A STUDY ON SHORTEST PATH PROBLEMS WITH COST-SHARING

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Abstract

The history of the shortest route issue is difficult to trace. It's easy to assume that even in the most rudimentary (even animal) society, finding quick pathways (for example, to food) is critical. In comparison to other combinatorial optimization issues such as shortest spanning tree, assignment, and transportation, mathematical study on the shortest route problem began comparatively late. This might be owing to the fact that the issue is basic and relatively simple, as shown by the fact that, at the time the topic became of interest, numerous researchers independently devised comparable approaches. Nonetheless, the issue has presented some significant challenges. Nonoptimal techniques for several considerable period heuristics have been examined (cf. for instance Rosenfeld [1956], who gave a heuristic approach for determining an optimal trucking route through a given traffic congestion pattern).Path finding, particularly searching in a labyrinth, is a traditional graph issue, with classical references including Wiener [1873], Lucas [1882] (decribing a technique owing to C.P. Tr'emaux), and Tarry [1895] - see Biggs, Lloyd, and Wilson [1976]. They serve as the foundation for depth-first search algorithms.

Introduction

Path difficulties were also addressed in the context of 'alternative routing' at the beginning of the 1950s, that is, finding a second shortest path if the shortest route is blocked. This is true for motorway use (Trueblood [1952]), but it also applies to telephone call routing. Making long-distance calls in the United States of America was automated at the time, and other routes for telephone calls via the United States' telephone network should be identified automatically. According to Jacobitti [1955]:

When a phone consumer places a long-distance call, the operator's main concern is getting the call to its destination. In certain circumstances, each toll operator has two major channels via which the call to this destination might be initiated. The most direct path is, of course, the first pick. If this is a busy time, the second option is selected, followed by any other available options at the operator's discretion. When dealing with such a call, telephone operators have the option of choosing amongst other routes. When it comes to operator or customer toll dialling, however, the route selection must be left to a machine. Because a machine's "intelligence" is confined to previously "programmed" functions, the route selection must be agreed upon and included into an automated alternative routing scheme.

1946–1953: matrix approaches for finding the shortest unit-length route

Matrix techniques were created to analyse network connections, such as discovering the transitive closure of a relation; that is, identifying the pairs of locations s, t in a directed graph such that t is accessible from s. Such approaches were investigated because of their applicability to communication networks (including neural networks) and animal sociology (e.g. peck rights).

The matrix approaches include modelling the directed graph with a matrix and then calculating the transitive closure using iterative matrix products. Landahl and Runge [1946], Landahl [1947], Luce and Perry [1949], Luce [1950], Lunts [1950, 1952], and A. Shimbel investigated this.

ALGORITHM OF SHORTEST PATH PROBLEMS

The aim of the algorithm is to find a group of paths which can take us to the destination from the source node. The group contains the shorter paths. It may contain the shortest path. If the group does not contain the shortest path then we can fuse two shorter paths to achieve the shortest path. The shorter path will be treated as chromosomes and by implementing Genetic Algorithm we can deduce the shortest path. In the world of multiplicity we need multiple options to perform a task so that our trail could not fail. If the shortest path fails to provide the optimum output then it can be substituted by other member in the derived group. Now, failure of the shortest path means congestion in the path, or the path is blocked, or disruption of the path for any natural calamity. According to meet today's requirement we cannot wait until the renewal of the damaged path. At this moment we can use other shorter paths to save time, cost, inconvenience and loss (loss could be National, if it is a vital and national route). This work saves the delay time of a unit (say packet in network line, unit of liquid in a pipeline or a vehicle on road) and the inconvenience in the network.

Traversing a smaller graph of less number of nodes will take less times. Less time means high performance. Creation of sub graph is possible only if we select those nodes which will provide the set of shorter paths. Nodes provided by “Algorithm for creation of Simplified Graph from the Sub Graph” and “Back Track Algorithm” are the responsible nodes for shorter paths.

In the run of multiplicity we need multiple options to accomplice our task. If a resource cannot work or busy servicing other users, then an alternative option or solution should be required to save the waiting time which causes delay in the execution of the whole process. When we consider multiplicity in searching the shortest path, then we should think of an algorithm which can provide not only the shortest path but should also provide the alternative path. Single path (shortest path by Dijkstra’s) can promote traffic and induce waiting time causing a delay and there are possibilities of damage of the available paths. If damage occurs then no transmission or transportation is possible. At this point of time of the proposed system have an alternative option then it can transmit or transfer it’s required from source to destination.

Binary Encoding

Crossover

Single point crossover - one crossover point is selected, binary string from beginning of chromosome to the crossover point is copied from one parent, the rest is copied from the second parent

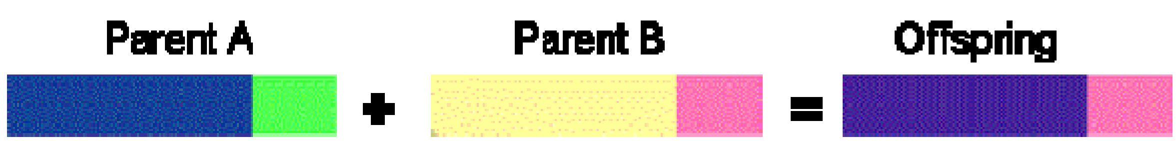


Figure 2.5: Crossover at Single Point

11001011+11011111 = 11001111

Two point crossover - two crossover point are selected, binary string from beginning of chromosome to the first crossover point is copied from one parent, the part from the first to the second crossover point is copied from the second parent and the rest is copied from the first parent

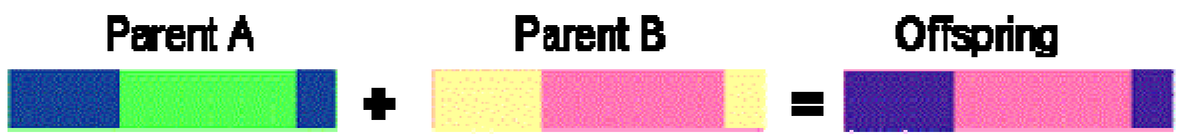


Figure 2.6: Crossover at Two Point

11001011 + 11011111 = 11011111

Uniform crossover - bits are randomly copied from the first or from the second parent

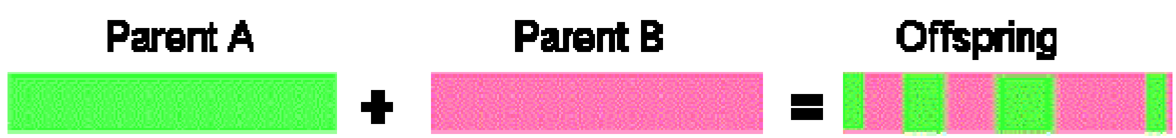
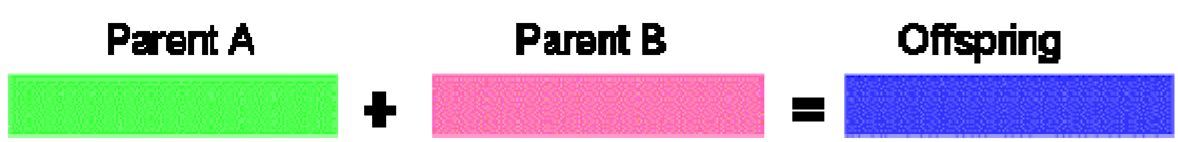


Figure 2.7: Uniform Crossover

11001011 + 11011101 = 11011111



Application

GIS was applied for problems in the network, environmental and life sciences, in particular ecology, geology etc. It has traversed almost all industries including computers, defense, intelligence, utilities, Natural Resources (Oil and Gas, River etc.), last but not the least Public Safety (i.e. emergency management and criminology) and many more.

Data Representation

Geographic Information System represents real objects such as roads, elevation, water bodies, land use, trees etc with digital data representation. Real objects can be divided into two abstractions – discrete objects as house and continuous fields as rainfall amount, elevations. Traditionally, there are two broad methods used to store data in GIS for both kinds of abstractions mapping references with raster images and vector [5] images. Points, lines and polygons are hybrid method of storing data is that of identifying point clouds which combine three dimensional points with Red Green and Blue information.

Geometric Networks

Geometric Networks are basically linear networks of objects that are used to represent interconnected features and to perform special analysis. It is composed of vertexes

which are connected, similar to graph in computer science. Just like graph it can have weight and flow assigned to its edges, which are used to represent different interconnected features. Geometric networks are used to represent road networks and public utility networks, such as electric, gas, water, Ethernet etc.

Operations

In vector based GIS map, it combines two or more maps according to predefined rules. Simple buffering describes regions of a map within a specified distance of one or more features (such as towns, roads, rivers etc.). This reflects spatial analysis within the Open Geospatial Consortium (OGC). For raster-based GIS, environmental sciences and remote sensing, applied to one or more maps often involving filtering and /or algebraic operations (map algebra). These processes involve processing of one or more raster layers fitting into simple rules resulting in a new map. Descriptive statistics (cell counts, means variances, maxima, minima, cumulative values, frequencies etc.) are often included in the generic term spatial analysis. Geospatial analysis words beyond Two Dimentional mapping operations and spatial analysis. It involves:

Surface Analysis: Analyzing the properties of physical surfaces (gradient, aspect and visibility).

