Generalized Family of Memory Type Exponential Estimators using EWMA for Enhanced Estimation of Population Mean

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Abstract

The estimation of population parameters comes across many scientific investigations. The traditional way of estimation utilises the current available information, however this estimation might be improved using current and previous information if available in the form of time-based surveys. The exponentially weighted moving average (EWMA) statistic estimates the population parameters using current as well as past information. This study presents an effective estimator for estimation of population mean in the occurrence of time series data. The use of EWMA statistic has provides an additional advantage of improving the effectiveness of the mean estimate, rather than classical statistics under SRS by adjusting the significance of the smoothing constant λ. The cost of the smoothing constant λ generally decreases from one to zero, which increases the efficiency of the suggested estimator. For $\lambda = 1$, the suggested estimator performs as par with their comparative estimator. This study utilizes EWMA statistic to propose a generalized form of memory type exponential estimator which is more useful in the context of time scaled surveys. The Bias and mean squared error (MSE) of proposed estimators are derived and efficiency of the suggested estimator along with some members and /or competing estimators have been reported.

Keywords: *Bias, MSE, EWMA, SRS, Smoothing constant, Memory type exponential estimators*

1. Introduction

The precision of an estimate of the population mean in sample surveys was significantly increased by the addition of auxiliary variable or variables at the estimation stage. For example, Murthy (1967), Singh (1986, 2003), and the references cited therein were just some of the authors who have contributed on estimating the population mean $\bar{Y}$ of the study variable y using information on an auxiliary variable. Authors have proposed various estimators along with their properties in simple random sampling without (or with) replacement schemes. There is sufficient information on several auxiliary variables in many survey contexts of practical significance. Auxiliary variable has been widely used by various authors to improve and refine estimators under different sampling schemes. These contributions emphasize the important part that auxiliary variables play in enhancing the precision and dependability of population parameter estimates and show how they can be applied to a variety of methodological developments under real-world situations. When there is a positive linear relationship between the study and auxiliary variable, the ratio estimator proposed by Cochran (1940) is usually employed. This estimate allows for changes that are consistent with the direct relationship between the two variables by utilizing their proportionality. The ratio estimator effectively reduces variability and enhances estimate dependability by incorporating the strength of positive correlation. However, when the linear relationship is negative, Robson's (1957) product estimator is more appropriate. By utilizing their inverse relationship, this estimator produces adjustments that represent the opposing patterns of the study and auxiliary variables. By accounting for the negative correlation, the product estimator ensures more accurate population mean forecasts despite the different directional trends. Several improved and modified classes of ratio, exponential, product, regression, and logarithmic types of estimators were created by numerous esteemed authors to estimate parameters using the available auxiliary information. For example, Bahl and Tuteja (1991), Sisodia and Dwivedi (1981), Singh and Espejo (2003), Khosnivasan (2007), Singh (2007,2008), Grover and Kaur (2011,2014), Jerrauddin and Kishun (2016), Subramani (2016), Yadav and Baghel (2021), H.P. Singh et al. (2022), Yadav and Kumar (2025) etc. have developed estimators which utilize auxiliary variables under simple random sampling scheme.

This study suggests exponential ratio type estimators for time-scaled surveys that utilize the use of both the sample's current as well as with the past information from the sample. A large number of surveys are carried out on a regular basis within specific time intervals. For example, NSS (National Sample Survey), NFHS (National Family Health Survey) both conducted every five years by the government of India. Additionally, SRS (Simple Registration System) was conducted every 5 years regularly, PLFS (Periodic Labour Force Survey) was also conducted every year, provide Critical insights into demographics. To estimate the population parameter from these time-scaled surveys by using traditional estimators is a significant challenge. Because the traditional estimators are designed for cross-sectional research and do not take into consideration the variations in time, present in longitudinal data, these produce ordinary results that do not effectively represent the complexity of the data. To tackle these difficulties EWMA based estimation is more appropriate. Memory-type estimators reduce variance by utilizing data from prior survey rounds by taking advantage of the time-series correlation. These estimators can greatly increase the efficiency of survey estimates by utilizing auxiliary information along with past information, which frequently leads to reduced variability when compared to conventional estimators. Robert (2000) was first who proposed EWMA statistic to observe the process mean. Noor-ul Amin (2019) suggested memory type ratio and product estimators incorporating EWMA. Further Noor-ul Amin (2020) utilized the hybrid exponentially weight moving average (HEWMA) statistics to suggest ratio and product estimator for surveys using time scale of data. Shashi et al. (2023) introduced the mean estimation for time based surveys using memory type logarithmic estimators. Further, Aslam et al (2024) presented a new memory type ratio estimator in survey sampling. Sharma and kumari (2024) suggested an estimation procedure for population mean for time scaled survey using EWMA. Singh et al. (2024) introduced an advanced memory type exponential estimators enhancing accuracy in population mean estimator. further Sharma and kumari (2025) introduced a variance estimation using Bernoulli auxiliary variable for time-scaled survey. In the existing literature, it has been observed that memory type estimators based on EWMA statistics perform better than traditional estimators.

The EWMA statistic is given as

$$Z_i = \lambda\bar{y} + (1-\lambda)Z_{i-1} \tag{1.1}$$

$$Q_i = \lambda\bar{x} + (1-\lambda)Q_{i-1} \tag{1.2}$$

Here $\bar{y}$ is the sample mean of study variable and $\bar{x}$ is the sample mean of auxiliary variable, and λ is the smoothing constant which is the weight assign to the observations and lies between 0 and 1 i.e. $0 \leq \lambda \leq 1$. If λ =1, the total weight is assigned to current data moreover values of λ towards 1, current values given more weight and λ towards 0, past values given more weight and λ =0, the total weight is based on past information. The mean and variance of EWMA statistic is given by

$$E(Z_i) = \mu_y \tag{1.3}$$

$$V(Z_i) = \frac{\sigma_y^2}{n}\left(\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)\right) \tag{1.4}$$

The limiting case of the variance is takes the form,

$$V(Z_i) = \frac{\sigma_y^2}{n}\left(\frac{\lambda}{2-\lambda}\right) \tag{1.5}$$

2. Notations and Some Existing Estimators

Let us consider $\mho = (\mho_1, \mho_2, \mho_3 \dots \dots \mho_N)$ be a finite population of size N and a sample of size n is drawn by SRSWOR from the population. Let Y and X be the study and auxiliary variables with in population having N units. Let $\bar{y}$ and $\bar{x}$ be the sample means of study and auxiliary variables respectively. Let

$$Z_i = \lambda\bar{y} + (1-\lambda)Z_{i-1}$$

$$Q_i = \lambda\bar{x} + (1-\lambda)Q_{i-1}$$

be the EWMA statistics of study and auxiliary variables, respectively, based on above population.

The following terms are used to derive mean square error expression:

$e_y = \frac{Z_i - \bar{Y}}{\bar{Y}} \quad e_x = \frac{Q_i - \bar{X}}{\bar{X}}$

$E(e_y) = E(e_x) = 0$

$E(e_y^2) = \frac{f_1 Var(Z_i)}{\bar{Y}^2} = \left(\frac{\lambda}{2-\lambda}\right) f_1 C_y^2$

$E(e_x^2) = \frac{f_1 Var(Q_i)}{\bar{Y}^2} = \left(\frac{\lambda}{2-\lambda}\right) f_1 C_x^2$

$E(e_y e_x) = \frac{f_1 Cov(Z_i, Q_i)}{\bar{Y}^2} = \left(\frac{\lambda}{2-\lambda}\right) f_1 \rho C_y C_x$

Noor-ul-Amin (2019) suggested a memory type of ratio and product estimator under SRS by using EWMA is given by

$$\bar{y}_{mri} = \frac{Z_i}{Q_i}\mu_x \tag{2.1}$$

$$\bar{y}_{mpi} = \frac{Z_i}{\mu_x}Q_i \tag{2.2}$$

The bias and MSE for the estimators are given below,

$$\text{Bias}\,(\bar{y}_{mri}) = -\mu_y \gamma \rho C_x C_y \left(\frac{\lambda}{2-\lambda}\right) \tag{2.3}$$

$$\text{MSE}(\bar{y}_{mri}) = \mu_y^2 \gamma \left(\frac{\lambda}{2-\lambda}\right)(C_y^2 + C_x^2 - 2\rho C_y C_x) \tag{2.4}$$

$$\text{Bias}\,(\bar{y}_{mpi}) = \mu_y \gamma \rho C_x C_y \left(\frac{\lambda}{2-\lambda}\right) \tag{2.5}$$

$$\text{MSE}(\bar{y}_{mpi}) = \mu_y^2 \gamma \left(\frac{\lambda}{2-\lambda}\right)(C_y^2 + C_x^2 + 2\rho C_y C_x) \tag{2.6}$$

The exponential ratio type estimator by employing EWMA is given by

$$\bar{y}_{mexi} = Z_i \exp\left(\frac{\bar{X}-Q_i}{\bar{X}+Q_i}\right) \tag{2.7}$$

The bias and MSE are given as,

$$\text{Bias}\ (\bar{y}_{mexi}) = \mu_y\gamma\left(\frac{\lambda}{2-\lambda}\right)\left[\frac{3}{8}C_x^2 - \frac{1}{2}\rho C_x C_y\right] \tag{2.8}$$

$$\text{MSE}(\bar{y}_{mexi}) = \mu_y^2\gamma\left(\frac{\lambda}{2-\lambda}\right)(C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x) \tag{2.9}$$

Singh et al. (2023) introduced a generalized estimators using EWMA statistics for estimation of population mean, which are given as

$$t_{mrr} = [\theta_1 Z_i + \theta_2(Q_i - \mu_x)]\left[\frac{\alpha\mu_x+\beta C}{\alpha Q_i+\beta C}\right] \tag{2.10}$$

$$t_{mrp} = [\theta_1 Z_i + \theta_2(Q_i - \mu_x)]\left[\frac{\alpha Q_i+\beta C}{\alpha\mu_x+\beta C}\right] \tag{2.11}$$

The corresponding MSE of above estimators are given as

$$\text{MSE}(t_{mrr}) = \mu_y^2[1 + \theta_{1mr}^2 A_{1mr} + \theta_{2mr}^2 B_{1mr} + 2\theta_{1mr}\theta_{2mmr}C_{1r} - 2\theta_{1mr}D_{1mr} - 2\theta_{1mr}D_{m1r} - 2\theta_{2mr}E_{1mr}] \tag{2.12}$$

$$\text{MSE}\big(t_{mrp}\big) = \big[1 + \theta_{1mp}^2 A_{1mp} + \theta_{2mp}^2 B_{1mp} + 2\theta_{1mp}\theta_{2mp}C_{1mp} - 2\theta_{1mp}D_{1mp} - 2\theta_{1mp}D_{1mp} - 2\theta_{2mmp}E_{1p}\big] \tag{2.13}$$

Bhushan et al (2023) presented a logarithmic estimator of population mean based on time surveys by using memory type estimator

$$\varepsilon_a^m = Z_t\left[1 + \log\left(\frac{Q_t}{\bar{X}}\right)\right]^{\alpha} \tag{2.14}$$

The Bias and MSE respectively are given as

$$\text{Bias}(\varepsilon_a^m) = \theta\bar{Y}\gamma\left\{\left(\frac{\alpha^2}{2} - \alpha\right)C_x^2 + \alpha\rho_{xy}C_x C_y\right) \tag{2.15}$$

$$\text{MSE}(\ \varepsilon_a^m) = \ \theta\bar{Y}\gamma\big(C_y^2 + \alpha^2 C_x^2 - 2\alpha\rho_{xy}C_x C_y\big)\ \ \gamma = \frac{\lambda}{\lambda-2} \tag{2.16}$$

Sharma and Kumari (2024) presented an estimation procedure for population mean using EWMA for time based survey, the estimator is given as

$$t_{pr} = \left[w_1 Z_i\left(\frac{\mu_x}{Q_i}\right)^{\alpha}\exp\left(\frac{\eta(\mu_x-Q_i)}{\eta(\mu_x-Q_i)+2\theta}\right)\right] + w_2 Q_i + (1 - w_1 - w_2)\mu_x \tag{2.17}$$

The Bias and MSE corresponding takes the form

$$\text{Bias}\big(t_{pr}\big) = (w_1 - 1)b + w_1\mu_y\theta\gamma(dC_x^2 - \alpha\rho C_y C_x) \tag{2.18}$$

$$\text{MSE}\big(t_{pr}\big) = (1 - 2w_1)b^2 + w_1^2(b^2\gamma\theta\big(C_y^2 + \alpha C_x^2 - 2\alpha\rho C_y C_x\big) + w_2\mu_x\theta C_x^2\gamma + 2w_1 w_2\mu_x\mu_y\theta\gamma(\rho C_y C_x - \alpha C_y^2) \tag{2.19}$$

$b = k + a\,,\ k = \frac{\eta\mu_x}{2(\eta\mu_x+\theta)}$

Singh et al. (2025) proposed an advanced memory type exponential estimators for enhancing accuracy in population mean estimation, the suggested estimators presented below,

$$t_{mpe_i} = [\nu_1\, Z_i + \nu_2\left(\frac{Z_i}{Q_i}\right)\bar{X}\,]\exp\left(\frac{\alpha(\bar{X}-Q_i)}{a(\bar{X}+Q_i)+2\beta}\right) \tag{2.20}$$

The Bias and MSE are given as respectively

$$Bias\left(t_{mpe_i}\right) = \bar{Y}\nu_1\left\{1 - f\gamma\left(\Upsilon C_y^2 - \frac{3}{2}\Upsilon^2 C_x^2\right)\right\} + \bar{Y}\nu_2\left[\left\{1 + f_1\gamma\left(1+\Upsilon+\frac{3}{2}\Upsilon^2\right)C_x^2 - (1+\Upsilon)\rho C_x C_y\right\} - 1\right] \tag{2.21}$$

$$MSE\left(t_{mpe_i}\right) = A_m{}'\nu_1^2 + B_m'\nu_2^2 + 2C_m'\nu_1\nu_2 + 2D_m'\nu_1 + 2E_m{}'\nu_2 + F_m{}' \tag{2.22}$$

