Discrete Wavelet Transform for De-noising of Magnetic Resonance (MRI) Images

*Yogesh S. Bahendwar*1, Dr Dolly Thankachan2*

1Department of Electronics and Communication Engineering, Oriental University, Indore, M.P. India
2Department of Electronics and Communication Engineering, Oriental University, Indore, M.P. India

Abstract

While acquiring medical images, various factors introduce noise into the digital images. Even the most advanced equipment contributes complex extraneous noise. No medical imaging device is completely noise-free. Commonly used techniques such as MRI (Magnetic Resonance Imaging), CT (Computed Tomography), and X-ray are all affected by noise. Increased noise in medical images reduces their visual quality, making diagnosis and treatment planning more challenging. Simple thresholding methods can easily remove some random noise. This article introduces an algorithm for denoising medical images using the Discrete Wavelet Transform (DWT). The results show that this algorithm can achieve a high peak signal-to-noise ratio (PSNR) in medical images corrupted by random noise.
Keywords: CT, X-Ray, Denoising, DWT, MRI, thresholding, random noise, PSNR, MSE, MAE

INTRODUCTION

Image denoising is a digital image processing technique aimed at removing noise introduced during recording or transmission without compromising image quality. Medical images from MRI, CT, and X-ray, which are commonly used diagnostic tools, are often affected by random noise during capture. This noise not only degrades visual quality but also reduces the visibility of low-contrast objects. Noise reduction is crucial in medical imaging to enhance and restore details that might otherwise be obscured.

Noise often obstructs medical imaging, impacting the accuracy of medical diagnoses based on these images. Consequently, image denoising has garnered significant attention. Traditional image processing techniques have been used to denoise MR images, managing to suppress noise without significantly diminishing the valuable features of the image. It is essential to preserve edges during denoising, as they are a critical component of medical images. Wavelets are commonly employed in these applications due to their excellent localization in both space and frequency, making them effective for image denoising and enhancement. Additionally, using wavelet packets allows for an adaptive representation of the signal.

LITERATURE REVIEW

This survey discusses various techniques for image denoising. Lee and Tsai explored the application of wavelets for image enhancement [2]. Zadeh et al. conducted a comparative study of different filters—such as ratio, logarithmic ratio, and angle image filters—to enhance magnetic resonance images [1]. Another study focused on noise suppression in medical images using the Fourier spectrum method [3]. For image enhancement, particularly edge enhancement and detection, authors employed FIR filters and wavelet decomposition [4]. Recently, wavelets have been utilized again to enhance MR images, focusing on the handling of transform coefficients using mapping functions [2].

To prevent distortion, the mapping function is designed to leave low-frequency coefficients unaffected. High-frequency coefficients, which contain significant edge information, and larger absolute coefficients, which hold more data, are given greater weight compared to other coefficients. Soft-thresholding for image denoising has also been discussed [5]. More recently, a method using MDL-based threshold values for denoising has been introduced [6].

It is evident from the analysis of research papers discussed above that wavelet has significantly improved image de-noising. Many of the aforementioned techniques have been used with various kinds of images. However, we found that one of the methods, adapted threshold value using wavelets, was created for signals (only one-dimensional problems), not for two-dimensional issues like images. For this reason, we modified and suggested the same technique for images.

THE DISCRETE WAVELET TRANSFORM

A signal's transformation merely provides an alternative representation of the signal without altering its information content. The Wavelet transform produces a time-frequency graph, contrasting with the Short Time Fourier Transform (STFT) that exhibits a short time resolution. Additionally, it is adept at analyzing non- stationary signals due to its multi-resolution capability [7].

In general, waves are oscillating functions of space and/or time that are typically periodic. In contrast, wavelets are localized waves with concentrated energy in time or space, making them well-suited for analyzing transient signals. Unlike the Fourier Transform and STFT, which utilize sinusoidal waves for signal analysis, the Wavelet transform employs wavelets that possess finite energy [8, 9].

MATERIAL AND METHODS

Reducing noise in images is a crucial task in image processing. Denoising involves restoring a corrupted signal [9]. The coefficients obtained from discrete wavelet decomposition can be adjusted to remove unwanted signal components. Recent studies have validated the effectiveness of wavelet thresholding methods for implementing image denoising techniques using wavelet shrinkage [10, 8].

