Optimal Power Flow solution for IEEE 14 and 30 bus system using Lagrange Function

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***Abstract*—This paper proposes a method to find optimal power flow for a power system in electricity market. The optimal power flow is a very large and very difficult mathematical programming problem. Almost every mathematical programming approach that can be applied to this problem has been attempted and it has taken developers many decades to develop computer codes that will solve the OPF problem reliably. The power flow in generators and load are characterized by demand. The different methods to find power flow and optimize it are based on mathematical equations formulation and solving the linear algebra. These methods are robust complex and tine taking as the size of power system increases. The proposed method solves the Lagrange function of System Cost function using Particle Swarm Optimization (PSO) for achieving ac optimal power flow (ACOPF) gives solution to power system of any order. The effectiveness of the proposed approach is tested on IEEE 14 and 30 bus system. Two bus system data is considered for analysis. The test results are accurate and this method can be applied to higher order power system to obtain ac optimal power flow.**

***Index Terms*— Optimal Power Flow, Lagrange Function, Particle Swarm Optimization, ACOPF, IEEE standard bus system**

I. INTRODUCTION

The Optimal Power Flow (OPF) is important for the improvement and efficiency enhancement of the existent electric power systems as well as the proper planning of the systems to be established in the future. In the past two decades, the problem of optimal power flow (OPF) has received much attention. It is of current interest of many utilities and it has been marked as one of the most operational needs. The OPF problem solution aims to optimize a selected objective function such as fuel cost via optimal adjustment of the power system control variables, while at the same time satisfying various equality and inequality constraints. The equality constraints are the power flow equations, while the inequality constraints are the limits on control variables and the operating limits of power system dependent variables. The problem control variables include the generator real powers, the generator bus voltages, the transformer tap settings, and the reactive power of switchable VAR sources, while the problem dependent variables include the load bus voltages, the generator reactive powers, and the line flows. Generally, the OPF problem is a large-scale highly co nstrained nonlinear nonconvex optimization problem.

The OPF based on Particle Swarm Optimization (PSO) method in which total generation cost function is considered as the objective function in [1]. The proposed OPF formulation contains detailed generator constraints including active and reactive power generation limits and also valve point loading effects. PSO developed in MATLAB is examined and tested on the standard IEEE 14 and 30 bus test systems. In [2] the incorporation of PSO as a derivative-free optimization technique in solving OPF problem significantly relieves the assumptions imposed on the optimized objective functions is done. [3] presents an optimal power flow algorithm that utilizes the complementary linear programming technique to find the severe nodal violations that lead variables of other nodes to violate their inequality constraints. The relation between sets of load nodes and sets of generator nodes is analyzed, and the contribution of each generator to individual loads is calculated under proper assumptions.

In [4] intelligent search evolution algorithm is proposed in which a two step initialization process have been adopted which eliminates the mutation operation and also it gives optimal solution with less number of generations which results in the reduction of the computation time. [5] provides an introduction to OPF from an operations research perspective that describes a complete and concise basis of knowledge for beginning OPF research. [6] paper presents an enhanced genetic algorithm for the solution of the optimal power flow (OPF) with both continuous and discrete control variables. Advanced and problem-specific operators are introduced in order to enhance the algorithm's efficiency and accuracy. In [7] the proposed approach introduces an index called the single contingency sensitivity (SCS) index to rank the system branches according to their suitability for installing Thyristor Controlled Series Capacitors(TCSCs). Once the locations are determined, the problem of identifying the optimal TCSC parameters is formulated as an optimization problem and a GA based approach is applied to solve the OPF problem. [8] paper proposes a new evolutionary approach named as multi agent particle swarm optimization algorithm for solving economic dispatch with security constraints (line flow and bus voltage). This method integrates multiagent systems and particle swarm optimization (PSO) to form a new algorithm, multiagent particle swarm optimization algorithm. In [9] the proposed algorithm computes the optimal generation schedule and effectively relieves bus voltage and line flow violations under single line outage contingencies. Security constraints such as bus voltage and line flow violations are effectively handled in the optimization problem using penalty function approach. [10] paper pivots on executions of optimal powerflow problem incorporating with stochastic wind power. This stochastic nature of wind power is modelled with the help of weibull probability distribution. In [11] recent work to develop a real-time algorithm for AC optimal power flow, based on quasi-Newton methods is implemented. The algorithm uses second order information to provide suboptimal solutions on a fast timescale, and can be shown to track the optimal power flow solution when the estimated second order information is sufficiently accurate. [12] paper addresses the optimal power flow (OPF) problem in AC-DC networks to jointly minimize the total electricity generation cost of the network and the cost of transferring active power from the AC grid to the DC microgrids. The AC-DC OPF problem is reformulated as an equivalent traditional AC OPF problem and due to the non-convexity of the AC OPF problem, convex relaxation techniques is used and this transforms the problem to a semidefinite program and the relaxation gap zero is achieved. The sufficient condition for the zero relaxation gap is satisfied, and the proposed approach enables us to find the global optimal solution efficiently.

