# Fixed Point Results for a New Three Step Iterative Process

Anju Panwar*, Ravi Parkash Bhokal**

*Assistant Professor, Dept. of Mathematics, M.D.U. Rohtak,

** Assistant Professor, Govt. College Dujana, Jhajjar,

Abstract:-

Aim of present paper is to study some features like convergence, stability and data dependency of a newly introduced iterative process for a class of nonlinear mapping. Numerical example is used to claim that the new iterative process has better rate of convergence than some of the existing iterative processes in the literature. These results may be interpreted as refinement and improvement in the previous known results.

Keywords:- new iterative process, data dependency, quasi-strictly contractive operators.

Mathematics Subject Classification : 47H09, 47H10.

## 1. Introduction and Preliminaries:-

There are numerous problems in science that can be modeled by fixed point. Problems where solutions cannot be obtained analytically, fixed point iteration procedures play a vital role in solution of such problems. Hence iterative procedures have gain popularity in obtaining the fixed points of nonlinear mappings. In recent time some studies are conducted by [8-11].

Let $H$ be a real normed linear space and $T : H \to H$ be a mapping. A point $\theta$ is called the fixed point if $T(\theta) = \theta.$ Throughout the paper $F(T)$ will represent the set of fixed points of mapping $T.$

In 2013, Karakaya et al.[2] introduced a three step iterative process by:

$$u_{n+1} = (1-\alpha_n - \beta_n)v_n + \alpha_n T v_n + \beta_n T w_n$$
$$v_n = (1-\rho_n - \theta_n)w_n + \rho_n T u_n + \theta_n T w_n,$$
$$w_n = (1-\omega_n)u_n + \omega_n T u_n, \;\; n \in N$$

Where $u_0 \in H$ and $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}\{\gamma_n\}_{n=0}^{\infty}\{\theta_n\}_{n=0}^{\infty}\{\rho_n\}_{n=0}^{\infty}, \{\omega_n\}_{n=0}^{\infty} \in [0,1]$. (1.1)

Gursoy and Karakaya [1] introduced the Picard-S Iteration process as follows :

$$u_{n+1} = T v_n,$$
$$v_n = (1-\rho_n)T u_n + \rho_n T w_n,$$
$$w_n = (1-\omega_n)u_n + \omega_n T u_n, \;\; n \in N \tag{1.2}$$

Where $u_0 \in H$ and $\{\rho_n\}_{n=0}^{\infty}, \{\omega_n\}_{n=0}^{\infty} \in [0,1]$.

Recently Dogan and Karakaya [3] introduced S-Picard iterative process by :

$$u_{n+1} = (1-\omega_n)T w_n + \omega_n T v_n,$$
$$v_n = (1-\rho_n)T u_n + \rho_n T w_n,$$
$$w_n = T u_n, \tag{1.3}$$

Where $u_0 \in H$ and $\{\rho_n\}_{n=0}^{\infty}, \{\omega_n\}_{n=0}^{\infty} \in [0,1]$.

It iss claimed by Dogan and Karakaya [3] that S-Picard iterative process has almost same rate of convergence as Picard-S Iterative process.

**Problem 1.1 :-** Is it possible to develop an iteration process whose rate of convergence is better than the iterative processes (1.2) and (1.3) ?

To answer this we introduce a new iteration process as :-

$$z_n = T x_n,$$
$$y_n = T x_n,$$
$$x_{n+1} = (1-\alpha_n)y_n + \alpha_n T y_n, \tag{1.4}$$

with sequence $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$ and $x_0 \in H$

In this paper we establish some convergence and stability results for our newly introduced iterative process.

Scherzer [7] introduced the class of class of quasi-strictly contractive operators by the condition:

$$\| p - Ty \| \le \delta \; \| p - y \|, \delta \in [0,1) \text{ and for all } y \in H \tag{1.5}$$

Chidume and Olaleru [12] gave several examples to show that (1.5) is more general than contraction mapping and they claimed that every contraction mapping with a fixed point satisfies inequality (1.5).