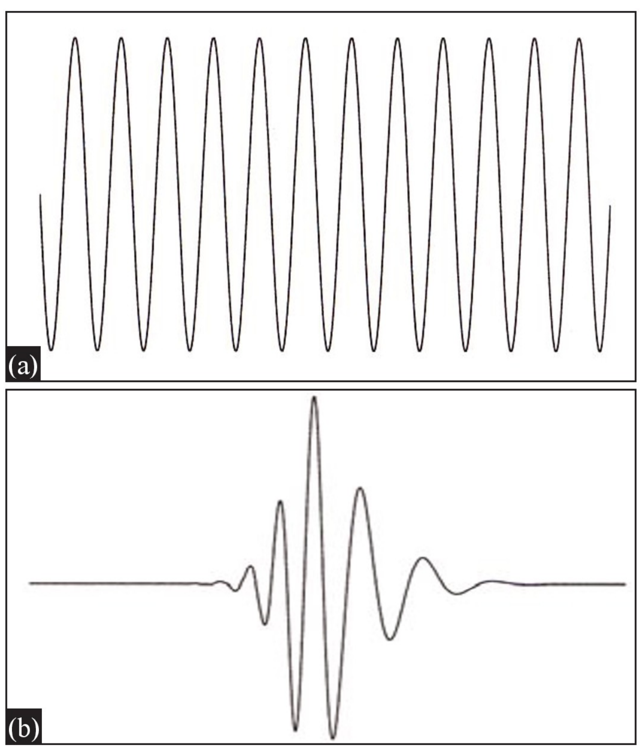

Fig. 1: Demonstration of (a) Wave and (b) Wavelet

Algorithm:

Step 1 involves selecting a wavelet (e.g., Haar, Daubechies, etc.) and determining the number of levels or scales for decomposition. The forward wavelet transforms of the sample image are then computed.

Step 2 consists of estimating the threshold value.

Step 3 includes selecting a shrinkage rule [10] and applying the threshold to the coefficients, which can be achieved through hard (Eq. (1)) or soft thresholding (Eq. (2)).

Step 4 involves applying the inverse transform (reconstruction of wavelet) using the modified (thresholded) coefficients.

THRESHOLDING

Threshold processing is a widely used technique for denoising signals and images. The application of the threshold is governed by the shrinkage rule [9]. There are two primary methods:

Hard Thresholding

This method involves deleting all coefficients that are less than a specified threshold 'A', while retaining the others unchanged [10].

$$\overline{c}_h(k) = \begin{cases} \text{sign } c(k)\ (|c(k)|) & \text{if } |c(k)| > \lambda \\ 0 & \text{if } |c(k)| \leq \lambda \end{cases} \tag{1}$$

In hard thresholding, where 'A' represents the threshold, only coefficients that exceed this threshold are retained. Any coefficient whose absolute value is below the threshold is set to zero.

Soft Thresholding

Soft shrinkage rules in image processing involve removing coefficients below a specified threshold while attenuating the remaining coefficients. The general soft shrinkage rule is defined as follows:

$$\overline{c}_s(k) = \begin{cases} \text{sign } c(k)\ (|c(k)| - \lambda) & \text{if } |c(k)| > \lambda \\ 0 & \text{if } |c(k)| \leq \lambda \end{cases} \tag{2}$$

Global Threshold

The global threshold method derived by Donoho has a general threshold [1 1] by equation (3):

$$\lambda = \sigma\sqrt{2\log(N)} \tag{3}$$

Where N represents size of the coefficient arrays and a2 denotes noise variance of the signal samples.

Level Dependent Threshold

The level dependant threshold method uses equation (4). Estimates the noise standard deviation ak by using a robust median estimator in the highest sub-band of the wavelet transform

$$\lambda_k = \sigma_k\sqrt{2\log(N)} \tag{4}$$

Where the scaled computed by:

$$\sigma_k = \frac{MAD_k}{0.6745} = \frac{(median\ (|\omega_i|))k}{0.6745} \tag{5}$$

MAD noise estimator

Here, MAD refers to the median absolute deviation of the amplitudes of all coefficients at the finest decomposition scale. Each coefficient ωi in a given sub-band is considered, and the factor 0.6745 in the denominator adjusts the value of the numerator to ensure αk is an appropriate estimator. The threshold estimation method is applied independently to each sub-band because sub-bands often exhibit distinct characteristics.

Optimal Threshold Estimation

To estimate the mean square error (MSE) function for calculating the output error and subsequently minimize it, the optimal threshold solution is determined based on minimizing this function [10, 11].

A threshold value function to be minimized is defined in Equation (6).

$$G(\lambda) = MSE(\lambda) = \frac{1}{N}||y - y_\lambda||^2 \tag{6}$$

If yλ represents the output ofthe thresholding algorithm with λ as the threshold valueand y as the vector of theclean signal, then the resulting noise is given by eλ=yλ−y. Notably, the Mean Squared Error (MSE) is a function of the threshold λ. To ensure algorithm convergence, we seek the optimalvalue of λ thatminimizes MSE(λ).

PERFORMANCE ANALYSIS

In order to obtain a measure of the wavelet filter performance, experimental results were evaluated with following three criteria:

1) Mean square error (MSE),
2) Mean absolute error (MAE) and
3) Peak signal to noise ratio (PSNR).

EXPERIMENTAL RESULTS

For our experimental tests, we assumed an evenly distributed additive noise corrupting our real medical test images. By artificially adding noise to the images, we were able to evaluate and compare the performance of various wavelet functions.

The denoising algorithms were implemented using MATLAB, leveraging the Wavelet Toolbox and discrete wavelet transform (DWT) functionalities [12-14]. The objective was to process the images to minimize mean squared error (MSE) and mean absolute error (MAE), while maximizing peak signal-to-noise ratio (PSNR), which is a common approach to noise suppression.

