STUDY ON LABELING OF CERTAIN CLASS OF GRAPHS

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ABSTRACT

A *graph* is a collection of nodes and lines that we call vertices and edges, respectively. A graph can be labeled or unlabeled. In this paper, we are interested in labeled graphs. In many labeled graphs, the labels are used for identification only. The kind of labeling we are interested in can serve dual purposes: Labeling can be used not only to identify vertices and edges, but also to signify some additional properties, depending on the particular labeling. The study of graph theory was born in the 18th century when the citizens of Konigsberg tried to solve the problem of traversing the 7 bridges on the Pregel river. They wondered if they could walk and cross every bridge exactly once and finish back at the starting point. Using graph representation, the well known mathematician Euler found that it is impossible to do it. However, it is fair to say that the formal and systematical study of graph theory began with D. Konig's book in 1935.

Keywords: Prime Distance Graphs, Prime Distance Labeling, Distinct Prime Distance Labeling, Generalized Petersen Graphs.

Introduction

Graph theory has rigorous applications in diversified fields like operations research, genetics, computer technology, physics, chemistry, communication networks, electrical network, economics and social sciences. There are many research topics in graph theory. Some of the major themes in graph theory are Graph coloring, Spanning trees, Planar graphs, Networks, Eulerian tours, Hamiltonian cycles, Matching, Domination theory and Graph labeling. Most of these topics have been widely discussed in the literature [13, 20,73].

LABELING OF GRAPHS

In the theory of graph labeling the labels are always mathematical objects which may be integers, prime numbers, modular integers, or elements of a group. A lot of research has been done in the topic of labeling of graphs. The study of graph labeling has focused on finding classes of graphs which admit a particular type of labeling. Many practical problems in real life situations have motivated the study of labeling of a graph subject to certain conditions. A systematic presentation of applications of graph labeling is given in [17].

The concept of labeling of graphs has recently gained a lot of popularity in the area of graph theory. Most of the graph labeling method trace their origin to one introduced by Rosa [36]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models in a broad range of applications. Over the past five decades various labeling of graphs such as cordial labeling, prime labeling, magic labeling, antimagic labeling, bimagic labeling, mean labeling, arithmetic labeling, graceful labeling, harmonious labeling etc., have been studied extensively in the literature [17]. Some graph labeling for the class of competition graph is also studied in the literature [10].Throughout the paper, we consider finite simple and undirected graphs with p vertices and q edges.

1.3 GRACEFUL GRAPHS

A. Rosa introduced a labeling function f from a set of vertices in a graph G to the set of integers $\{0, 1, 2,\ldots, q\}$, where q is the number of edges in G, so that each edge xy is assigned the label $| f(x) - f(y) |$, with all labels distinct. Rosa called this labeling as *ü valuation*. Independently, Golomb [19] studied the same type of labeling and called the labeling as *graceful labeling* and this is now the popular term.

The Ringel-Kotzig conjecture (GTC) that all trees are graceful has been the focus of many papers. Apart from the theoretical developments, researchers have been trying to find applications of graph labeling. Applications of graph labeling have been found in X-ray crystallography, coding theory, radar, circuit design, astronomy and communication design. Some interesting applications of graph labeling can be found in the literature.

Generally, there are two main basic reasons for developing a theory. First, we may need a new theory to solve a problem. An example of this is graceful labeling that was developed to solve the problem of the decomposition of a complete graph into isomorphic subgraphs. Second, the development of a new theory comes from human curiosity. An example of this is magic labeling. In this paper, we are mainly driven by the second reason.

MAGIC AND ANTIMAGIC GRAPHS

Interestingly, in 1963 Sedlacek published a paper about another kind of graph labeling[38]. He called the labeling *magic labeling*. It was motivated by the magic square notion in number theory. The weight of a vertex v in G under an edge labeling is the sum of edge labels corresponding to all edges incident with v. If all vertices in G have the same weight k, we call the labeling as vertex-magic edge labeling. If all vertices in G have different weights, then the labeling is called vertex antimagic edge labeling. The weight of a vertex *v* in G under a total labeling is defined as the sum of the label of *v* and the edge labels corresponding to all the edges incident with *v*. If all vertices in G have the same weight *k*, we call the labeling vertex-magic total labeling. If all vertices in G have different weights, then the labeling is called vertex-antimagic total labeling.

