively. We can either fix or make the clipper's parameters learnable based on the known sampling kernel. In cases where the sample nucleus is unknown and the FRI signal cannot be reconstructed using the traditional method, this entails using calcium imaging [24, 25]. As the former offers the normalization impact of the encoder network output, work loss accounts for the error of the reconstructed work and the resynthesized discrete model.

### 2. EXISTING SYSTEM

FRI signal time coding is a recent study. Alexandru and Dragotti tackled this issue initially, looking at an exponentially replicating kernel-based event-driven model of the Dirac pulse stream. They created a brandnew reconstruction method that consistently returns the form to its original state [46, 47]. Samples used in reconstruction algorithms must sustain less than the time interval between two successive pulses. Although this is useful for reconfiguring, it can become expensive because the choice of available cores depends on the signal. The new modification must also serve as an example of acknowledging limitations. Arrangement of By modifying the IF-TEM's parameters, the reconstruction algorithm will produce the sample necessary for flawless reconstruction by using the alpha synapse function to filter the Dirac pulse stream. In order to overcome this issue, we developed the Fourier domain reconstruction approach, which serves as the foundation for this paper.

2. Real-time FRI signal encoding was recently demonstrated by Naiman et al. using a hardware prototype. Their approach encodes the event time—which is the TEM's output—using a counter. The FRI signal is returned using the start time [48, 49, 50]. We stumbled upon the works of Naiman and his buddies while researching our paper. IFTEM's reconstruction issue in the presence of noise was addressed. Their method is comparable to, but they innovate by getting rid of words that make transitions forward unstable. Its drawback is that the signal might be repeated incessantly.

3. Following the concepts introduced in, we ascertain the temporal input of the FRI signal and refine the idea of Fourier domain reconstruction. We take into account two time-shifting techniques (C-TEM and IF-TEM) and design a model based on reconstruction and the model. Unlike other approaches, our method is less constrictive and the kernel model selection is not only reliant on the signal. FRI signals that can be represented as pulses with different weights and times can use this approach. We also demonstrate how various changes over time can be accommodated by adapting the development process. We also examine how measurement noise affects forward pass and signal measurement and devise a robust signal reconstruction method.. The approach can be used to construct correct structures, according on experimental data. The primary file contains the results for Dirac pulses, whereas the supplementary material contains the results for B-spline flow.

The erosion of traditional symbols has been accompanied by the emergence of several new, alternative channels for ideas. Apart from the above listed aspects, the higher classes have shown interest in luxury (Zanette, Pueschel, and Touzani 2022), convenience (Currid-Halkett 2017), and business (Bellezza, Paharia, and Keinan 2017). These subjects are alternative because they deviate from the luxury consumption standard in terms of energy use, variety, and advancement in at least one area (Dubois et al., 2017). 2021).

As a physical layer on the CR network, research on the Multiple-input Multiple-output (MIMO)-OFDM system, one of the best hybrid multi-carrier systems. In order to maximize the CR network's overall capacity, the ideal power distribution algorithm has been thoroughly examined under various circumstances [26, 27]. Theoretically, this method can maintain PU band interference at a reasonable level while optimizing overall capacity. We also provide the ideal solution in order to reduce the algorithm's complexity [28, 29, 30]. The continuous location estimation problem is frequently converted into a frequency estimation problem by the current FRI signal reconstruction techniques.

### **Disadvantages:**

The idea of the Gaussian method is not correct.

- In this model, the number of subcarriers used by users is limited.
- The total capacity of the CR network is limited.

• This limits the model to only consider a MIMO wireless system with the same antenna instead of narrow Haze.