Network Analysis: Analyzing the attributes of natural and man-made networks in order to understand the behavior of flows within and around the network and location. This may be used to address a wide range of practical problems-route selection, facility location and problems involving flows like those found in hydrology and transportation problem.

Geovisualization: The creation and manipulation of maps, images, charts, diagrams, 3D views and their associated data. This provides a range of tools, which provide static or rotating views, providing animations, dynamic linking, brushing and spatio temporal visualizations etc. this tool is the least developed and reflects limited range of suitable and compatible datasets and the limited set of analytical methods available. All these facilities increases the core tools utilized in spatial analysis throughout the patterns and relationships, construction of models and communication in results.

Algorithm to Create Leveled Graph

The re-created and re-distributed into levels. Whose first and last level contains the source node and the destination nodes. And the middle level will contain the possible and responsible nodes for shorter and shortest paths between source and destination node. Below the pseudo code will describe the phenomenon to create a leveled graph.

Pseudo Code for Creation of Leveled – Graph from the main Graph

1. Select the source node.

2. Keep the source node in the first level [level 0 / level 1- The graph will be created on the basis of levels]

3. Consider those nodes which are connected to the present level for the next level.

4. Traverse every node in the present level.

5. Eliminate those nodes whose connecting edge with the nodes in the same level or below level and mark the new node for the next level.

6. Terminate if destination found, else goto step 4.

7. Keep the destination node in the nth level and eliminate other nodes [With the help of Back Track Algorithm (Elimination Algorithm)].

As discussed in the pseudo code the algorithm have been developed and designed as follows: -

ALGORITHM 3.1: Creation of Sub – Graph from the Main – Graph

Step 1: Choose Source Node.

TSource = Source…….[TSource = Temporary Source].

Step 2: Create First Level.

Step 3: Place TSource in the Level.

Step 4: Find connecting nodes to TSource and Add it to Queue.

Step 5: If all Nodes of the Same Level Traversed.

TSource = Next un – Traversed node in the Same Level.

Goto Step 6

else

Goto Step 3

Step 6: Create the next Level (Level++).

Step 7: Add the Queue to the Level.

Step 8: If Destination Found

Goto Step 9

else

TSource = Next un – Traversed node in the Same Level.

Goto Step 3

Step 9: End.

After implementation of the algorithm we will get the leveled graph defined below.

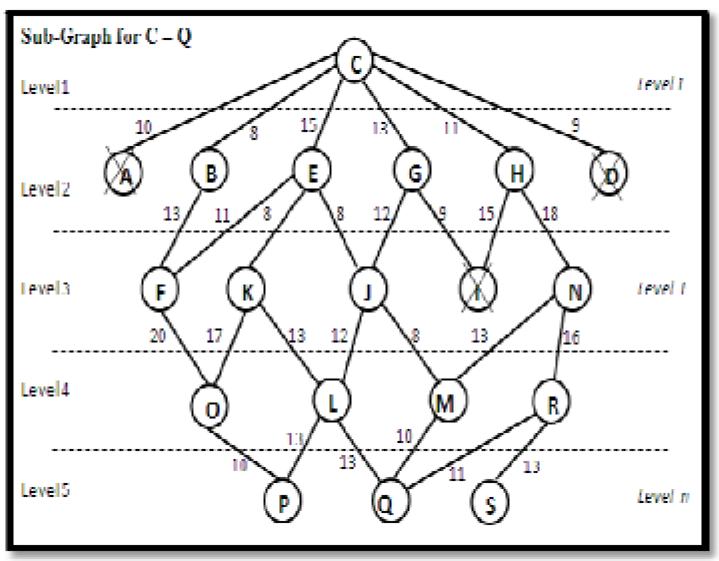


Figure 3.1: Leveled Graph with end dead nodes.

In the above graph Node A, D and I are useless because their connecting edges connects the nodes which lies on the same level or below level, it don't have the connecting edge connecting next level. It creates a dead end. So we don't require those Nodes in the Sub – Graph. So the graph will look like as follows.

Conclusion

This research opens up opportunities for further study into and development of shared shortest paths. We can incorporate our optimal solution for two journeys into potentially better heuristics that add two journeys at a time for problems that use more than two journeys.

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