The weight of an edge *e* under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with *e*. If all edges in G have the same weight *k*, we call the labeling as edge-magic vertex labeling. If all vertices in G have different weights, then the labeling is called edge antimagic vertex labeling. The weight of an edge *e* under a total labeling is defined as the sum of the label of *e* and the vertex labels corresponding to every vertex incident with *e*. If all edges in G have the same weight *k*, we call the labeling as edge-magic total labeling. If all vertices in G have different weights, then the labeling is called edge antimagic total labeling.

LABELINGS IN CAYLEY DIGRAPHS

In 1878, Cayley constructed a graph for a given group with a generating set which is now popularly known as Cayley graphs. A directed graph or digraph G(V,E) consists of a finite set of points called vertices and a set of directed arrows between the vertices. Let G be a finite group and S be a generating subset of G. The Cayley digraph denoted by Cay(G,S), is the digraph whose vertices are the elements of G, and there is an arc from *g* to *gs* whenever g ℵ G and s ℵ S. If $S = S^{-1}$ then there is an arc from *g* to *gs* if and only if there is an arc from *gs* to *g*. The Cayley graphs and Cayley digraphs are excellent models for interconnection networks [2].

For example hypercube, butterfly, and cube-connected cycle's networks are Cayley graphs [22].

Though different kinds of labeling were studied and many conjectures were made for different subclasses of graphs, the labeling of well known graph namely Cayley graphs has not been investigated until a new labeling namely super vertex (a,d)–antimagic labeling for digraphs was introduced in [68]. Moreover the existence of super vertex (a,d)-antimagic labeling and vertex magic total labeling for a certain class of Cayley digraphs has been investigated in the literature [68]. Thamizharasi and Rajeswari studied magic labelings of Cayley digraphs and its line digraphs[60]. K.Thirusangu et al. studied super vertex (a,d) antimagic labelling and vertex magic total labelling of certain classes of Cayley digraphs[66]. K.Thirusangu and E.Bala obtained magic and antimagic labelings in Cayley digraphs of 2-generated p-groups [65] . The Product antimagic labelings in Cayley digraphs of 2-generated 2-groups has also been studied in the literature [67].

COMPLEMENTARY SUPER EDGE MAGIC LABELING

we concentrate on super- edge magic labeling and complementary super- edge magic labeling for connected graphs. The existing section is followed by introduction and section 2.2 bears the introductory part of complementary super- edge magic labeling which shows the existing of this labeling for certain graphs such as for cycle Cn for odd n and for the ladder graph. Previously, the results of this section have been published in International Journal of Applied Science & Technology Research Excellence and IJAET. In section 2.3, we deal with complementary super-edge magic labeling for the generalized prism Cm X Pn and G $\cong$T (n, n, n-1, n, 2n-1). The results of this section have been published in IJIRSET. In last section, we define complementary super edge magic labeling for (n, t)- kite (where t=2) graph and union of complete graph K2 and cycle Cn (where n is even and n $\neq$10) i.e. G = P2 $\cup$ Cn .

In 1976, Kotzing and Rosa [118] introduced edge-magic labelings. Interest in magic labelings has been lately rekindled by a paper on the subject (EMT labeling) due to Ringel and Llado [167].

The notion of magic strength introduced by Kong, Lee and Sun [116] was extended to edge- magic graphs and super edge- magic graphs by S. Avadayappan, P. Jayanthi and R. Vasuki [18, 19]. They proved the following theorem

In next two Figures, we depict SEM labeling and CSEM labeling of the ladder graph L5 with sems = 28 and csems = 44 respectively.

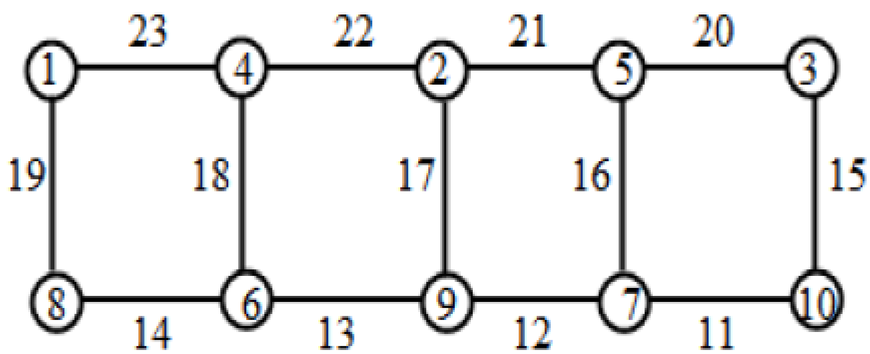

Figure 2.8 : Super edge magic labeling for L5 with sems = 28

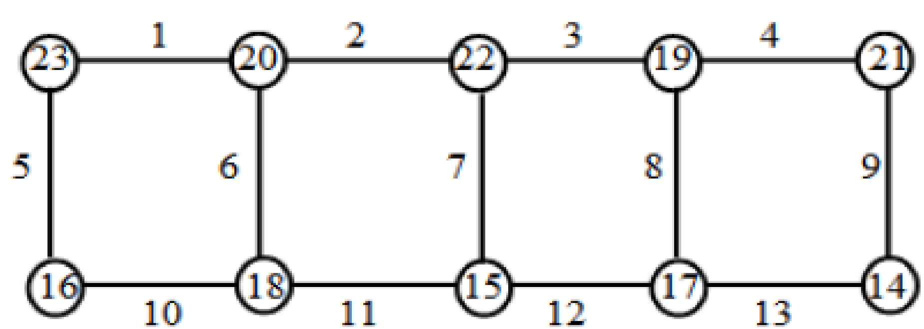

Figure 2.9: Complementary super edge magic labeling for L5 with csems = 44

ZERO EDGE MAGIC AND n-EDGE MATIC GRAPHS

PRELIMINARIES

In this section we give the basic notions relevant to this paper. Let G = G(V,E) be a ¿nite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels (usually the set of integers). In this paper, we deal with vertex labeling with domain as the set of all vertices.

The vertex-weight of a vertex v in G under an edge labeling to be the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex weight of v is de¿ned as the sum of the label of v and the edge labels corresponding to all the edges incident with v. If all vertices in G have the same weight k, we call the labeling vertex-magic edge labeling or vertex-magic total labeling respectively and we call k a magic constant. If all vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex-antimagic total labeling respectively.

The edge-weight of an edge e under a vertex labeling is de¿ned as the sum of the vertex labels corresponding to every vertex incident with e. Under a total labeling, we also add the label of e. Using edge-weight, we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagic total labeling.

A (p,q)-graph G is said to be (1,0) edge-magic with the common edge count k if there exists a bijection f : V (G) Í {1,2,...,p} such that for all e = (u,v) ℵ E(G), f(u) + f(v) = k. It is said to be (1,0) edge-antimagic if for all e = (u,v) ℵ E(G),f(u) + f(v) are distinct.

ZERO EDGE MAGIC GRAPHS

A complete n-ary pseudo tree is 0-edge magic.

Proof: Let G be a complete n-ary pseudo tree of height h. Clearly the vertices in level l_i are adjacent to some vertices in level (i - 1) and some vertices in level (i + 1) for $1 \leq i \leq (h - 1)$. This is true for all level except the root level l_0 and the leaf level l_h. Label the vertices in i^{th} level as -1 when $i \equiv 1(\text{mod}2)$ and label the remaining vertices as 1.

Therefore, all the vertices in level i have label -1 when i is odd and the vertices in $i - 1^{th}$ level have label 1. The edges incident at i^{th} level vertices has the label zero. If the height of the tree h is odd , The $(h – 1)^{th}$ level is even and the vertices at level h is adjacent to the vertices in $(h – 1)^{th}$ level.