**Definition 1.2[4] :-** Let $H$ be a real normed linear space and $P_1,\ P_2 : H \to H$ be two operators. Then $P_1$ is called the approximate operator of $P_2$ if for all $x \in H$ and for fixed $\varepsilon > 0$, we have

$$\| P_1 x - P_2 x \| \leq \varepsilon \qquad (1.6)$$

**Definition 1.3[4]:-** Let $H$ be a real normed linear space and Let $\{x_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$ be the sequence converging to $l_1$ and $l_2$ respectively. Assume that $\lim_{n \to \infty} \frac{|sx_n - l_1|}{|u_n - l_2|} = l$. Then

1. If $l = 0$ then the sequence $\{x_n\}_{n=0}^{\infty}$ converges faster to $l_1$ than $\{u_n\}_{n=0}^{\infty}$ to $l_2$.
2. $0 < l < \infty$, then both the iterative process have same rate of convergence.

**Lemma 1.4 [5]:-** If $l$ be a real number satisfying $0 \leq l < \infty$ and $\{\vartheta_n\}_{n=0}^{\infty}$ be the sequence of positive numbers such that $\lim_{n\to\infty} \vartheta_n = 0$ and for $u_{n+1} \leq l\, u_n + \vartheta_n, n = 1,2, \ldots$ then $\lim_{n\to\infty} u_n = 0$.

**Lemma1.5 [6]:-** $\{\vartheta_n\}_{n=0}^{\infty}$ be the sequence of positive numbers for which there exists $n_0 \in N$ such that for all $n \geq n_0$ following inequality is satisfied:

$$\vartheta_{n+1} \leq (1 - \lambda_n)\vartheta_n + \lambda_n k_n$$

where $k_n \in (0,1), \forall\, n \in N, \sum_{n=1}^{\infty} \lambda_n = \infty$ and then we have $0 \leq \limsup_{n\to\infty} \vartheta_n \leq \limsup_{n\to\infty} k_n$.

**Lemma1.6 [3]:-** Let $H$ be a real normed linear space and $T : H \to H$ be a mapping satisfying condition (1.5) with a fixed point $p$. Assume that $\check{T}$ be approximate operator of $T$ for given $\varepsilon$. Then

$$\| Tx - \check{T}y \| \leq 2\delta \| x - p \| + \delta \| y - x \| + \varepsilon. \qquad (1.7)$$

## 2. Main Results

**Theorem 2.1**:- Let $H$ be a real normed linear space and $T : H \to H$ be a mapping satisfying condition (1.5). Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by the iterative process (1.4) with sequence $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$ and satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then the iterative process (1.4) converges to fixed point of the operator $T$.

Proof:- Using (1.4) and (1.5) we have,

$$\| z_n - p \| = \| Tx_n - p \| \leq \delta \| x_n - p \| \quad (2.1)$$

Again by (1.4), (1.5) and (2.1) we have

$$\| y_n - p \| = \| Tz_n - p \| \leq \delta \| z_n - p \| \leq \delta^2 \| x_n - p \| \quad (2.2)$$

Now using (1.4), (1.5) and (2.2) we have

$$\| x_{n+1} - p \| = \| (1-\alpha_n)y_n + \alpha_n Ty_n - p \|$$

$$\leq (1-\alpha_n) \| y_n - p \| + \alpha_n \| Ty_n - p \|$$

$$\leq (1-\alpha_n) \| y_n - p \| + \alpha_n\delta \| y_n - p \|$$

$$\leq (1-\alpha_n + \alpha_n\delta) \| y_n - p \|$$

$$\leq (1-\alpha_n(1-\delta))\delta^2 \| x_n - p \| \quad (2.3)$$

Now $(1-\alpha_n(1-\delta))\delta^2 < 1$, hence by lemma (1.5) we have,

$\lim_{n \to \infty} \| x_n - p \| = 0$. This completes the proof.

**Theorem 2.2**:- Let $H$ be a real normed linear space and $T : H \to H$ be a mapping satisfying condition (1.5) with fixed point $q$. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by the iterative process (1.4) with sequence $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$ and satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Let $\{u_n\}_{n=0}^{\infty} \subseteq H$ be the sequence generated by

$$w_n = Tu_n,$$

$$v_n = Tu_n,$$

$$u_{n+1} = (1-\alpha_n)v_n + \alpha_n Tv_n \quad (2.4)$$

and let $\kappa_n = \| u_{n+1} - f(T, u_n) \|$ with $\lim_{n\to\infty} \delta_n = 0$. Then the iterative process (1.4) is T-stable.