# **Objectives:**

The Finite Rate of Innovation (FRI) sampling theory makes it possible to reconstruct classes of non-band restricted broadcast signals from low-rate discrete samples with very small amounts of free parameters[41,42, 43]. A common solution to this problem is to estimate the spectrum using methods that estimate the signal subspace that tends to disappear at a given signal-to-noise ratio (PSNR). To avoid this problem, we are considering an alternative strategy that utilizes information in tagged files. We propose two model-based learning methods: denoising depth unfolded spectrum estimation and constructing encoder-decoder deep neural networks to characterize the detection process. There is a discernible improvement in breaking PSNR when comparing the simulation results for both learning algorithms to the conventional subspace-based methods ...While classical FRI techniques perform similarly at low noise levels and outperform deep unrolled networks, encoder-decoder networks may reconstruct FRI signals even when a sample kernel is not recognized [44, 45]. We also obtained competitive results in terms of true and false positive rates while providing more accurate estimations in the pulse analysis of in vivo calcium imaging data.

### **PROPOSED SYSTEM**

We show that K Diracs flows may be used to reconstruct the Diracs function using two different deep modeling approaches. When reconstructing finite rate of innovation (FRI) data, Fast Fourier Transform (FFT) facilitates peak localization and spectrum analysis while Rectified Linear Units (ReLU) provide nonlinearity and adaptability that improve signal quality and accuracy [31, 32, 33]. While FFT uses frequency component analysis to assist in determining the heart's location, ReLU shows irregularities by utilizing the unique characteristics of the FRI signal, which streamlines the signal representation and improves the reconstruction procedure overall. Deep expansion Wirtinger gradient descent and FRIED-Net handle the situation regardless of whether the sample kernel is known [34, 35]. Frequency estimation problems can be converted from continuous estimation problems using spectrum estimation techniques like matrix pen and Prony's Cadzow denoising method. Using numerical value decomposition (SVD) to estimate the signal subspace, this method entails signal subspace estimation [36, 37]. We illustrate this by contrasting our learning method, which is based on the creation of K Diracs flows, with the performance of the classical FRI method. The FRIED-Net that we have presented can learn and reconstruct the FRI signal

Flow diagram:



Fig. 1: Block diagram of proposed method

Our method uses a neural network to subtract the FRI noise from the noise model, which allows it to reconstruct the FRI signal directly, whereas the standard FRI method often requires knowledge of the kernel model [38, 39, 40]. Furthermore, without influencing the reconstruction, we can find the represented kernel by figuring out the  $\theta$  parameters of the back propagation. We apply FRIED Net to calcium imaging data to illustrate this.

# **3.1 Explanation of Modules**

# 3.1.1 Data Selection

A dataset is an assemblage of data assembled for the purpose of machine learning research. Digital images used to assess, educate, and test machine learning and artificial intelligence (AI) algorithms—mainly computer vision algorithms—are included in the category of image data [32]. The process of locating relevant data, sources, and data collection instruments is known as data selection. The process of choosing data comes before data collecting. One kind of composite file is an image. Binary format is typically used for image files.

# **3.2.2 Subcarrier allocation**

By allocating subcarriers among rival users, subcarrier assignment determines the master user and the corresponding listener of each subcarrier. In order to offer additional transmission, a secondary signal known as a subcarrier has its frequency modified to match the main frequency, or carrier. It permits several

divisions to be carried by a single transmission. Sidebands of RF carriers that have been altered to send more data are known as subcarriers. Stereo sound on a mono radio or black and white on a TV are two examples. It is possible to break down the subcarrier allocation problem at the subcarrier level and solve it concurrently [41, 42]. The user with the highest subcarrier signal-to-noise ratio (SNR) in a secure transmission may possess the subcarrierThe signal-to-noise ratio (SNR) of background noise systems is contingent upon the subcarrier's output power and gain. The only difference between the SNR and subcarrier gain is that each user transmits at the same power. Consequently, the first user has the most subcarrier gain, while the eavesdropper has the second-best gain. The best subcarrier allocation plan can be implemented independently of power distribution and even before power distribution.