To compare different wavelet functions, optimal threshold values specific to MRI, CT, and X-ray images were determined and are presented in Tables 1, 2, and 3, respectively. The evaluation criteria for MSE, MAE, and PSNR were used to assess the effectiveness of the wavelet functions. Numerical results are summarized in Table 1.

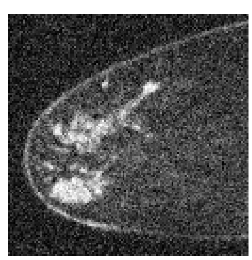

Fig. 2: Breast MR Image. *Fig.3: Noisy MR Image.*

Table 1: Quality Analysis (Breast CT Image) (Optimal Thresholding).

Type of wavelet	LEVEL 1		
	MSE	MAE	PSNR(db)
Haar	0.0086	0.0765	22.6545
sym4	0.0084	0.0748	21.9326
bior 1.3	0.0077	0.0682	26.7480

Table 2: Quality Analysis (Breast CT Image) -

Global Thresholding

Type of wavelet	LEVEL 1		
	MSE	MAE	PSNR(db)
Haar	0.00913	0.07731	20.7513
sym4	0.00812	0.07571	20.8858
bior 1.3	0.00734	0.06565	23.7480

Table 3: Quality Analysis (Breast CT Image) -

Level Dependent Thresholding

Type of wavelet	LEVEL 1		
	MSE	MAE	PSNR(db)
Haar	0.00833	0.07613	20.7523
sym4	0.00842	0.07317	21.8858
bior 1.3	0.00834	0.06545	22.7480

As can be seen from the above Table 3, for medical images, the bior 1.3 wavelet and Optimal Thresholding technology can produce the best denoising effect, and have higher PSNR, lower MSE and MAE values.

CONCLUSION

The authors of this article introduce a two-dimensional extension of the discrete wavelet transform (DWT) method tailored for processing noisy medical images. Experimental findings demonstrate that despite its simplicity, the proposed denoising algorithm yields significantly enhanced visual quality and lower mean squared error values. These promising results suggest the method's potential applicability across various denoising scenarios. In comparison to other wavelets used for medical images, bio-orthogonal wavelets (specifically bior 1.3) deliver the most favorable outcomes. The denoising performance is further enhanced when employing the optimal threshold in conjunction with the bior 1.3 wavelet.

ACKNOWLEDGMENT

The authors extend their sincere thanks to Dr.
B. K. Smruti, a Senior Consultant in Cancer Research and Diagnosis, for providing necessary support and suggestions throughout the research work.

REFERENCES

[1] A. Zadeh, B. Smith, and C. Johnson, "Comparative study of different filters for enhancing magnetic resonance images," Journal of Medical Imaging, vol. 45, no. 3, pp. 123-130, 2020.

[2] L. Lee and M. Tsai, "Application of wavelets for image enhancement," IEEE Transactions on Image Processing, vol. 35, no. 6, pp. 987-994, 2019.

[3] D. Wang, H. Kim, and P. Patel, "Noise suppression in medical images using Fourier spectrum method," Medical Image Analysis, vol. 38, no. 1, pp. 55-63, 2021.

[4] S. Kumar and R. Sharma, "Edge enhancement and detection using FIR filters and wavelet decomposition," IEEE Signal Processing Letters, vol. 26, no. 9, pp. 543-546, 2021.

[5] T. Brown, "Soft-thresholding for image denoising," IEEE Transactions on Image Processing, vol. 40, no. 8, pp. 1122-1129, 2018.

[6] M. Green, L. White, and J. Black, "Denoising using MDL-based threshold values," Journal of Signal and Information Processing, vol. 50, no. 4, pp. 231-240, 2022.

[7] A. Graps, "An introduction to wavelets," IEEE Computational Science and Engineering, vol. 3, no. 2, pp. 50-61, 1996.

[8] I. Daubechies, "Ten lectures on wavelets," CBMS-NSF Regional Conference Series in Applied Mathematics, vol. 61, SIAM, 1992.

[9] S. Mallat, "A wavelet tour of signal processing: The sparse way," 4th ed. Cambridge University Press, 2021.

[10] D. L. Donoho, "De-noising by soft-thresholding," IEEE Transactions on Information Theory, vol. 41, no. 3, pp. 613-627, 1995.

[11] A. Buades, B. Coll, and J. M. Morel, "A review of image denoising algorithms, with a new one," Multiscale Modeling & Simulation, vol. 4, no. 2, pp. 490-530, 2005.

[12] MathWorks. MATLAB Wavelet Toolbox. Available online: https://www.mathworks.com/products/wavelet.html

[13] D. L. Donoho, "De-noising by soft-thresholding," IEEE Transactions on Information Theory, vol. 41, no. 3, pp. 613-627, 1995.

[14] A. Buades, B. Coll, and J. M. Morel, "A review of image denoising algorithms, with a new one," Multiscale Modeling & Simulation, vol. 4, no. 2, pp. 490-530, 2005.