Proof :- We have $\kappa_n = \| u_{n+1} - f(T, u_n) \|$ with $\lim_{n\to\infty} \kappa_n = 0$. Using (1.5) and (2.4) we have

$$\| u_{n+1} - q \| \leq \| u_{n+1} - f(T, u_n) \| + \| f(T, u_n) - q \|$$

$$\leq \kappa_n + \| (1-\alpha_n)T(T(u_n)) + \alpha_n T(T(T(v_n))) - q \|$$

$$\leq \kappa_n + (1-\alpha_n) \| T(T(u_n)) - q \| + \alpha_n \| T(T(T(u_n))) - q \|$$

$$\leq \kappa_n + \delta(1-\alpha_n) \| T(u_n) - q \| + \alpha_n \delta \| T(T(u_n)) - q \|$$

$$\leq \kappa_n + \delta^2(1-\alpha_n) \| u_n - q \| + \alpha_n \delta^2 \| T(u_n) - q \|$$

$$\leq \kappa_n + \delta^2(1-\alpha_n) \| u_n - q \| + \alpha_n \delta^3 \| u_n - q \|$$

$$\leq \kappa_n + (1-\alpha_n(1-\delta))\delta^2 \| u_n - q \| \qquad (2.5)$$

Since $1-\alpha_n(1-\delta) < 1$ and $\delta^2 < \delta,$ hence (5) becomes

$$\| u_{n+1} - q \| \leq \kappa_n + \delta \| u_n - q \| \qquad (2.6)$$

Now (2.6) satisfies all the requirements of lemma 1.4, so we have $\lim_{n\to\infty} u_n = q$. Hence the iterative process (1.4) is T-stable.

**Example 2.3** :- Let $H = [0,1]$ and $T : H \to H$ be a mapping defined by $Tx = \frac{x}{2}$. Then $T$ has a unique fixed point 0. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined by $x_n = \frac{1}{n}$. Then $\lim_{n\to\infty} x_n = 0$.

Now we have

$$\kappa_n = \| x_{n+1} - f(T, x_n) \|$$

$$\leq \| x_{n+1} - ((1-\alpha_n)T(T(x_n)) + \alpha_n T(T(T(x_n))) \|$$

$$\leq \| x_{n+1} - (1-\alpha_n)\frac{x_n}{4} - \alpha_n \frac{x_n}{8} \|$$

$$\leq \| \frac{1}{n+1} - \frac{1}{8n} - \frac{1}{16n} \|$$

We have $\lim_{n\to\infty} \kappa_n = 0$. Hence the iterative process (1.4) T-stable.

**Theorem 2.4**:- Let $H$ be a real normed linear space and $T : H \to H$ be a mapping satisfying condition (1.5) with fixed point $q$. Let $\{x_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$be the sequence generated by the iterative process (1.4) and (1.3) respectively with sequence $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty} \in [0,1]$ and satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$, $\sum_{n=0}^{\infty} \alpha_n \beta_n = \infty$. Also assume that $\alpha_1 \leq \alpha_n \leq 1$ and $\beta_1 \leq \beta_n \leq 1$. Then the iterative process (1.4) converges faster than the iterative process (1.3) provided both have same initial approximation.

Proof :- From (3) we have

$$\| x_{n+1} - p \| \leq (1 - \alpha_n(1 - \delta))\delta^2 \| x_n - p \|$$

$$\| x_n - p \| \leq (1 - \alpha_{n-1}(1 - \delta))\delta^2 \| x_{n-1} - p \|$$

$$\| x_{n-1} - p \| \leq (1 - \alpha_{n-2}(1 - \delta))\delta^2 \| x_{n-2} - p \|$$

……………………………………………………..

$$\| x_1 - p \| \leq (1 - \alpha_0(1 - \delta))\delta^2 \| x_0 - p \|$$

Combining all the above inequalities we have,

$$\| x_{n+1} - p \| \leq \delta^{2(n+1)} \| x_0 - p \| \prod_{k=0}^{n+1}(1 - \alpha_k(1 - \delta))$$