# 3.3.3 Transmitter

An significant consideration in many technical domains is bandwidth. It clarifies the distinction between high and low frequencies in signal transmission, including radio transmissions, in signal processing. Hertz (Hz) is used to measure signal bandwidth. The difference between an electromagnetic wave signal's highest and lowest frequencies is known as its signal bandwidth. In short, it's the range of frequencies within which an electromagnetic wave (signal) functions. Hertz (symbol Hz)[44] is its unit. Reducing signal power to a manageable level outside of the intended frequency range is known as band limitation. The fuzzy principle distinguishes between two kinds of signals: A signal is said to as band-limited (BL) if its Fourier transform is zero outside of the last time; otherwise, it is referred to as non-bandlimited (NBL).

# 3.3.4 Breakdown PSNR

While prior research like this one indicates that Cadzow's denoising improves the performance of classical subspace-based reconstruction as described by the Cramer-Rao constraint, they will still fall short of the PSNR criterion [47]. It is hypothesized that during spectrum estimation, confusion between the signal and noise subspaces leads to the collapse of subspace-based approaches.

### **Receivers:**

Receivers use electronic filters to isolate the desired RF signal from all other receivers. The signal is isolated. Electrical generators can be powered via antennas

### **3.3.5 Functional Analysis**

A mathematical tool for comprehending and representing nonlinear issues is Fourier analysis. It enables us to decompose a periodic function into sine and cosine waves, two basic functions. As part of the performance breakdown, we first suggest deep unfolding the denoising procedure, which is applied in standard FRI approaches prior to solving the transformed frequency estimation problem.

### Advantages:

- > SU can receive subcarriers and PU can achieve good transmission without interference from SU.
- > Excellent performance in non-collaborative and collaborative models to achieve SU success.

### **3.2 Implementation Process**

# 3.2.1 Learning-Based FRI Reconstruction

With learning parameters, express the algorithm deeply into the network layer while maintaining data integrity. Article This section presents FRI reconstruction using two works as a basis: FRI encoder-decoder network (FRIED-Net) and depth expansion projection Wirtinger gradient descent. While the latter takes into account the former FRI problem and permits reconstruction without knowing the pulse picture, the former seeks to improve the denoising process of the frequency estimation problem to limit the incidence of subspace variation events in the conventional FRI process.A. Unwound Depth Projection Gradient Distance Wirtinger We first present the depth unwound methodology of the noise reduction process prior

to the Pronys method. A method called algorithm expansion is used to turn algorithms back into deep neural networks.

The unwinding deep network essentially follows a parameter optimization technique by using training data to allow the program's parameters to be learned via backpropagation [48]. The Cadzow denoising technique, which substitutes the K-level matrix set for the projection of the Toeplitz matrix set, is the most often used iterative denoising procedure. As a result, we are looking into an alternative method that does not use subspace estimation—namely, directly connecting the FRI detection process information to our network design. We created the FRI Encoder-Decoder Network (FRIED-Net), an autoencoder-like model, because FRI signals are described by several parameters.

# **3.2.2 In Classic FRI Techniques:**

In our discussion on Data Requirement, Effectiveness, and Professional Development, we explain how learning-based strategies can triumph over setbacks. The primary outcomes of the K = 2 simulations are displayed in Table II, along with the data that we used to replicate the pulses. Since both traditional FRI and Deep Unfolded PWGD use Prony's approach in sampling kernel  $\varphi(t)$ , they need to know this information in order to convert the FRI reconstruction problem into a spectrum estimation. In FRIED-Net, the decoder can either be learned or fixed based on whether it is known, whereas the encoder can be trained independently without knowing  $\varphi(t)$ . Since we just need the encoder to rebuild the locations,  $\varphi(t)$  is not needed in this case during the evaluation stage. Because FRIED-Net uses more than 100 times as many free parameters than Deep Unfolded PWGD, it is therefore more suitable for applications where the pulse is uncertain, such as calcium imaging [49,50]. Nevertheless, this is offset by the high complexity required by one SVD every iteration for both Deep Unfolded PWGD and classical FRI. Next, by splitting the reconstruction performance into two cases—before and after the standard FRI techniques fail—we may talk about the reconstruction performance. While the performance of FRIED-Net plateaus despite the decoder refining the estimates, both the Pronys approach with Cadzow and Deep Unfolded PWGD closely match the Cramer-Rao bound when the PSNR is high and the locations are far apart. All of our suggested learning-based methods can overcome the breakdown PSNR for the breakdown region. This is due to the fact that our learning-based techniques often make good use of the ground truth labels in the training data as a system's past knowledge. For this specific FRI reconstruction issue, traditional FRI techniques.

