

# Fractional Order IMC-TDD Controller for Integrating process using Firefly Optimization Algorithm

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## Abstract:

A novel approach incorporating a modified Internal Model Control (IMC) framework with a Fractional-Order Tilt Double Derivative (FOTDD) controller is introduced specifically tailored for integrating processes by using Firefly Optimization Algorithm. This technique is expanded to encompass all three categories of process models: integrating with time delay (IPTD), double integrating with time delay (DIPTD), and integrating with first order and time delay (IFOPTD). The feedback controller is devised within an IMC framework, swiftly derived from the plant parameters. Subsequently, the fractional IMC filter can be adjusted using the explicit expressions obtained for practical convenience. A numerical analysis is carried out, examining various examples from existing literature across all integrating model types by using Firefly Optimization Algorithm (FOA) can be effectively utilized to solve optimization problems and enhance system performance across various domains. The outcomes are contrasted with existing methodologies concerning setpoint tracking, disturbance rejection, and parameter perturbations. Through the utilization of a fractional order IMC-TDD Controller for Integrating processes employing the Firefly Optimization Algorithm, the findings exhibit a straightforward and resilient structure with minimal tuning parameters, showcasing the system's performance are acquired through the utilization of the MATLAB/Simulink software.

**Keywords:** Fractional IMC filter, Integrating process, Double integrating process, Disturbance rejection, Tilt derivative, Firefly Optimization Algorithm (FOA).

## 1.Introduction:

An integrating process is represented by a first-order differential equation, indicating that the output of the process is the integral of its input. This leads to a steady-state response that continues to increase or decrease indefinitely, depending on the sign of the input. Integrating processes are common in various systems, such as level control systems, where the output is the integral of the flow rate into or out of a tank, or in velocity control systems, where the output is the integral of the acceleration. Integrating processes pose challenges for control systems design, as they are inherently non-minimum phase systems, meaning that their zeros are located in the right-half plane. This characteristic can lead to instability or poor performance if not properly addressed in the control system design. Specialized control strategies, such as the IMC-TDD controller mentioned earlier, are often employed to effectively control integrating processes.

The Fractional order (IMC-TDD) Controller is a specialized control strategy designed to address the control challenges posed by integrating processes with time delays. It combines the concepts of fractional calculus, which allows for non-integer differentiation and integration orders, with the IMC-TDD framework, which is tailored for systems characterized by integrating

dynamics and time delays.

The FOA serves as a powerful optimization tool to fine-tuning the parameters of the Fractional order IMC-TDD Controller. Inspired by the flashing behaviour of fireflies, the FOA efficiently explores the parameter space, seeking optimal solutions that minimize specified performance criteria. Integrating the FOA into the controller design process enhances its effectiveness in regulating integrating processes under diverse conditions.

Designing an enhanced IMC-PID controller with a lead-lag filter to handle unstable and integrating processes with time delay [1]. Proposing an enhanced fractional filter-based fractional IMC-TDD controller design and analysis to elevate the performance of integrating processes [2]. Internal model-based fractional order controller for all methods of integrating process like fractional order plus time delay processes [3]. When designing controllers for integrating processes with time delay, it has been observed that they yield superior performance in oscillation, overshoot, and reduced settling time [4]. Improved IMC design for enhancing load disturbance rejection in integrating and unstable processes characterized by slow dynamics [5]. In contrast to inherently stable processes, integrating processes can

exhibit instability despite bounded inputs. Authors have devised multiple methods to effectively control such processes [6,7]. The approaches offered reduced overshoot and achieved faster settling times through appropriate selection of tuning parameters. The fractional order based IMC-TDD controller for integrating processes was devised based on user-defined tuning parameters [8,9]. The optimal  $H_2$  minimization framework was extended with IMC by Begum et al. [10].

This paper presents a comprehensive investigation of the Fractional Filter IMC-TDD Controller utilizing the FOA. Simulation results and Case studies are provided to illustrate the efficacy and robustness of the proposed approach in controlling integrating processes.

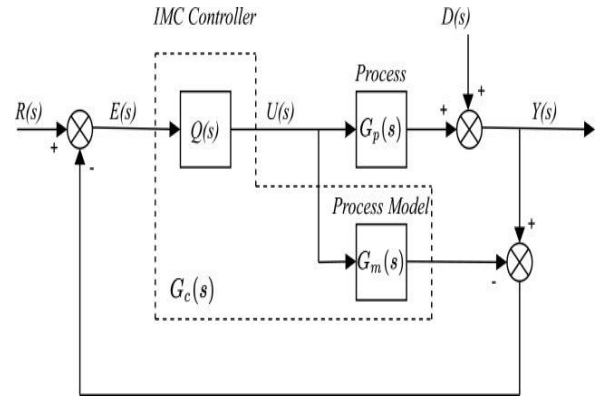


Figure 1 Structure of IMC Controller

$$G_{p1}(s) = \frac{Ke^{-\theta s}}{s} = \frac{Ke^{-\theta s}}{s + \frac{1}{\gamma}} = \frac{\gamma Ke^{-\theta s}}{\gamma s + 1}$$

Considering  $\gamma$  as a constant with a high value, let's assume  $\gamma = 100$ . Now, by employing the FIMC as

$$f(s) = \frac{1}{\gamma s^\beta + 1}$$

## 2. MODELLING:

An integrating process is a type of dynamic system that exhibits integral behavior, meaning that its output is proportional to the integral of its input over time. In mathematical terms, an integrating process can be described by a first-order differential equation with an integrating (or integrating action) term. This integral action allows the system to eliminate steady-state errors and achieve accurate tracking of reference signals or setpoints. This approach is expanded for below mentioned methods.

### Type 1: Integrating plus time delay process:

The Integrating Plus Time Delay (IPTD) process model is a representation commonly used in process control and system dynamics. It describes dynamic systems that exhibit both integrating behavior (integral action) and time delay. In mathematical terms, the IPTD process model can be expressed as a transfer function:

$$G_{p1}(s) = \frac{Ke^{-\theta s}}{s} \tag{1}$$

$K$  is the steady-state gain or process gain, representing the degree of integrating action.  $\theta$  is the time delay parameter.  $s$  is the Laplace variable, representing the complex frequency domain.

Here,  $\lambda$  denotes a fractional filter time constant ( $\lambda > 0$ ), and  $\beta$  is a positive real number, where  $\beta \in (0,1)$  for all integrating plants. The novel FOTDD is introduced as:

$$FOTDD = K_t \frac{1}{s^n} + K_d s^\mu + K_{dd} s^{\mu_1} \tag{2}$$

The given equations are compared with the FOTDD controller parameters. In this case we get the formula for  $G_{p1}(s)$  is:

$$K_t = \frac{1}{\gamma k}; K_d = \frac{1}{K}; \beta = \frac{1}{n}; \mu = 1 - \beta$$

It is evident for an IPTD process, In this context, where  $K_t$  represents the tilted gain and  $K_d$  denotes the derivative gain, the double derivative is not necessary, thus  $K_{dd} = 0$ . Additionally, the definition of the new cascaded fractional filter is as follows:

$$F_f = \frac{1+0.5\theta s}{\lambda+0.5\theta\lambda s+\theta s^{1-\beta}} \tag{3}$$

Applications of IPTD models can be found in various real-world systems, including chemical processes, biological systems, and control systems with transport delays. Understanding and modeling the behavior of systems using the IPTD model is crucial for designing effective control strategies, predicting system responses, and tuning controllers to achieve desired performance. Control engineers often employ techniques such as PID (Proportional-Integral-Derivative) control to regulate systems described by IPTD models.

**Type 2: Double Integrating plus time delay process:**

The Double Integrating Plus Time Delay Process (DIPTD) model incorporating two levels of integration along with a time delay. This model is used to represent systems with more complex dynamics, particularly those that exhibit higher-order integrative behaviour. In mathematical terms,

$$G_{p2}(s) = \frac{K e^{-\theta s}}{s^2} \quad (4)$$

K is the steady-state gain or process gain.  $\theta$  is the time delay parameter. s is the Laplace variable. These equations are compared with the FOTDD controller parameters. In this case we get the formula for  $G_{p2}(s)$  is:

$$K_t = \frac{1}{\gamma^2 K}; \quad K_d = \frac{2}{\gamma K}; \quad K_{dd} = \frac{1}{K}; \quad \beta = \frac{1}{n};$$

$$\mu = 1 - \beta; \quad \mu_1 = 2 - \beta$$

DIPTD models are commonly encountered in systems with more complex dynamics, such as mechanical systems with inertia or second-order differential equations governing their behavior. Understanding and modeling these systems using DIPTD models are crucial for designing effective control strategies and predicting system responses accurately. Control engineers often employ advanced control techniques to regulate systems described by DIPTD models, ensuring stability and desired performance.

**Type 3: Integrating plus first order plus time delay process:**

The Integrating Plus First Order Plus Time Delay (IFOPTD) process model is another variation commonly used in process control and system dynamics. It combines an integrating element, a first-order element, and a time delay, making it more complex than the IPTD model. In mathematical terms,

$$G_{p3}(s) = \frac{K e^{-\theta s}}{s(s+1)} \quad (5)$$

is the transfer function of the process. K is the steady-state gain.  $\theta$  is time delay parameter. s is Laplace variable. These equations are compared with the FOTDD controller parameters. In this case we get the formula for  $G_{p3}(s)$  is:

$$K_t = \frac{1}{\gamma K}; \quad K_d = \frac{\gamma + 1}{\gamma K}; \quad K_{dd} = \frac{1}{K}; \quad \beta = \frac{1}{n};$$

$$\mu = 1 - \beta; \quad \mu_1 = 2 - \beta$$

IFOPTD models are often used to represent systems with intermediate complexity, where both integrating and first-order dynamics play significant roles in system behavior. Understanding and modeling these systems using IFOPTD models are essential for designing effective control strategies and accurately predicting system responses to regulate systems described by IFOPTD models and achieve desired performance.

Fractional order IMC-TDD Controller is designed for system for mentioned above models. The IMC (Internal Model Control) Tilt Double Derivative Controller is a type of advanced control strategy used in process control systems. It's a variation of the IMC controller, which is designed to provide robust performance in the presence of process disturbances and uncertainties.

Double derivative controller is an enhancement to the standard IMC controller. It adds an additional derivative action to the controller, which can improve the control system's response to sudden changes or disturbances in the process. The IMC-Tilt approach introduces a tilt parameter that allows tuning the controller to prioritize either disturbance rejection or setpoint tracking. By adjusting this parameter, the controller can be optimized for specific process requirements. