A Study on Fuzzy Transportation Problem Using Different Types of Fuzzy Membership Functions

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Abstract:

In fuzzy logic and fuzzy set theory, fuzzy membership functions are one of the main concepts for problem solving. Simply define the member elements by using the characteristic function, in which 1 indicates membership and 0 non-membership. For a fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set. In this research article, we study different types of membership functions, such as triangular, trapezoidal, Gaussian, sigmoidal, shape-oriented, and probabilistic, to analyze and solve the fuzzy transportation problems with the help of numerical examples.

Keywords: Fuzzy logic, Fuzzy set theory, Membership function, Fuzzy Transportation Problem (FTP), Triangular, Trapezoidal.

Introduction:

The first concept of fuzzy set theory was introduced by Zadeh [1965]. It deals with uncertain objects and situations. Similarly, H.-J. Zimmermann [1980] researches fuzzy situations that follow; the majority of the traditional modeling, reasoning, and computing techniques are characterized by crispness, determinism, and precision. By crisp, we mean binary type as opposed to approximation type. For example, in traditional dual logic, a statement can be true or false, with no between ground. Similar to how an element in set theory can either belong to a set or not, a solution in optimization can either be feasible or not. Accuracy presupposes that a model's parameters accurately capture either our understanding of the phenomenon being represented or the characteristics of the actual system being modeled. In general, precision also suggests that there are no ambiguities in the model or that it is unequivocal.

In this paper, we study different types of membership functions, such as triangular membership functions, trapezoidal membership functions, Gaussian membership functions, sigmoidal membership functions, generalized bell-shaped membership functions, evaluate fuzzy membership functions, difference between two sigmoidal membership functions, Gaussian combination membership functions, pi-shaped membership functions, product of two sigmoidal membership functions, s-shaped membership functions, z-shaped membership functions, perform fuzzy arithmetic membership functions, defuzzify membership functions, and probabilistic OR membership functions, to analyze and solve the fuzzy transportation problems with the help of numerical examples to know about the effects of different types of fuzzy membership functions on the fuzzy transportation problem and their solutions.

Fuzzy Membership Functions:

In fuzzy logic and fuzzy set theory, fuzzy membership functions are one of the main concepts for problem solving. Simply define the member elements by using the characteristic function, in which 1 indicates membership and 0 non-membership. For a fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set.

If X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

$\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\tilde{A}$ that maps X to the membership space M (When M contains only the two points 0 and 1, $\tilde{A}$ is non-fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a fuzzy set). The range of the membership function is a subset of the non-negative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

Numerical Example:

The transportation problem to solved using different types of fuzzy membership functions.

Table.1 Transportation problem

Destinations / Sources	D1	D2	D3	D4	Supply
S1	14	56	48	27	70
S2	82	35	21	81	47
S3	99	31	71	63	93
Demand	70	35	45	60	

Triangular membership function [trimf] or Evaluate fuzzy membership function [evalmf]:

$$\mu(x;a,b,c) = \begin{cases} 0 & if\ x \le a \\ \frac{x-a}{b-a} & if\ a < x \le b \\ \frac{c-x}{c-b} & if\ b < x < c \\ 0 & if\ x \ge c \end{cases}$$

x is the input value

a, b, c are parameters of the triangle's start, peak and end respectively.

Table.2 Membership values of the Table.1 for Triangular membership function [trimf] or Evaluate fuzzy membership function [evalmf]

0.4	0.8	0.8	0.7	1
0.8	0.5	0.9	0.9	0.7
0.1	0.9	0.9	0.7	0.7
1	0.5	1	0	

Gaussian membership function [gaussmf]:

$$\mu(x;c,\sigma) = e^{-\frac{(x-c)^2}{2\sigma^2}}$$

x is the input value

c is the center (mean) of the Gaussian function

σ is the standard deviation, which controls the width of the bell curve.

Table.3 Membership values of the Table.1 for Gaussian membership function [gaussmf]

0.1353	0.8353	0.9802	0.8353	0.1353
0.006	0.3247	0.1979	0.0082	0.955
0.015	0.164	0.995	0.783	0.071
1	0.3247	0.882	0.6065	

Generalized bell-shaped membership function [gbellmf]:

$$\mu(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$

x is the input value

a is a parameter that controls the width of the bell shape

b is a parameter that controls the slope of the bell shape

c is the center of the bell.