#### 4. EXPERIMENTAL RESULTS



Figure 2: (a)Original FRI Signal (b)Node Initallization (c)Digital Signal Input (d)Symbol of M-array QAM modulation (e) M-array QAM modulation (f) Signal at Transmitter

Fig.2(a)shows input Signals with the sub carries allocation to transmitted the signal from source to destination and A rectified linear unit (ReLU) is an activation function that introduces the property of non-linearity to a deep learning model and solves the vanishing gradients issue. "It interprets the positive part of its argument. It is one of the most popular activation functions in deep learning. Fig .2(b) sows the node is a point of intersection/connection within a data communication network. In an environmentwhere all devices are accessible through the network, these devices are all considered nodes. The individual definition of each node depends on the type of network it refers to. Fig.2(c)shows here a digital signal is a signal that exists in one of two states: high or low, open or closed, on or off.

Fig.2(d) and Fig.2(e&f) M-array quadrature amplitude modulation (M-QAM) is a modulation scheme that conveys data by modulating the data transmission onto the amplitude via two carrier signals.



# Figure3: (a) Rayleigh CDF with sig = 1.00 (b) Rayleigh Distribution (c) Signal at Transmission channel.

Fig.3(a) Represents the by using the Rayleigh distribution and rectified linear unit algorithms are to calculate the depth of information in a signal to don't loss any information in the receiver channel.

Fig.3(b) shows the rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communication channel).

Fig.3(c) Represents the it will vary randomly, or fade, according to a Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables



Figure4 : (a)Symbol after M-array QAM demodulation (b) M-array QAM demodulation (c) Received Data signal from Channel

Fig.4(a) Represents the M-array demodulation at the receiver side with quadreature amplitude modulation technique to get information without loss and accurate information .Fig.4(b) It shows we can get information in the receiver side M-array quadrature amplitude modulation (M-QAM).Fig.4(c) It represents the modulation scheme that conveys data by modulating the data transmission onto the amplitude via two carrier signals



# Figure 5 : (a)FFT Signal (b)Binary Reconstructed message (c)Average block error probability (d) BER Plot (e) BER Plot for proposed work (f) Carrier-To-interference Ratio

Fig.5(a) Represents the FFTs are mainly used to visualize signals. However, there are also applications where FFT results are used in calculations. For example, very simple levels of defined frequency bands can be calculated by adding them via an RSS (Root Sum Square) algorithm. Fig.5(b) shows the another application is the comparison of spectra. These signals are called "binary" signals. We give different names to the binary signal levels; "1" and "0"; "true" and "false"; "on" and "off". Fig.5(c) shows these names may represent any particular voltage or current level. Typically, we assign the logic value "1" to the higher output voltage and "0" to the lower output voltage. Fig.5(d and e) Represents the signal-to-interference ratio (SIR or S/I), also known as the carrier-to-interference ratio (CIR or C/I), is the quotient between the average received modulated carrier power S or C. Fig.5(f) Shows the average received co-channel interference power I, i.e. crosstalk, from other transmitters than the useful signal.