3. Proposed Memory Type Exponential Estimators

We proposed a generalised family of memory type estimators for estimation of population mean. Suppose y and x are the study and auxiliary variable, then the suggested estimators are given as,

$$t_{1e} = k_1\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\bar{y} + k_2\bar{y}\exp\left(\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right) \tag{3.1}$$

$$t_{2e} = [k_1\,\bar{y} + k_2\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\bar{y}\,]\exp\left(\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right) \tag{3.2}$$

Now employing above both estimators in memory type exponential estimator form given as follows

$$t_{1me_i} = k_1 Z_i\left(\frac{\bar{X}}{Q_i}\right)^{\alpha} + k_2 Z_i\exp\left(\frac{a(\bar{X}-Q_i)}{a(\bar{X}+Q_i)+2b}\right) \tag{3.3}$$

$$t_{2me_i} = [k_1\,Z_i + k_2 Z_i\left(\frac{\bar{X}}{Q_i}\right)^{\alpha}\,]\exp\left(\frac{a(\bar{X}-Q_i)}{a(\bar{X}+Q_i)+2b}\right) \tag{3.4}$$

Where

$$Z_i = \lambda\bar{y} + (1-\lambda)Z_{i-1}$$

$$Q_i = \lambda\bar{x} + (1-\lambda)Q_{i-1}$$

As we know and used, $e_y = \frac{Z_i-\bar{Y}}{\bar{Y}}$ $\quad e_x = \frac{Q_i-\bar{X}}{\bar{X}}$

$E\left(e_y\right) = E(e_x) = 0$

$E\left(e_y^2\right) = \frac{f_1 Var(Z_i)}{\bar{Y}^2} = \left(\frac{\lambda}{2-\lambda}\right)f_1 C_y^2$

$E(e_x^2) = \frac{f_1 Var(Q_i)}{\bar{Y}^2} = \left(\frac{\lambda}{2-\lambda}\right)f_1 C_x^2$

$E\left(e_y e_x\right) = \frac{f_1 Cov(Z_i Q_i)}{\bar{Y}^2} = \left(\frac{\lambda}{2-\lambda}\right)f_1\rho C_y C_x$

Eq. (3.3) can be re-written as

$$t_{1me_i} = k_1\overline{Y}(1+e_y)\left(\frac{\overline{X}}{\overline{X}(1+e_x)}\right)^{\alpha} + k_2\overline{Y}(1+e_y)\exp\left(\frac{a(\overline{X}-\overline{X}(1+e_x))}{a(\overline{X}+\overline{X}(1+e_x))+2b}\right) \qquad (3.5)$$

$$t_{1me_i} - \overline{Y} = (k_1-1)\overline{Y} + k_1\overline{Y}\{e_y - \alpha e_x + \frac{\alpha(\alpha+1)}{2}e_x^2 - \alpha e_y e_x\} + k_2\overline{Y}\{(1+e_x)(1-\frac{\theta e_x}{2} + \frac{3}{8}\theta^2 e_x^2)\} \qquad (3.6)$$

$$Bias(t_{1me_i}) = (k_1-1)\overline{Y} + k_1\overline{Y}\{\frac{\alpha(\alpha+1)}{2}\left(\frac{\lambda}{2-\lambda}\right)f_1C_x^2 - \alpha\left(\frac{\lambda}{2-\lambda}\right)f_1\rho C_yC_x\} + k_2\overline{Y}\{\left(1+\frac{3}{8}\theta^2\left(\frac{\lambda}{2-\lambda}\right)f_1C_x^2\right) - \frac{\theta}{2}\left(\frac{\lambda}{2-\lambda}\right)f_1\rho C_yC_x\} \qquad (3.7)$$

Or $$Bias(t_{1me_i}) = (k_1+k_2-1)\overline{Y} + k_1\overline{Y}\{\frac{\alpha(\alpha+1)}{2}\gamma f_1C_x^2 - \alpha\gamma f_1\rho C_yC_x\} + k_2\overline{Y}\{\left(\frac{3}{8}\theta^2\gamma f_1C_x^2\right) - \frac{\theta}{2}\gamma f_1\rho C_yC_x\}] \qquad (3.8)$$

$$MSE(t_{1me_i}) = \overline{Y}^2[1 + k_1^2A_m + k_2^2B_m - 2K_1C_m - 2K_2D_m + 2k_1k_2E_m] \qquad (3.9)$$

We get the optimum values of k_1 and k_2

$k_{1(opt)} = \frac{D_mE_m - B_mC_m}{E_m^2 - A_mB_m}$ and $k_{2(opt)} = \frac{E_mC_m - A_mD_m}{E_m^2 - A_mB_m}$

The optimum $MSE(t_{1me_i})$

$$MSE\left(t_{1me_i}(opt)\right) = \overline{Y}^2\left[1 + \frac{C_m^2B_m - 2C_mD_mE_m + A_mD_m^2}{E_m^2 - A_mB_m}\right] \qquad (3.10)$$

Where,

$A_m = 1 + \gamma f_1C_y^2 + \alpha(2\alpha+1)\gamma f_1C_x^2 - 4\alpha\gamma f_1\rho C_yC_x$,

$B_m = 1 + \gamma f_1C_y^2 + \theta^2\gamma f_1C_x^2 - 2\theta\gamma f_1\rho C_yC_x$, $C_m = 1 + \frac{\alpha(\alpha+1)}{2}\gamma f_1C_x^2 - \alpha\gamma f_1\rho C_yC_x$

$D_m = 1 + \frac{3}{8}\theta^2\gamma f_1C_x^2 - \frac{\theta}{2}\gamma f_1\rho C_yC_x$

$E_m = 1 + \gamma f_1C_y^2 + \left\{\frac{\alpha(\alpha+1)}{2} + \frac{3\theta^2}{8} + \frac{\alpha\theta}{2}\right\}\gamma f_1C_x^2 - (2\alpha+\theta)\gamma f_1\rho C_yC_x$

Eq. (3.4) can be re-written as

$$t_{2me_i} = [k_1\overline{Y}(1+e_y) + k_2\overline{Y}(1+e_y)\left(\frac{\overline{X}}{\overline{X}(1+e_x)}\right)^{\alpha}]\exp\left(\frac{a(\overline{X}-\overline{X}(1+e_x))}{a(\overline{X}+\overline{X}(1+e_x))+2b}\right) \qquad (3.11)$$

$$t_{2me_i} - \overline{Y} = (k_1+k_2-1)\overline{Y} + k_1\overline{Y}\left[e_y + \frac{\theta}{2}e_x + \frac{3}{8}\theta^2e_x^2 - \frac{\theta}{2}e_ye_x\right] + k_2\overline{Y}\left[e_y - e_x\left(\alpha+\frac{\theta}{2}\right) + e_x^2\left\{\frac{3}{8}\theta^2 + \frac{\alpha\theta}{2} + \frac{\alpha(\alpha+1)}{2}\right\} - e_ye_x\left(\alpha+\frac{\theta}{2}\right)\right]$$ [[