Table.4 Membership values of the Table.1 for Generalized bell-shaped membership function [gbellmf]

0.325	0.884	0.998	0.992	0.5
0.998	0.943	0.604	0.9999	0.992
0.604	0.9999	0.9999	0.992	0.992
1	0.943	0.943	1	

Difference between two sigmoidal membership function [dsigmf]:

$$S(x; a, c) = \frac{1}{1 + e^{-a(x-c)}}$$

x is the input value

a is a parameter that determines the slope of the sigmoid curve

c is the center (the point of inflection) of the sigmoid curve

$$D(x; a_1, c_1, a_2, c_2) = S(x; a_1, c_1) - S(x; a_2, c_2)$$

a_1 and c_1 are the parameters of the first sigmoidal function

a_2 and c_2 are the parameters of the second sigmoidal function

Table.5 Membership values of the Table.1 for Difference between two sigmoidal membership function [dsigmf]

0.244	0.432	0.459	0.451	0.462
0.450	0.440	0.395	0.367	0.451
0.159	0.620	0.461	0.453	0.211
0.462	0.440	0.440	0.462	

Gaussian combination membership function [gauss2mf]:

$$G(x; c, \sigma) = e^{-\frac{(x-c)^2}{2\sigma^2}}$$

x is the input value

c is the center (mean) of the Gaussian function

σ is the standard deviation, which controls the width of the bell curve.

Product of two Gaussian functions

$$G_{produ\sigma t}(x; c_1, \sigma_1, c_2, \sigma_2) = G(x; c_1, \sigma_1) \times G(x; c_2, \sigma_2)$$

Sum of two Gaussian functions

$$G_{sum}(x; c_1, \sigma_1, c_2, \sigma_2) = G(x; c_1, \sigma_1) + G(x; c_2, \sigma_2)$$

Table.6 Membership values of the Table.1 for Gaussian combination membership function [gauss2mf]

0.6543	0.651	0.4656	0.3912	0.457
0.4374	0.51596	0.7629	0.4448	0.2962
0.6065	0.2282	0.6065	0.5708	0.5708
0.607	0.5336	0.466	0.607	

Pi shaped membership function [pimf]:

$$\mu(x;a,b,c,d)=\begin{cases}0 & if\ x\le a\\ 2\left(\frac{x-a}{b-a}\right)^2 & if\ a<x\le\frac{a+b}{2}\\ 1-2\left(\frac{x-b}{b-a}\right)^2 & if\ \frac{a+b}{2}<x\le b\\ 1 & if\ b<x\le c\\ 1-2\left(\frac{x-c}{d-c}\right)^2 & if\ c<x\le\frac{c+d}{2}\\ 2\left(\frac{x-d}{d-c}\right)^2 & if\ \frac{c+d}{2}<x\le d\\ 0 & if\ x>d\end{cases}$$

a and d are the points where the function begins and ends respectively.

b and c are the points where the function reaches the value of 1.

Table.7 Membership values of the Table.1 for Pi shaped membership function [pimf]

0.32	0.68	0.68	1	1
0.92	1	0.98	0.98	1
0	0.98	1	1	0
1	1	1	1	

Product of two sigmoidal membership functions [psigmf]:

$$S(x;a,c)=\frac{1}{1+e^{-a(x-c)}}$$

x is the input value

a is a parameter that determines the slope of the sigmoid curve

c is the center (the point of inflection) of the sigmoid curve

$$S_{product}(x; a_1, c_1, a_2, c_2) = S(x; a_1, c_1) \times S(x; a_2, c_2)$$

Table.8 Membership values of the Table.1 for Product of two sigmoidal membership functions [psigmf]

0.2128	0.2597	0.1903	0.1426	0.1963
0.2389	0.2356	0.0683	0.2174	0.1424
0.6177	0.0975	0.0682	0.1918	0.7140
0.2358	0.2356	0.2358	0.2358	

Sigmoidal membership function [sigmf]:

$$S(x; a, c) = \frac{1}{1 + e^{-a(x-c)}}$$

x is the input value

a is a parameter that determines the slope of the sigmoid curve

c is the center (the point of inflection) of the sigmoid curve

Table.9 Membership values of the Table.1 for Sigmoidal membership function [sigmf]

0.168	0.931	0.858	0.426	0.982
0.995	0.622	0.289	0.994	0.848
0.999	0.525	0.983	0.964	0.998
0.982	0.622	0.818	0.952	

S-shaped membership function [smf]:

$$S(x; a, b) = \begin{cases} 0 & if\ x \le a \\ 2\left(\frac{x-a}{b-a}\right)^2 & if\ a < x \le \frac{a+b}{2} \\ 1 - 2\left(\frac{b-x}{b-a}\right)^2 & if\ \frac{a+b}{2} < x < b \\ 1 & if\ x \ge b \end{cases}$$

a is the point where the function starts to increase

b is the point where the function reaches 1.