### 4.1 Application to Neuroscience - Calcium Imaging

In this part, we demonstrate the practical application of our suggested FRIED-net to spike detection from calcium imaging data. Understanding how neural networks function in people and animals depends in large part on the monitoring of brain activity. Fluorescent calcium sensors offer a means of concurrently monitoring cell size since brain activity alters intracellular calcium concentrations. Previous research had adequately reproduced this using FRI theory, suggesting that calcium transients simulate a decreasing exponential flow. Here, we demonstrate the application of FRIED-Net in a manner akin to the simulations, but without defining the kernel model as a distortion parameter.

### 4.2 Method

### 1) Calcium Imaging Dataset

We made use of the rule-1 dataset, which includes recordings of loosely closed cell attachment in GCaMP6f-expressing neurons as well as contemporaneous imaging. In this case, the calcium imaging data corresponds to the noisy samples  $\tilde{y}[n]$ , while the ground truth spikes tk for the training data are obtained from the contemporaneous cell-attached recording. The temporal resolution of the spikes is 100 ms, and each image has a duration of 240 seconds and is sampled at 60 Hz. We then choose nine files from that same cell, eight of which contain training data and one contains testing data.

# 2) Data Preprocessing and Spike Detection

We must preprocess the data to make sure they behave as FRI signals and are appropriate for usage in DNNs before using them for FRIED-Net. First, we remove the surrounding fiber of the signal in the region of interest in order to perform nerve fiber correction. This keeps environmental contamination out of the data and guarantees that calcium transients come from the recording alone, simulating the FRI signal.

### **3) FRIED-Net training:**

While the direct network (which uses simply the FRIED-Net encoder) is better at approximating the size and number of spikes, the entire FRIED-Net is better at identifying small numbers. The squared error of the reconstructed model and the anticipated position add up to the loss function. We merely train FRIED-Net's encoder as a direct link for the long window; the loss function is simply the squared error of the total area.

# Advantages:

- The SU can know the available subcarriers and the PU can achieve good transmission without the intervention of the SU.
- > Best practices in non-collaborative and collaborative models to achieve all successful goals of SU.
- This combination has three advantages. First, this network does not need to know the FRI pulse pattern, which is not the case in existing methods. Second, the network does not face the different problems that arise in training examples. Additionally, the proposed algorithm can adapt to changes in the sample. Numerical results show that for a given number of samples, the structured design reduces and cleans the reconstruction of the FRI signal relative to the independent data-driven design for weak samples.

# 4.3 Application

23Finite Rate of Innovation (FRI) signals, radar, lidar, sonar, ultrasound, and so forth. Perfect for time-offlight imaging uses like The FRI signal receives a sampling rate below Nyquist because of its limited flexibility.

Audio, video, sensor data, picture, and many other forms of data can all be found in a FRI signal. Applications for electronics include speech recognition, management, image, audio and video, and communications.

# **5. CONCLUSIONS**

Rectified Linear Unit (ReLU) offers nonlinearity and adaptability, while Fast Fourier Transform (FFT) assists with spectrum analysis and peak finding in the setting of finite cost innovation (FRI) signals. When combined, these aid in the precise and effective creation of signals. While FFT uses frequency component analysis to help establish the position of the heart, ReLU uses the peculiarities of the FRI signal to show irregularities, simplifying the signal representation and enhancing the reconstruction process as a whole. Whether or not the sample kernel is known, the circumstances are handled via FRIED-Net and deep expansion Wirtinger gradient descent. We illustrate this by contrasting our learning method, which is based on the creation of K Diracs flows, with the performance of the classical FRI method. The suggested FRIED-Net can learn and reconstruct the FRI signal even in the absence of sample background knowledge. Our method uses a neural network to subtract the negative FRI from the noise model, which allows it to reconstruct the FRI signal directly, whereas the standard FRI method often requires knowledge of the kernel model. Furthermore, without influencing the reconstruction, we can learn the represented kernel by figuring out the  $\theta$  parameters of the backpropagation. We apply FRIED Net to calcium imaging data to illustrate this.

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