(3.12)

$$Bias(t_{2me_i}) = (k_1+k_2-1)\overline{Y} + k_1\overline{Y}\left[\frac{3}{8}\theta^2\left(\frac{\lambda}{2-\lambda}\right)f_1C_x^2 - \frac{\theta}{2}\left(\frac{\lambda}{2-\lambda}\right)f_1\rho C_yC_x\right] + k_2\overline{Y}\left[\left\{\frac{3}{8}\theta^2 + \frac{\alpha\theta}{2} + \frac{\alpha(\alpha+1)}{2}\right\}\left(\frac{\lambda}{2-\lambda}\right)f_1C_x^2 - \left(\alpha+\frac{\theta}{2}\right)\left(\frac{\lambda}{2-\lambda}\right)f_1\rho C_yC_x\right] \qquad (3.13)$$

Or $\mathrm{Bias}(t_{2me_i}) = (k_1 + k_2 - 1)\bar{Y} + k_1\bar{Y}\left[\frac{3}{8}\theta^2\gamma f_1 C_x^2 - \frac{\theta}{2}\gamma f_1 \rho C_y C_x\right] +$

$$k_2\bar{Y}\left[\left\{\frac{3}{8}\theta^2 + \frac{\alpha\theta}{2} + \frac{\alpha(\alpha+1)}{2}\right\}\gamma f_1 C_x^2 - \left(\alpha + \frac{\theta}{2}\right)\gamma f_1 \rho C_y C_x\right] \quad (3.14)$$

$$\mathrm{MSE}(t_{2me_i}) = \bar{Y}^2[1 + k_1^2 A_{1m} + k_2^2 A_{2m} - 2k_1 A_{3m} - 2k_2 A_{4m} + 2k_1 k_2 A_{5m}] \quad (3.15)$$

We get the optimum values of k_1 and k_2

$k_{1(opt)} = \frac{A_{4m}A_{5m} - A_{2m}A_{3m}}{A_{5m}^2 - A_{1m}A_{2m}}$ and $k_{2(opt)} = \frac{A_{5m}A_{3m} - A_{1m}A_{4m}}{A_{5m}^2 - A_{1m}A_{2m}}$

The optimum $\mathrm{MSE}(t_{2me_i})$

$$\mathrm{MSE}\left(t_{2me_i}(opt)\right) = \bar{Y}^2\left[1 + \frac{A_{3m}^2 A_{2m} - 2A_{3m}A_{4m}A_{5m} + A_{1m}A_{4m}^2}{A_{5m}^2 - A_{1m}A_{2m}}\right] \quad (3.16)$$

Where, $A_{1m} = 1 + \gamma f_1 C_y^2 + \theta^2\gamma f_1 C_x^2 - 2\theta\gamma f_1 \rho C_y C_y C_x$

$A_{2m} = 1 + \gamma f_1 C_y^2 + \left\{\left(\alpha + \frac{\theta}{2}\right)^2 + \frac{3}{4}\theta_x^2 + \alpha\theta + \frac{\alpha(\alpha+1)}{2}\right\}\gamma f_1 C_x^2 - 4\left(\alpha + \frac{\theta}{2}\right)\gamma f_1 \rho C_y C_x$

$A_{3m} = 1 + \frac{3}{8}\theta^2\gamma f_1 C_x^2 - \frac{\theta}{2}\gamma f_1 \rho C_y C_x,$

$A_{4m} = 1 + \left\{\frac{3}{8}\theta^2 + \frac{\alpha\theta}{2} + \frac{\alpha(\alpha+1)}{2}\right\}\gamma f_1 C_x^2 - \left(\alpha + \frac{\theta}{2}\right)\gamma f_1 \rho C_y C_x$

$A_{5m} = 1 + \left\{\frac{3}{4}\theta^2 + \frac{\alpha\theta}{2} + \frac{\alpha(\alpha+1)}{2} - \frac{\theta}{2}(\alpha + \frac{\theta}{2})\right\}\gamma f_1 C_x^2 - \left(2\alpha + \frac{3}{2}\theta\right)\gamma f_1 \rho C_y C_x \quad \gamma = \frac{\lambda}{2-\lambda}$

Table 1 Some members of the suggested estimators for different chosen values of k_1 and k_2 and .

α	a	b	k_1	k_2	Estimators
0	0	0	1	0	$t_{1me_{i(1)}} = Z_i$
1	1	0	1	0	$t_{1me_{i(2)}} = Z_i\left(\frac{\bar{X}}{Q_i}\right)$
1	1	0	0	1	$t_{1me_{i(3)}} = Z_i \exp\left(\frac{(\bar{X} - Q_i)}{(\bar{X} + Q_i)}\right)$
1	1	0	1	1	$t_{1me_{i(4)}} = Z_i\left(\frac{\bar{X}}{Q_i}\right) + Z_i \exp\left(\frac{(\bar{X} - Q_i)}{(\bar{X} + Q_i)}\right)$
-1	1	0	1	1	$t_{1me_{i(5)}} = Z_i\left(\frac{Q_i}{\bar{X}}\right) + Z_i \exp\left(\frac{(\bar{X} - Q_i)}{(\bar{X} + Q_i)}\right)$
1	a	b	1	1	$t_{1me_i(6)} = Z_i\left(\frac{\bar{X}}{Q_i}\right) + Z_i \exp\left(\frac{a(\bar{X} - Q_i)}{a(\bar{X} + Q_i) + 2b}\right)$
-1	a	b	1	1	$t_{1me_i(7)} = Z_i\left(\frac{Q_i}{\bar{X}}\right) + Z_i \exp\left(\frac{a(\bar{X} - Q_i)}{a(\bar{X} + Q_i) + 2b}\right)$
1	a	b	k_1	k_2	$t_{1me_{i(8)}} = k_1 Z_i \left(\frac{\bar{X}}{Q_i}\right) + k_2 Z_i \exp\left(\frac{a(\bar{X} - Q_i)}{a(\bar{X} + Q_i) + 2b}\right)$
-1	a	b	k_1	k_2	$t_{1me_{i(9)}} = k_1 Z_i \left(\frac{Q_i}{\bar{X}}\right) + k_2 Z_i \exp\left(\frac{a(\bar{X} - Q_i)}{a(\bar{X} + Q_i) + 2b}\right)$

1	1	0	k_1	k_2	$t_{2me_{i(1)}} = [k_1 Z_i + k_2 Z_i \left(\frac{\bar{X}}{Q_i}\right)]\exp\left(\frac{(\bar{X} - Q_i)}{(\bar{X} + Q_i)}\right)$
-1	1	0	k_1	k_2	$t_{2me_{i(2)}} = [k_1 Z_i + k_2 Z_i \left(\frac{Q_i}{\bar{X}}\right)]\exp\left(\frac{(\bar{X} - Q_i)}{(\bar{X} + Q_i)}\right)$
α	1	0	k_1	k_2	$t_{2me_{i(3)}} = [k_1 Z_i + k_2 Z_i \left(\frac{\bar{X}}{Q_i}\right)^{\alpha}]\exp\left(\frac{(\bar{X} - Q_i)}{(\bar{X} + Q_i)}\right)$
1	a	b	k_1	k_2	$t_{2me_{i(4)}} = [k_1 Z_i + k_2 Z_i \left(\frac{\bar{X}}{Q_i}\right)]\exp\left(\frac{a(\bar{X} - Q_i)}{a(\bar{X} + Q_i) + 2b}\right)$ Singh et al. (2024)
-1	a	b	k_1	k_2	$t_{2me_{i(5)}} = [k_1 Z_i + k_2 Z_i \left(\frac{Q_i}{\bar{X}}\right)]\exp\left(\frac{a(\bar{X} - Q_i)}{a(\bar{X} + Q_i) + 2b}\right)$

4. Simulation Study:

The effectiveness of the suggested memory type estimators is evaluated by conducting simulation study. The MSE values of the suggested estimators are calculated by using the following steps.

1. Generated a population of size 1000 by using the bivariate normal distribution with parameters as (Y, X) ~ BVN (2, 10, 1, 2, ρ) .
2. The different values of correlation coefficient are varied as $\rho = 0.75, 0.80, 0.85, 0.90, 0.95$ and also values of smoothing constant are taken as $\lambda = 0.1, 0.3, 0.5, 0.7, 0.9)$.
3. Select 10000 samples of size $n = 100,200,300,500$ respectively and compute the estimator value t for each samples.
4. The MSE for each estimator is obtained by using following formula

$$MSE(tme_i) = \frac{1}{10,000}\sum_{j=1}^{10,000}(t_i - \bar{Y})^2$$

Where $t_i = \bar{y}_0$, $\bar{y}_{mri}$, $\bar{y}_{mexpi}$, t_{1mei} , t_{2mei}

Table 2. MSE of suggested memory type estimators at various values of n ρ λ

ρ	n	λ	$\bar{y}_o$	$\bar{y}_{mri}$	$\bar{y}_{mexpi}$	t_{1mei}	t_{2mei}
0.75	100	0.1	0.008166503	0.000268569	0.000332640	0.000213052	0.000222847
		0.3	0.008173239	0.000948538	0.001163321	0.000770214	0.000795854
		0.5	0.008312435	0.001825371	0.002228867	0.001527800	0.001541377
		0.7	0.008250831	0.002897890	0.003569009	0.002347591	0.002408365
		0.9	0.008316885	0.004457427	0.005487760	0.003538159	0.003720245
	200	0.1	0.003652369	0.000132470	0.000161764	0.000106671	0.000111541
		0.3	0.003707866	0.000428323	0.000525748	0.000351604	0.000359368
		0.5	0.003783434	0.000841313	0.001036565	0.000654983	0.000700693
		0.7	0.003616406	0.001295324	0.001585708	0.001085788	0.001091277
		0.9	0.003699319	0.001964133	0.002431416	0.001588129	0.001632731
	300	0.1	0.002201819	0.000082585	0.000101793	0.000059410	0.000068384
		0.3	0.002102786	0.000234706	0.000291644	0.000186582	0.000194161
		0.5	0.002111216	0.000476881	0.000584328	0.000384256	0.000400248
		0.7	0.002180146	0.000769127	0.000944860	0.000617714	0.000643730
		0.9	0.002142438	0.001141529	0.001409476	0.000919591	0.000951105
	500	0.1	0.000896695	0.000028624	0.000035571	0.000024374	0.000023804
		0.3	0.000915163	0.000101826	0.000125243	0.000086645	0.000085496
		0.5	0.000912720	0.000198111	0.000245028	0.000162732	0.000165022
		0.7	0.000921343	0.000322355	0.000397285	0.000260327	0.000269092
		0.9	0.000909066	0.000488510	0.000599953	0.000101750	0.000410346
0.80	100	0.1	0.009045468	0.000302555	0.000380499	0.000189571	0.000243511
		0.3	0.009062809	0.000989044	0.001248891	0.000609006	0.000792006
		0.5	0.008966663	0.001927484	0.002408721	0.001229912	0.001563237
		0.7	0.008872651	0.003035599	0.003816164	0.001928749	0.002446531
		0.9	0.008860972	0.004587235	0.005771110	0.002939071	0.003696039
	200	0.1	0.003968924	0.000128965	0.000163012	0.000084196	0.000103520
		0.3	0.003944270	0.000444567	0.000559884	0.000282035	0.000357602
		0.5	0.003940473	0.000847503	0.001064002	0.000538260	0.000683972
		0.7	0.003938992	0.001345965	0.001694220	0.000842010	0.001082365
		0.9	0.003994425	0.002084165	0.002611454	0.001309041	0.001684055
	300	0.1	0.002220480	0.000075952	0.000094949	0.000499661	0.000061685
		0.3	0.002238282	0.000250994	0.000315942	0.000160094	0.000202048
		0.5	0.002261330	0.000483347	0.000606557	0.000310401	0.000390472
		0.7	0.002278068	0.000780378	0.000978596	0.000499220	0.000630714
		0.9	0.002303904	0.001188093	0.001498541	0.000751330	0.000953978
	500	0.1	0.000979233	0.000032089	0.000040300	0.000020049	0.000025860
		0.3	0.000983060	0.000109436	0.000137610	0.000069036	0.000088132
		0.5	0.000986324	0.000207089	0.000259103	0.000132489	0.000167759
		0.7	0.000982410	0.000338125	0.000423773	0.000215131	0.000273360
		0.9	0.000953619	0.000497573	0.000658904	0.000322237	0.000402689
0.85	100	0.1	0.009063687	0.000271682	0.000359913	0.000133081	0.000204074
		0.3	0.008725909	0.000875701	0.001158638	0.000428160	0.000658720
		0.5	0.008904293	0.001747993	0.002303505	0.000837200	0.001319322
		0.7	0.009056795	0.002843490	0.003756955	0.001329345	0.002137407
		0.9	0.009123442	0.004337980	0.005736045	0.002073553	0.003261709
	200	0.1	0.003890635	0.000117568	0.000155784	0.000057025	0.000088241
		0.3	0.003949721	0.000403809	0.000532826	0.000193127	0.000304336
		0.5	0.003968613	0.000781543	0.001028500	0.000379781	0.000591255
		0.7	0.003938205	0.001244321	0.001636776	0.000598418	0.000941351
		0.9	0.003936331	0.001890712	0.002485942	0.000908182	0.001430991

ρ	n	λ	$\bar{y}_o$	$\bar{y}_{mri}$	$\bar{y}_{mexpi}$	t_{1mei}	t_{2mei}
	300	0.1	0.002329720	0.000067250	0.000088200	0.000033829	0.000051154
		0.3	0.002301588	0.000235219	0.000309073	0.000117290	0.000178477
		0.5	0.002240017	0.000442572	0.000582411	0.000225272	0.000355593
		0.7	0.002295177	0.000723407	0.000954852	0.000350277	0.000545327
		0.9	0.002329251	0.001117214	0.001471960	0.000533120	0.000843365
	500	0.1	0.000955236	0.000028422	0.000037444	0.000015033	0.000021577
		0.3	0.000973850	0.000103115	0.000135944	0.000048559	0.000077359
		0.5	0.000996356	0.000193509	0.000255766	0.000091092	0.000145453
		0.7	0.000988385	0.000310071	0.000409818	0.000146505	0.000233121
		0.9	0.000983333	0.000469734	0.000620468	0.000227959	0.000353841
0.90	100	0.1	0.008897268	0.000267789	0.000357467	0.000092625	0.000196482
		0.3	0.008870172	0.000895105	0.001187416	0.000310282	0.000661647
		0.5	0.008955450	0.001739439	0.002318098	0.000592731	0.001278365
		0.7	0.008789487	0.002721919	0.003625309	0.000968479	0.002004490
		0.9	0.008743447	0.004168114	0.005533686	0.001469423	0.003080321
	200	0.1	0.003862899	0.000119466	0.000160316	0.000038529	0.000086909
		0.3	0.003845475	0.000387853	0.000516483	0.000137364	0.000285659
		0.5	0.003915140	0.000747387	0.000998130	0.000258624	0.000548157
		0.7	0.003914051	0.001225826	0.001634747	0.000423815	0.000900566
		0.9	0.003992881	0.001890992	0.002515590	0.000653779	0.001393190
	300	0.1	0.002286174	0.000071322	0.000094862	0.000024427	0.000052540
		0.3	0.002312194	0.000238502	0.000316370	0.000083870	0.000176408
		0.5	0.002293191	0.000447503	0.000593718	0.000158026	0.000330986
		0.7	0.002250879	0.000699324	0.000928813	0.000244465	0.000516414
		0.9	0.002263362	0.001075425	0.001427249	0.000383095	0.000795357
	500	0.1	0.001010817	0.000029764	0.000039240	0.000010231	0.000022154
		0.3	0.000987792	0.000099154	0.000132150	0.000033836	0.000072862
		0.5	0.000987261	0.000190175	0.000254171	0.000063226	0.000139160
		0.7	0.001023062	0.000318410	0.000423257	0.000107380	0.000234604
		0.9	0.001031216	0.000485518	0.000500798	0.000162862	0.000359392
0.95	100	0.1	0.007881209	0.000229331	0.000318264	0.000050276	0.000158192
		0.3	0.008021462	0.000759110	0.001055576	0.000157780	0.000521772
		0.5	0.008290599	0.001498832	0.002080964	0.000296493	0.001031135
		0.7	0.008002724	0.002330242	0.003226988	0.000482598	0.001610191
		0.9	0.009296536	0.003586581	0.004981080	0.000722600	0.002467540
	200	0.1	0.003566053	0.000100182	0.000139024	0.000018753	0.000068894
		0.3	0.003563544	0.000330551	0.000460581	0.000066477	0.000226426
		0.5	0.003522373	0.000639337	0.000889169	0.000127854	0.000439023
		0.7	0.003507252	0.001020025	0.001418800	0.000208746	0.000700575
		0.9	0.003576034	0.001574012	0.002188383	0.000319009	0.001081496
	300	0.1	0.002045749	0.000055447	0.000076890	0.000012071	0.000038293
		0.3	0.002108680	0.000197621	0.000274846	0.000037933	0.000135570
		0.5	0.002022686	0.000364200	0.000506900	0.000074101	0.000249920
		0.7	0.002103453	0.000606729	0.000844737	0.000121014	0.000415996
		0.9	0.002115859	0.000930261	0.001295976	0.000184073	0.000637209
	500	0.1	0.000890733	0.000025389	0.000035315	0.000005268	0.000017443
		0.3	0.000880770	0.000083060	0.000115532	0.000016834	0.000057039
		0.5	0.000889966	0.000163026	0.000227324	0.000031393	0.000111455
		0.7	0.000877983	0.000255072	0.000355017	0.000055236	0.000175056
		0.9	0.000907219	0.000396832	0.000552367	0.000077785	0.000272057