Now applying the assumption $\alpha_1 \leq \alpha_n \leq 1$ on above inequality we have

$$\| x_{n+1} - p \| \leq \delta^{2(n+1)} \| x_0 - p \| \prod_{k=0}^{n+1}(1 - \alpha_1(1 - \delta))$$

$$\leq \delta^{2(n+1)} \| x_0 - p \| (1 - \alpha_1(1 - \delta)^{n+1} \qquad (2.7)$$

Now from (1.5) and (1.3) we have

$$\| u_{n+1} - p \| = \| (1 - \alpha_n)Tw_n + \alpha_n Tv_n - p \|$$

$$\leq (1 - \alpha_n) \| Tw_n - p \| + \alpha_n \| Tv_n - p \|$$

$$\leq (1 - \alpha_n)\delta \| w_n - p \| + \alpha_n\delta \| v_n - p \| \qquad (2.8)$$

$$\| v_n - p \| = \| (1 - \beta_n)Tw_n + \beta_n Tv_n - p \|$$

$$\leq (1 - \beta_n) \| Tu_n - p \| + \beta_n \| Tw_n - p \|$$

$$\leq (1 - \beta_n)\delta \| u_n - p \| + \beta_n\delta \| w - p \| \qquad (2.9)$$

and

$$\| w_n - p \| = \| Tu_n - p \| \leq \delta \| u_n - p \| \tag{2.10}$$

Using (2.10) in (2.9) we have,

$$\begin{aligned} \| v_n - p \| &\leq (1-\beta_n)\delta \| u_n - p \| + \beta_n \delta^2 \| u_n - p \| \\ &\leq \delta\big(1 - \beta_n(1-\delta)\big) \| u_n - p \| \end{aligned} \tag{2.11}$$

Using (2.10) and (2.11) in (2.8) we obtain

$$\begin{aligned} \| u_{n+1} - p \| &\leq (1-\alpha_n)\delta^2 \| u_n - p \| + \alpha_n \delta^2 \big(1 - \beta_n(1-\delta)\big) \| u_n - p \| \\ &\leq \delta^2 [1 - \alpha_n + \alpha_n\big(1 - \beta_n(1-\delta)\big)] \| u_n - p \| \\ &\leq \delta^2 (1 - \alpha_n\beta_n(1-\delta)) \| u_n - p \| \end{aligned} \tag{2.12}$$

From (2.12) we have

$$\| u_{n+1} - p \| \leq \delta^2 (1 - \alpha_n\beta_n(1-\delta)) \| u_n - p \|$$

$$\| u_n - p \| \leq \delta^2 (1 - \alpha_{n-1}\beta_{n-1}(1-\delta)) \| u_{n-1} - p \|$$

$$\| u_{n-1} - p \| \leq \delta^2 (1 - \alpha_{n-2}\beta_{n-2}(1-\delta)) \| u_{n-2} - p \|$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\| u_1 - p \| \leq \delta^2 (1 - \alpha_0\beta_0(1-\delta)) \| u_0 - p \|$$

From the above inequality we have the following estimate

$$\| u_{n+1} - p \| \leq \delta^{2(n+1)} \| u_0 - p \| \prod_{k=0}^{n+1} (1 - \alpha_k\beta_k(1-\delta))$$

Now applying the assumption $\alpha_1 \leq \alpha_n \leq 1$ and $\beta_1 \leq \beta_n \leq 1$ on above inequality we have

$$\begin{aligned} \| u_{n+1} - p \| &\leq \delta^{2(n+1)} \| u_0 - p \| \prod_{k=0}^{n+1} (1 - \alpha_1\beta_1(1-\delta)) \\ &\leq \delta^{2(n+1)} \| u_0 - p \| (1 - \alpha_1\beta_1(1-\delta)^{n+1} \end{aligned} \tag{2.13}$$

Let $a_n = \delta^{2(n+1)} \| x_0 - p \| (1 - \alpha_1(1-\delta)^{n+1}$ and

$$b_n = \delta^{2(n+1)} \parallel u_0 - p \parallel (1 - \alpha_1\beta_1(1 - \delta)^{n+1}$$

Now $\frac{a_n}{b_n} = \frac{\delta^{2(n+1)}\|x_0 - p\|(1-\alpha_1(1-\delta)^{n+1}}{\delta^{2(n+1)}\|u_0 - p\|(1-\alpha_1\beta_1(1-\delta)^{n+1}}$

$$= (\frac{1-\alpha_1(1-\delta)}{1-\alpha_1\beta_1(1-\delta)})^{n+1}$$

Now $\alpha_n, \beta_n \in [0,1]$ and $\delta \in (0,1)$ so we may easily conclude that $\frac{1-\alpha_1(1-\delta)}{1-\alpha_1\beta_1(1-\delta)} < 1$ and hence

$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and hence using definition 1.3, we conclude that iterative process (1.4) converges faster than the iterative process (1.3).