Table.10 Membership values of the Table.1 for S-shaped membership function [smf]

0.32	0.755	0.32	0.245	0.5
0.595	0.5	0.18	0.439	0.245
0.558	0.595	0.151	0.375	0.636
0.5	0.5	0.5	0.5	

Trapezoidal membership function [trapmf]:

$$T(x;a,b,c,d) = \begin{cases} 0 & if\ x \le a \\ \frac{x-a}{b-a} & if\ a < x \le b \\ 1 & if\ b < x \le c \\ \frac{d-x}{d-c} & if\ c < x \le d \\ 0 & if\ x \ge d \end{cases}$$

a is the start of the rise

b is the start of plateau

c is the end of the plateau

d is the end of the fall.

Table.11 Membership values of the Table.1 for Trapezoidal membership function [trapmf]

0.4	0.4	1	0.7	0
0	1	1	0	1
0	1	1	0.7	0
0	1	1	1	

Z-shaped membership function [zmf]:

$$Z(x;a,b) = \begin{cases} 1 & if\ x \le a \\ 1 - 2\left(\frac{x-a}{b-a}\right)^2 & if\ a < x \le \frac{a+b}{2} \\ 2\left(\frac{b-x}{b-a}\right)^2 & if\ \frac{a+b}{2} < x < b \\ 0 & if\ x \ge b \end{cases}$$

a is the point where the function starts to decrease *b* is the point where the function reaches 0.

Table.12 Membership values of the Table.1 for Z-shaped membership function [zmf]

0.68	0.437	0.68	0.755	0.111
0	0.875	0.995	0.02	0.755
0	0.82	0.7312	0.6222	0.155
0.111	0.875	0.875	0	

Defuzzify membership function [defuzz]:

Table.13 Membership values of the Table.1 for Defuzzify membership function [defuzz]

0.6	0.62	0.9	0.08	1
0.4	0.25	0.1	0.55	0.85
1	0.05	0.95	0.35	0.7
1	0.25	0.75	0.5	

Probabilistic OR [probor]:

Table.14 Membership values of the Table.1 for Probabilistic OR [probor]

0.79	0.7536	0.7504	0.8029	0.79
0.8524	0.7725	0.8341	0.8461	0.7509
0.9901	0.7861	0.7941	0.7669	0.9349
0.79	0.7725	0.7525	0.76	

Perform fuzzy arithmetic [fuzarith]:

Table.15 Membership values of the Table.1 for Perform fuzzy arithmetic [fuzarith]

1	0.5	0.3	0.4	0.5
0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	

Conclusion:

From the above tables, we can see that different kinds of membership value for the same transportation table. But we commonly use triangular membership functions, trapezoidal membership functions, and sigmoidal membership functions for evaluating the fuzzy transportation problems and also evaluate fuzzy membership functions [evalmf] and triangular membership functions [trimf], giving similar results. Respectively, the Gaussian membership function and the Gaussian combination membership function are similar. And in sigmoidal, there are three types: difference between two sigmoidal membership functions [dsigmf], product of two sigmoidal membership functions [psigmf], and sigmoidal membership function [sigmf], which is practically similar.

Reference:

Dr. A. Rajkumar et.al., A Method for Solving Bottleneck-Cost Transportation Problem Using Fuzzy Optimization Trapezoidal fuzzy numbers with λ-Cut and Ranking Method. Advances and Applications in Mathematical Sciences Volume 21, Issue 8, June 2022, Pages 4563-4574 © 2022 Mili Publications, India. UGC care.

Dr. A. Rajkumar et.al., A MATLAB-Based Method for Solving Fuzzy Transportation Problems, Indian Journal of Natural Sciences, Vol.14 / Issue 79 / Aug / 2023, Pg.No:58613 – 58620.

Dr. A. Rajkumar et.al., “A Study on Solving Least-Cost Fuzzy Transportation Problem using MATLAB” Indian Journal of Natural Sciences Vol.14 / Issue 82 / Feb / 2024 Pg.No. 67473 – 67481.

Dubois, Didier, and Henri M. Prade. Fuzzy Sets and Systems: Theory and Applications. Mathematics in Science and Engineering, v. 144. New York: Academic Press, 1980.