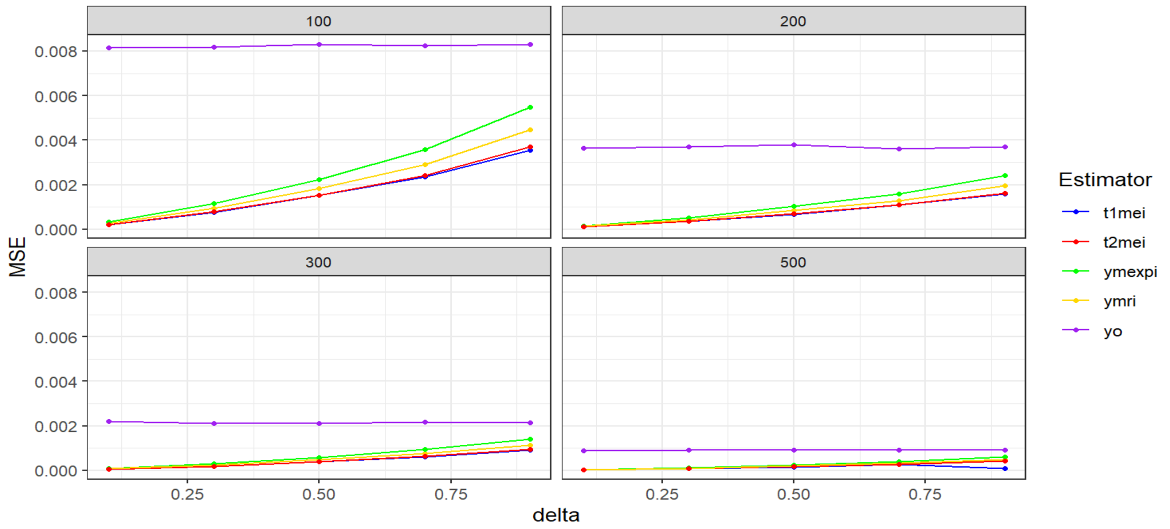

Fig. 1 MSE of suggested and existing estimators at 0.75 correlation coefficient

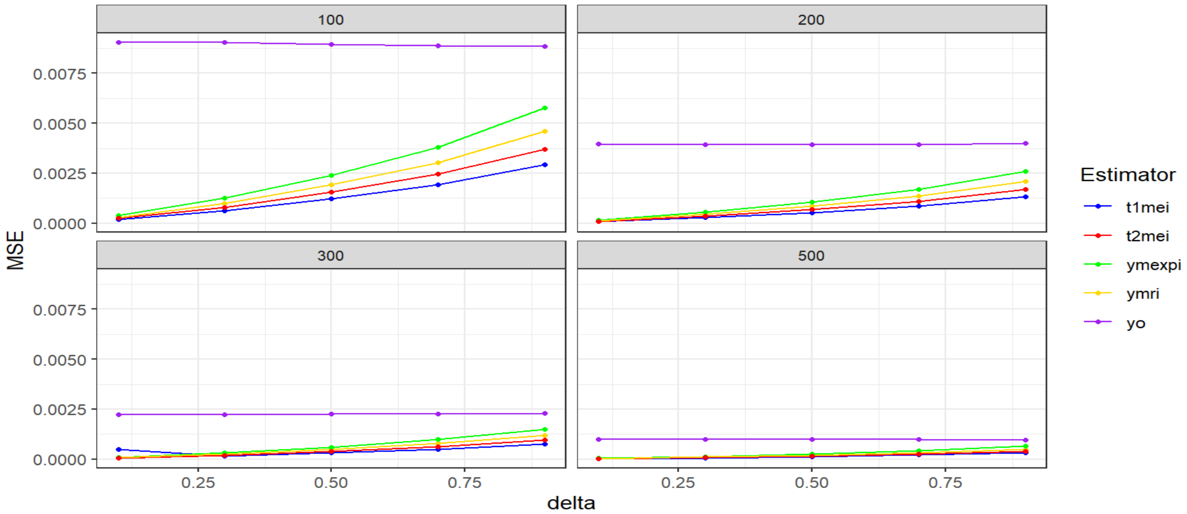

Fig. 2 MSE of suggested and existing estimators at 0.80 correlation coefficient

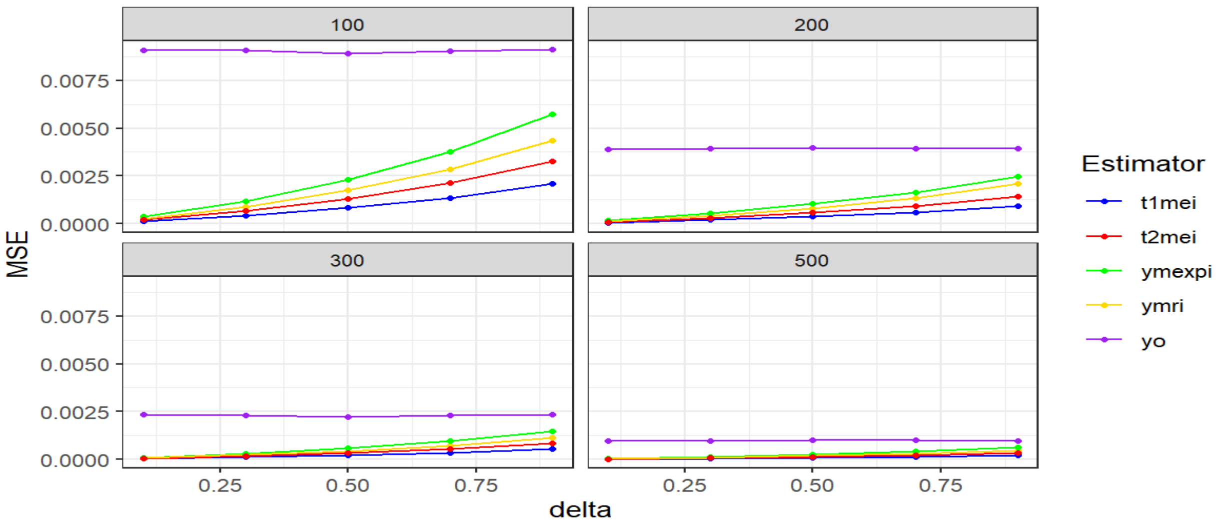

Fig. 3 MSE of suggested and existing estimators at 0.85 correlation coefficient

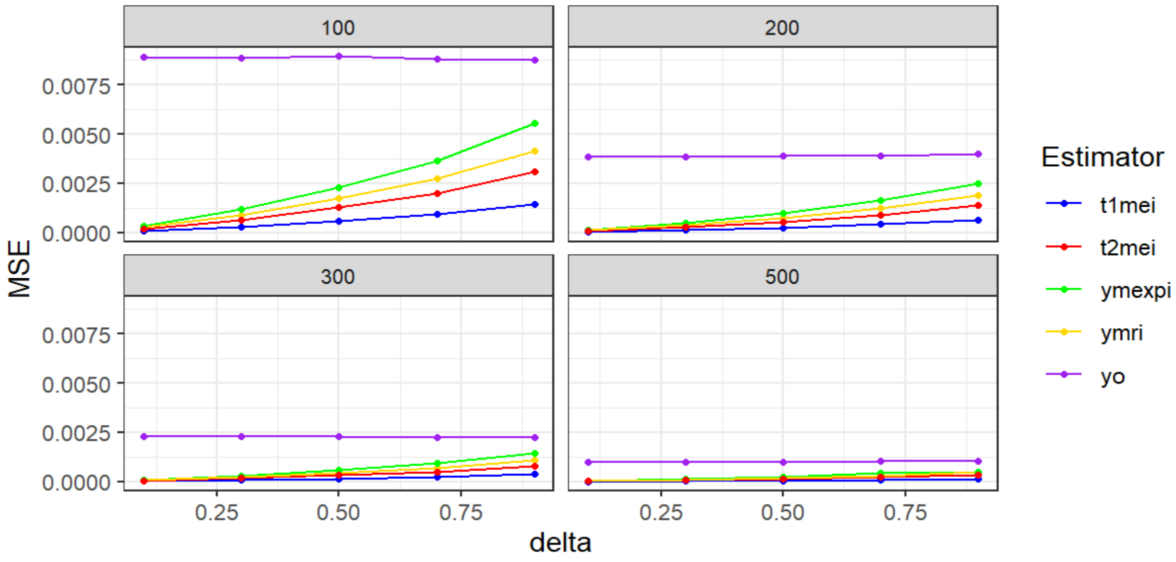

Fig. 4: MSE of suggested and existing estimators at 0.90 correlation coefficient

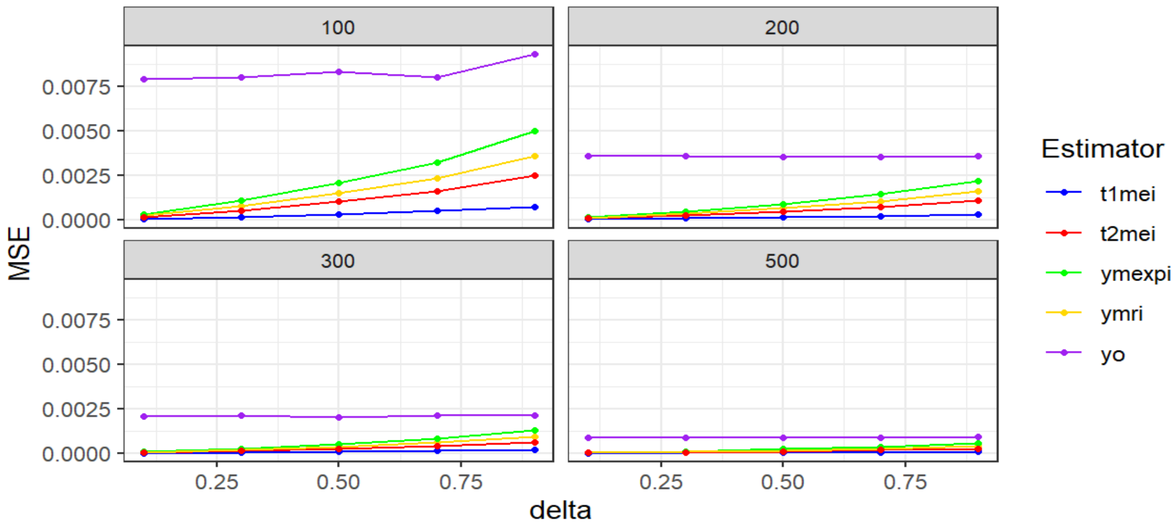

Fig. 5: MSE of suggested and existing estimators at 0.95 correlation coefficient

5. Findings and Discussion

Table 2 represents the MSE of the usual, time based ratio, time based exponential ratio and proposed estimators with smoothing constants λ across different values of ρ and n . Principal observations from the above table is presented below.

i. For fixed value of ρ, as smoothing constant λ increases from 0.1 to 0.9 there is increase in the value of MSE's. At 0.75 correlation coefficient, when λ is 0.1 then MSE's of suggested estimators are 0.000213052, 0.000222847 respectively and when it increases from 0.1 to 0.3 then MSE's are also increases to 0.000770214, 0.000795854 respectively.

ii. As the sample size n rises that is n = 100, 200, 300, 500, MSE of proposed estimators decreases. That is large sample sizes help the estimators to better reflect the underlying relationships and lower estimate variability. It is clearly observed that at ρ = 0.75 correlation coefficient, when sample size changes from 100 to 300 then MSE's of suggested estimators decrease from 0.000213052, 0.000222847 respectively to 0.000062488, 0.000064571 respectively and a similar trend observed for the other estimators as well.

iii. It is also observed that as correlation coefficient ($0.75 \leq \rho \leq 0.95$) between the study and the auxiliary variable increases then MSEs of suggested estimators' decreases. That is a higher value of ρ indicates that the auxiliary variable provides more pertinent data for estimating the study variable, leading to more accurate prediction. When correlation coefficient ρ has increased from 0.75 to 0.80 then MSE's of suggested estimators have decreased from 0.000025562, 0.000028118 respectively to 0.000020654, 0.000027031 respectively at 500 sample size and when correlation coefficient has increased from 0.90 to 0.95 then MSE's of suggested estimators have decreased from 0.000010927, 0.000021203 respectively to 0.000004798, 0.000017395 respectively at n = 500 sample size. The findings showed that the suggested estimators observed more efficient and effective as more data points become available.

In all the findings we depict that when smoothing constant λ grows from 0.1 to 0.9 , then MSE of all the estimators are rapidly increases for all sample sizes and correlation coefficient. And we found that MSE of the suggested estimator decreases with including past information. It also advocates that uses of past information along with current sample information the proposed EWMA estimator performs better than traditional estimators. They help the estimator to attain better accuracy and applicability in real worlds.

6. Conclusion:

In this study, we have presented generalized family of memory type exponential estimators using EWMA for enhanced estimation of population mean. Some other known estimator such as usual estimator, memory type ratio and memory type exponential ratio estimator have become members of these generalized estimators. In addition, we conclude that proposed estimators also performing better than non-time based counterparts. Furthermore, a comprehensive simulation study presented in Table 2 showed the wider applicability of the proposed estimators. The simulation study very well find that the estimators are more efficient as compared to other established estimators, such as $\bar{y}$, $\bar{y}_{mri}$, and $\bar{y}_{mexpi}$ and based on these results, we conclude that our proposed estimators advocates better accuracy and provide a higher level of efficiency in comparison to other estimators at various values of ρ n and λ.

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Appendix:

$A_{1mr} = 1 + f\left(\frac{\lambda}{2-\lambda}\right)(c_y^2 + 3\gamma'^2 c_x^2 - 4\gamma'\rho c_y c_x)$

$B_{1mr} = f\left(\frac{\lambda}{2-\lambda}\right)\gamma'^{*2} c_y^2$

$C_{1mr} = f\left(\frac{\lambda}{2-\lambda}\right)\gamma'^{*}(2\gamma c_x^2 - \rho c_y c_x)$

$D_{1mr} = 1 - f\left(\frac{\lambda}{2-\lambda}\right)(\gamma'\rho c_y c_x - \gamma'^2 c_x^2)$

$E_{1mr} = f\left(\frac{\lambda}{2-\lambda}\right)\gamma'\gamma'^{*} c_x^2$

$A_{1mp} = 1 + f\left(\frac{\lambda}{2-\lambda}\right)(c_y^2 + \gamma'^2 c_x^2 - 4\gamma'\rho c_y c_x)$

$B_{1mp} = f\left(\frac{\lambda}{2-\lambda}\right)\gamma'^{*2} c_y^2$

$C_{1mp} = -f\left(\frac{\lambda}{2-\lambda}\right)\gamma'^{*}(2\gamma c_x^2 - \rho c_y c_x)$

$D_{1mp} = 1 + f\left(\frac{\lambda}{2-\lambda}\right)(\gamma'\rho c_y c_x)$

$E_{1mp} = -f\left(\frac{\lambda}{2-\lambda}\right)\gamma'\gamma'^{*} c_x^2$, $\gamma' = \frac{\alpha\mu_x}{\alpha\mu_x + \beta c}$, $\gamma'^{*} = \frac{\mu_x}{\mu_y}$, and $\beta = 1$

$A'_m = \bar{Y}^2\left[1 + f_1\gamma\left\{C_y^2 + C_x^2 - 2\rho C_Y C_x\right\}\right]$,

$B'_m = \bar{Y}^2\left[1 + f_1\gamma\left\{C_y^2 + (3 + 4\Upsilon + 4\Upsilon^2)C_x^2 - 4(1 + \Upsilon)\rho C_y C_x\right\}\right]$

$C'_m = \bar{Y}^2\left[1 + f_1\gamma\left\{C_y^2 + (1 + 2\Upsilon + 4\Upsilon^2)C_x^2 - 2(1 + 2\Upsilon)\rho C_y C_x\right\}\right]$

$D'_m = \bar{Y}^2\left[1 + f_1\gamma\left\{\frac{3}{2}\Upsilon^2 C_x^2 - \rho C_y C_x\right\}\right]$

$E'_m = -\bar{Y}^2\left[1 + f_1\gamma\left\{\left(1 + \Upsilon + \frac{3}{4}\Upsilon^2\right)C_x^2 - (1 - \Upsilon)\rho C_y C_x\right\}\right]$

$F'_m = \bar{Y}^2$