In this paper, Lagrangerian Function solved by PSO is applied to IEEE 14 and 30 bus test systems. This new approach connects PSO to solve Power Flow equations to find the minimum generation cost values. At the end of the optimization process, the generation and load constraints and convergence time of each test systems are comparatively analyzed. Also, the optimal results obtained in this study are compared to the results of the similar studies reported in the literature. As a result, PSO shows better performance in terms of finding lower cost values than those in the literature in a shorter time. The potential and effectiveness of the proposed approach are demonstrated. The results are promising and show the effectiveness and robustness of the proposed approach.

II. Optimal Power Flow

In this paper, the objective of OPF problem is to identify minimum generation cost of generator units meeting equality and inequality constraints. The equality constraints represent conventional power flow equations and the inequality constraints represent the system operating and control limits. Mathematically, the OPF problem is formulated as a nonlinear optimization problem with equality and inequality constraints, as shown below:

Objective Function :	f(x,u)	(1)
Equality Constraints :	g(x,u)=0	(2)
Inequality Constraints :	h(x,u)≤0	(3)

A. Objective Function

Input-Output load characteristics of generator units exhibit a nonlinear, convex structure. It is possible to obtain cost curves that are closer to actual values considering Valve Point Loading Effect. The objective function of OPF including the effect of the Valve-Point is presented in equation (4):

$$F = \sum_{i=1} (a_i P_{Gi} + b_i P_{Gi} + c_i) \quad (4)$$

State variables are presented in equation (5):

$$x^T=[V_{L1}\dots V_{LNG}, P_{G1}, Q_{G1}\dots Q_{GNG}, S_{L1}\dots S_{LNG}] \quad (5)$$

V_L : load bus voltage
P_{G1} : slack bus active power
Q_G : generator reactive powers
S_L : transmission line loading

Control variables are shown in equation (6):

$$u^T=[V_{G1}\dots V_{GNG}, P_{G2}\dots P_{GNG}, T_1\dots T_{NT}] \quad (6)$$

V_G : generator voltages
P_G : generator real power outputs P_G except slack bus P_{G_1}
T : transformers tap settings

B. Limit Conditions

1) Load Flow Equations

$$P_{Gi} - P_{Li} - V_i \sum_{j=1}^{N} V_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) = 0 \quad (7)$$
$$(i = 1, \dots, N)$$

$$Q_{Gi} - Q_{Li} - V_i \sum_{j=1} (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) = 0 \quad (8)$$
$$(i = 1, \dots, N)$$

2) Generator Limits

$$P^{min} \le P_{Gi} \le P^{max} \quad i = 1 \dots N_G \quad (9)$$

$$Q^{min} \le Q_{Gi} \le Q^{max} \quad i = 1 \dots N_G \quad (10)$$

$$V^{min} \le V_{Gi} \le V^{max} \quad i = 1 \dots N_G \quad (11)$$

P^{min}_{Gi} : minimum active power of the generator ith
P^{max}_{Gi} : maximum active power of the generator ith
Q^{min}_{Gi} : minimum reactive power of the generator ith
Q^{max}_{Gi} : maximum reactive power of the generator ith
V^{min}_{Gi} : minimum voltage value of the generator ith
V^{max}_{Gi} : maximum voltage value of the generator ith

3) Transformer Limits

$$T^{min} \le T \le T^{max} \quad (12)$$

T^{min} : minimum level control of the transformer ith
T^{max} : maximum level control of the transformer ith

III. Lagrange Function

The calculations are done using the Lagrangian while ignoring the generation limits. This allows us to have only equality constraints, and the solutions of the Lagrangian are simply a solution of a set of linear equations.

The expression for the Lagrangian with the power flow equations written out becomes

$$\mathcal{L} = \sum_{i=1}^{N_{bus}} (a_i + b_i P_{gen_i} + c_i P_{gen_i}{}^2)$$
$$+\lambda_1(100B_{x11}\theta_1 + 100B_{x12}\theta_2 + 100B_{x13}\theta_3 - P_{gen_1}$$
$$+\lambda_2(100B_{x21}\theta_1 + 100B_{x22}\theta_2 + 100B_{x23}\theta_3 - P_{gen_2}$$
$$+\lambda_3(100B_{x31}\theta_1 + 100B_{x32}\theta_2 + 100B_{x33}\theta_3 - P_{gen_3}$$
$$+\lambda_4(\theta_1 - 0)$$

where we have now assumed the reference bus to be bus 1.

We now must solve the Lagrangian by taking the derivatives of L with respect to each independent variable in the problem. The independent variables are

$$P_{gen_1}, P_{gen_2}, P_{gen_3}, \theta_1, \theta_2, \theta_3, \lambda_1, \lambda_2, \lambda_3$$

Then we need derivatives of L with respect to each of these variables.