Hitchcock, F.L. (1941) The Distribution of a Product from Several Sources to Numerous Localities. Journal of Mathematics and Physics, 20, 224-230. http://dx.doi.org/10.1002/sapm1941201224

Kaufmann, A., and Madan M. Gupta. Introduction to Fuzzy Arithmetic: Theory and Applications. Van Nostrand Reinhold Electrical/Computer Science and Engineering Series. New York, N.Y: Van Nostrand Reinhold Co, 1985.

Koopmans T.C., Optimum utilization of the transportation system, Proceeding of the International Statistical Conference, Washington D.C., 1947

Lee, C.C. 'Fuzzy Logic in Control Systems: Fuzzy Logic Controller. I'. IEEE Transactions on Systems, Man, and Cybernetics 20, no. 2 (April 1990): 404–18. https://doi.org/10.1109/21.52551.

Lee, C.C. 'Fuzzy Logic in Control Systems: Fuzzy Logic Controller. II'. IEEE Transactions on Systems, Man, and Cybernetics 20, no. 2 (April 1990): 419–35. https://doi.org/10.1109/21.52552.

Mamdani, E.H., and S. Assilian. 'An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller'. International Journal of Man-Machine Studies 7, no. 1 (January 1975): 1–13. https://doi.org/10.1016/S0020-7373(75)80002-2.

Mamdani, E.H. 'Advances in the Linguistic Synthesis of Fuzzy Controllers'. International Journal of Man-Machine Studies 8, no. 6 (November 1976): 669–78. https://doi.org/10.1016/S0020-7373(76)80028-4.

Mamdani, E.H. 'Application of Fuzzy Logic to Approximate Reasoning Using Linguistic Synthesis'. IEEE Transactions on Computers C–26, no. 12 (December 1977): 1182–91. https://doi.org/10.1109/TC.1977.1674779.

Pawan Kumar Oberoi, Optimization Techniques, Second edition, 2015.

Rahman, Md & Akter, Afsana & Siddique, Md. Miskat. (2022). Cost Minimization: A Comparison Study on Various Types of Transportation Methods.

Schweizer, B. and A. Sklar, 'Associative functions and abstract semi-groups'. Publ. Math. Debrecen 10 (1963): 69–81.

Sugeno, M., "Fuzzy measures and fuzzy integrals: a survey," (M.M. Gupta, G. N. Saridis, and B.R. Gaines, editors) Fuzzy Automata and Decision Processes, pp. 89-102, North-Holland, NY, 1977.

Sugeno, Michio, ed. Industrial Applications of Fuzzy Control. Amsterdam ; New York : New York, N.Y., U.S.A: North-Holland ; Sole distributors for the U.S.A. and Canada, Elsevier Science Pub. Co, 1985.

Yager, Ronald R. 'On a General Class of Fuzzy Connectives'. Fuzzy Sets and Systems 4, no. 3 (November 1980): 235–42. https://doi.org/10.1016/0165-0114(80)90013-5.

Yager, Ronald R., and Dimitar P. Filev. 'Generation of Fuzzy Rules by Mountain Clustering'. Journal of Intelligent and Fuzzy Systems 2, no. 3 (1994): 209–19. https://doi.org/10.3233/IFS-1994-2301.

Zadeh, L.A. 'Fuzzy Sets'. Information and Control 8, no. 3 (June 1965): 338–53. https://doi.org/10.1016/S0019-9958(65)90241-X.

Zadeh, Lotfi A. 'Outline of a New Approach to the Analysis of Complex Systems and Decision Processes'. IEEE Transactions on Systems, Man, and Cybernetics SMC-3, no. 1 (1973): 28–44. https://doi.org/10.1109/TSMC.1973.5408575.

Zadeh, L.A. 'The concept of a linguistic variable and its application to approximate reasoning. I'.Information Sciences 8, no. 3 (1975): 199–249. https://doi.org/10.1016/0020-0255(75)90036-5

Zadeh, L.A. 'The concept of a linguistic variable and its application to approximate reasoning. II'. Information Sciences 8, no. 4 (1975): 301–357. https://doi.org/10.1016/0020-0255(75)90046-8

Zadeh, L.A. 'The concept of a linguistic variable and its application to approximate reasoning. III'. Information Sciences 9, no. 1 (1975):43–80. https://doi.org/10.1016/0020-0255(75)90017-1

Zadeh, L.A. 'Fuzzy Logic'. Computer 21, no. 4 (April 1988): 83–93. https://doi.org/10.1109/2.53.

Zadeh, L.A. 'Knowledge Representation in Fuzzy Logic'. IEEE Transactions on Knowledge and Data Engineering 1, no. 1 (March 1989): 89–100. https://doi.org/10.1109/69.43406.