The edges incident at level h has label 0. Clearly the root vertex is adjacent to the first level vertices, the edges incident with root vertex has label zero. Thus all the edges in n -ary pseudo tree has label zero and hence G is zero edge magic.

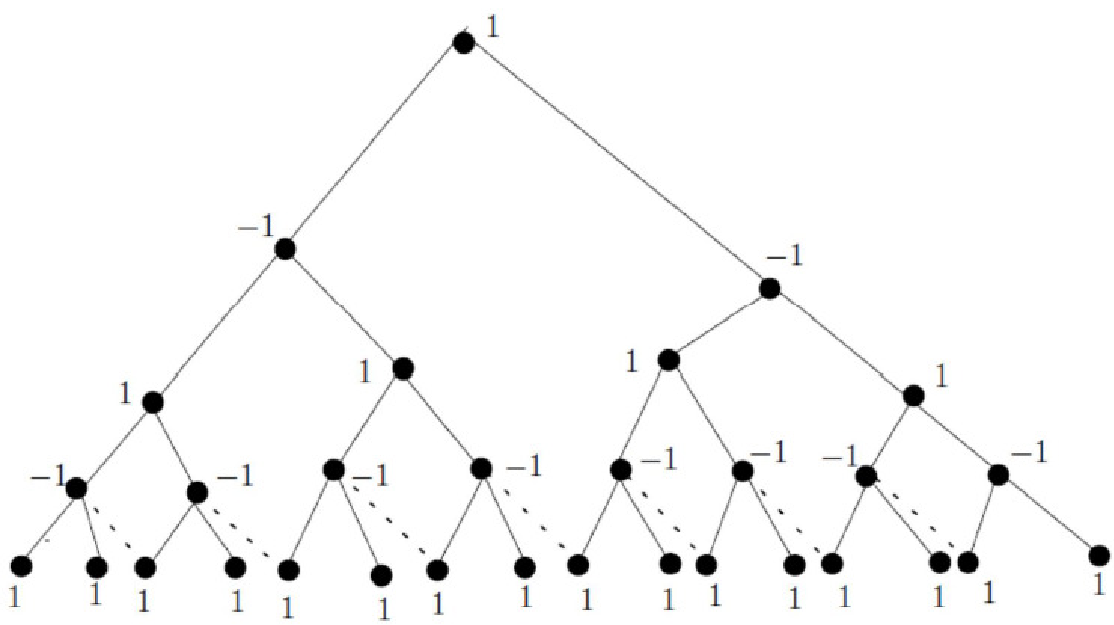

Figure 3.2.1: n-ary psudo tree

n - EDGE MAGIC GRAPHS

De¿nition 3.3.1

A (p,q)-graph G is said to be n- edge magic if there exists a surjective f : V (G) → {n + 1,-1} such that for each uv ℵ E(G),f(u) + f(v) = n

Theorem 3.3.2

A complete k- ary tree is n-edge magic.

Proof: Let G be a complete k- ary tree of height h. Clearly the vertices in level *li* are adjacent to some vertices in level (i - 1) and some vertices in level (i + 1) for $1 \leq i \leq (h - 1)$. This is true for all the vertices except the vertices in root level l_0 and the leaf level l_h. Label the vertices in ith level as -1 when $i \equiv 1 (\text{mod } 2)$ and label the remaining vertices as n + 1.

Therefore, all the vertices in level i have label -1 when i is odd and the vertices in $(i-1)^{th}$ level have label n+1. The edges incident at ith level vertices has the label n. If the height of the tree h is odd , the $(h -1)^{th}$ level is even and the vertices at level h is adjacent to the vertices in $(h - 1)^{th}$ level.

The edges incident at level h has label n. Clearly the root vertex is adjacent to the first level vertices, the edges incident with root vertex has label n. Thusall the edges in a complete k-ary tree has label n and hence G is ní edge magic.

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