IV. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a heuristic method inspired of the social model of bird swarms and fish schooling. PSO, designed for the solution of nonlinear problems with continuous variables, was developed by Kennedy and Eberhart in 1995. Each individual, which corresponds to a candidate solution, is referred as a particle in a multidimensional search space. The particles in the search space adjust their location and velocity according to their own experience and the experience of neighbors

The position and velocity vectors of a particle in an N-dimensional search space are expressed in the equations (13) and (14)

$$X_i = (x_{i1}, \dots, x_{in}) \quad (13)$$
$$V_i = (v_{i1}, \dots, v_{in}) \quad (14)$$

x_{ii} : position of particle ith in a search space with n particles

v_{ii} : velocity of particle ith in a search space with n particles

The best position obtained by a particle is expressed asfollows:

$$Pbest_i = (x_{i1}{}^{best}, \dots, x_{in}{}^{best}) \quad (15)$$

The particle that has the best position all among the otherparticles in the population is expressed in equation (16):

$$Gbest_i = (x_{i1}{}^{best}, \dots, x_{in}{}^{best}) \quad (16)$$

The velocity and position of each particle updated after *(k+1)* steps is formulated as follows:

$$X_i^{(k+1)} = X_i^k + V_i^{(k+1)} \quad (17)$$

The velocity of ith individual at (k+1) iteration is calculatedin equation (18):

$$V_i^{(k+1)} = \omega V^k + c_1\ rand_1 x(Pbest_i^k - X^k) + c_2\ rand_2 x(Gbest_i^k - X^k) \quad (18)$$

k	: number of iteration
V_i^k	: velocity of particle ith at iteration k
X_i^k	: position of particle ith at iteration k
c_1 and c_2	: acceleration coefficients
w	: inertia weight parameter
$rand_1$, $rand_2$	: random numbers between [0,1]

ω inertia weight parameter as a function of k iteration isexpressed as follows:

$$(k) = \omega_{max} - (^{\omega max - \omega min}/\text{Max.Iter})\ xk \quad (19)$$

At this point *Max.Iter* and *k* are different from each other and indicate the maximum number of iteration and the current number of iteration, respectively. Maximum velocity is expressed as follows [17]:

$$v^{max} = (x^{max} - x^{min})/N \quad (20)$$

N : number of intervals

V. Results and Discussion

In this section, a standard IEEE 14 and 30-bus system has been considered to demonstrate the effectiveness and robustness of proposed algorithm. In 30-bus test system, bus 1 is considered as slack bus, while bus 2,3,5,8,11 and 13 are taken as generator buses and other buses are load buses. A MATLAB program is implemented for the test system on a personal computer with Intel Pentium dual core 2 GHz processor and 8 GB RAM. Many runs have been performed for the test system. The optimal solution results over these five runs have been tabulated. For Standard IEEE bus system data is given as the input parameters for the test system. The solution for the OPF problem is obtained.

Table I and II summarizes the OPF results of both the bus systems.

IEEE 14 bus system	Result
No. of Buses	14
No. of Generators	3
No. of Lines	20
PGen1	28.244
QGen1	4.189
PGen2	46.307
QGen2	-29.335
PGen3	25.44
QGen3	36.356

IEEE 30 bus system	Result
No. of Buses	30
No. of Generators	10
No. of Lines	41
PGen1	65.194
QGen1	74.324
PGen2	10.528
QGen2	-0.838
PGen3	32.213
QGen3	0
PGen4	41.485
QGen4	0
PGen5	4.484
QGen5	0
PGen6	37.589
QGen6	0
PGen13	40.523
QGen13	0
PGen22	26.896
QGen22	0
PGen23	19.972
QGen23	0
PGen27	20.895
QGen27	0

It is clear that the PSO improved for this study provides smaller values in terms of total generation cost of generator units than those found by both conventional PSO and other methods developed in the literature. Moreover, the system losses calculated for this study are better than most of those calculated in the literature. Therefore, it can be inferred that the system losses are in the acceptable limits. Adequate information on the convergence times of the studies made for IEEE 14 and 30 bus system could not be found in the literature.

It is obvious that the proposed PSO algorithm for this study provide better results than those reported in the literature in terms of total generation cost of the generator units. Due to the target of the optimization problem developed for this study is the minimization of the total generation cost, the system losses are not considered as the objective functions. It is more suitable to show the effects of proposed algorithm on total system losses. Therefore, it can be induced that the system losses are in the acceptable limits reported in the conducted study. Adequate references related to IEEE 14 and 30 bus test system could not be found in the literature to be able to compare the convergence times. The PSO carried out on Lagrange Function for this study converges faster to solve same optimization problem.

VI, CONCLUSION

Lagrange Function solved with PSO was used in the OPF problem solution for IEEE 14 and 30 bus test systems, and the following conclusions are obtained:

- The proposed method developed particularly for this study was displayed a better performance than the OPF methods solved in the literature.
- The other methods except PSO used in the literature to solve the OPF problem provided more costly objective function values than the method improved for this study.
- Generally, total system loss values found as a consequence of OPF problem by the method developed for this study are lower than total system losses reported in the literature.

With the PSO developed for this study, the OPF objective function exhibited a relatively faster convergence than the methods used in the similar studies in the literature. This is particularly seen on the study made using IEEE 14 and 30 bus standard test system. No comparison could be made for other systems, since no suitable references could be found in the literature.

As a result, less objective function and generally less system loss values are achieved in a shorter convergence time using the method used, as an optimization tool on IEEE 14 and 30 test systems compared to similar studies in the literature.

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