**Example 2.5**:- $H = [0,\infty)$ and $\rho : H \to H$ be a mapping defined by $\rho(\theta) = \frac{\theta - \ln(\theta+1)}{2}$. Clearly $\rho$ satisfies the condition (1.5). The following table shows the convergence pattern of iterative process (1.3) and (1.4) for initial approximation $x_1 = u_1 = 1$ and $\alpha_n = \beta_n = \frac{n+1}{n+2}$.

| $x_n$ | S-Picard Iteration | New Iteration |
|---|---|---|
| $x_1$ | 0.002262870819999 | 0.001786312500 |
| $x_2$ | 0.000000000000121 | **0** |
| $x_3$ | **0** | **0** |

**Table 1 : Comparison of rate of convergence between two iterative schemes**

In the above table S-Picard iteration reaches the fixed point at third step while the new iteration process reaches the fixed point at second step hence the new iteration process has better rate of convergence than the S- Picard iterative process.

**Theorem 2.6**: - Let $\check{T}$ be approximate operator of $T$ and Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by the iterative process (1.4) for operator $T$. Consider the iterative process $\{\check{x}_n\}$ defined by

$$\check{z}_n = \check{T}\check{x}_n, \quad \check{y}_n = \check{T}\check{z}_n \quad \text{and} \quad \check{x}_{n+1} = (1-\alpha_n)\check{y}_n + \alpha_n\check{T}\check{y}_n \tag{2.14}$$

Let $Tp = P$and $\check{T}\check{p} = \check{p}$. Then $\parallel p - \check{p} \parallel \leq \frac{\delta(1+\varepsilon)}{1-\delta^2}$.

Proof :- From (1.4) and (2.14) and lemma 1.6, we have,

$$\parallel z_n - \check{z}_n \parallel = \parallel Tx_n - \check{T}\check{x}_n \parallel \leq 2\delta \parallel x_n - p \parallel + \delta \parallel x_n - \check{x}_n \parallel + \varepsilon \tag{2.15}$$

Again using (1.4) and (2.14) and lemma 1.6,

$$\| y_n - \check{y}_n \| = \| Tz_n - \check{T}\check{z}_n \| \leq 2\delta \| z_n - p \| + \delta \| z_n - \check{z}_n \| + \varepsilon \tag{2.16}$$

Also from (1.4) and (2.14) and lemma 1.6,

$$\| x_{n+1} - \check{x}_{n+1} \| = \| (1-\alpha_n)y_n + \alpha_n T y_n - ((1-\alpha_n)\check{y}_n + \alpha_n \check{T}\check{y}_n) \|$$

$$\leq (1-\alpha_n) \| y_n - \check{y}_n \| + \alpha_n \| Ty_n - \check{T}\check{y}_n \|$$

$$\leq (1-\alpha_n) \| y_n - \check{y}_n \| + \alpha_n (2\delta \| y_n - p \| + \delta \| y_n - \check{y}_n \| + \varepsilon)$$

$$\leq (1-\alpha_n + \alpha_n\delta) \| y_n - \check{y}_n \| + 2\alpha_n\delta \| y_n - p \| + \alpha_n\varepsilon \tag{2.17}$$

Using (2.15) in (2.16) we have

$$\| y_n - \check{y}_n \| \leq 2\delta \| z_n - p \| + \delta(2\delta \| x_n - p \| + \delta \| x_n - \check{x}_n \| + \varepsilon) + \varepsilon \qquad (2.s18)$$

From (2.18) and (2.17) we have

$$\| x_{n+1} - \check{x}_{n+1} \| \leq (1-\alpha_n+\alpha_n\delta)(2\delta \| z_n - p \| + \delta(2\delta \| x_n - p \| + \delta \| x_n - \check{x}_n \| + \varepsilon) + \varepsilon) + 2\alpha_n\delta \| y_n - p \| + \alpha_n\varepsilon$$

$$\leq 2(1-\alpha_n(1-\delta))\delta \| z_n - p \| + 2(1-\alpha_n(1-\delta)\delta^2 \| x_n - p \|$$

$$+ (1-\alpha_n(1-\delta))\delta^2 \| x_n - \check{x}_n \| + 2\alpha_n\delta \| y_n - p \|$$

$$+ (1-\alpha_n(1-\delta)\delta(\varepsilon+1) + \alpha_n\varepsilon$$

Now $\alpha_n \in [0,1], \delta \in (0,1)$ hence $1-\alpha_n(1-\delta) < 1$ , therefore we have

$$\| x_{n+1} - \check{x}_{n+1} \| \leq 2\delta \| z_n - p \| + 2\delta^2 \| x_n - p \| + \delta^2 \| x_n - \check{x}_n \| + 2\alpha_n\delta \| y_n - p \|$$

$$+ \delta(\varepsilon+1) + \alpha_n\varepsilon$$

Let $\varrho = 1-\delta^2 \in (0,1)$, $\mu_{n+1} = \| x_{n+1} - \check{x}_{n+1} \|$ and

$$\xi_n = \frac{2\delta \| z_n - p \| + 2\delta^2 \| x_n - p \| + 2\alpha_n\delta \| y_n - p \| + \delta(\varepsilon+1) + \alpha_n\varepsilon}{\varrho} \geq 0$$

Hence

$$\| x_{n+1} - \check{x}_{n+1} \| \leq (1 - \varrho) \| x_n - \check{x}_n \| + 2\delta \| z_n - p \| + 2\delta^2 \| x_n - p \| + 2\alpha_n \delta \| y_n - p \| + \delta(\varepsilon + 1) + \alpha_n \varepsilon \quad (2.19)$$

Now (19) satisfies all the requirements of lemma (1.5), hence we obtain

$$\| p - \check{p} \| \leq \frac{\delta(1+\varepsilon)}{1-\delta^2}.$$

## 3 Conclusions:-

In this paper, some fixed point results are obtained with the help of newly defined iterative procedure and claimed that its rate of convergence is better than the others which are referred to this paper. Some fixed point results have been supported with the help of non trivial examples.

## References:-


1. Gursoy, F., Karakaya, V., A Picard-S hybrid type iteration method for solving a differential equation with related argument, Xiv: 1403.2546 (2014).
2. Karakaya. V., Dogan, K., Gursoy, F.,Erturk, M., Fixed point of a new three step iteration algorithm under contractive like operators over normed spaces, Abstract and Applied analysis, 2013, Article ID 560258, 1-9.
3. Dogan, K., Karakaya, V., A study in fixed point theory for a new iterative scheme and a class of generalized mappings, Creat. Math. Inform., 27(2018), 151-160.
4. Berinde, V., Picard iteration converges faster than Mann iteration for a class of quasi-contractive operators, Fixed Point theory and Applications, 2(2014), 97-105.
5. Weng, X., Fixed point iteration for local strictly pseudo contractive mappings, Proc. Amer. Math. Soc., 113(1991), 727-731.
6. Soltuz, S.M. Grosan, T., Data dependence for Ishikawa iteration when dealing with contractive like operators, Fixed Point Theory, 2008, 1-7.
7. Scherzer, O., Convergence criteria of iterative methods based on Landweber iteration for solving non-linear problems, J.Math. Anal. Appl. 194 (1995), 911-923.
8. Erturk, M., Gursoy, F., Some convergence, stability and data dependency results for Picard-S iteration method of quasi-strictly contractive operators, Mathematica Bohemica, 144(2019), 69-83.

9. Panwar, A., Bhokal, R.P., Fixed Point Results for K-iteration process using nonlinear type mappings, Open Access Library Journal, 2019, 1-13.
10. Akewe, H., Okeke, G.A., Convergence and stability theorems for the Picard-Mann hybrid iterative scheme for a general class of contractive like operators. Fixed Point theory & Application 2015, 1-8.
11. Berinde, V., On a notion of rapidity of convergence used in the study of fixed point iterative methods. Creat. Math. Inform. 2016, 29-40.
12. Chidume, C.E., Olaleru, J.O., Picard iteration process for a general class of contractive mappings. J. Niger. Math. Soc., 2014, 19-23.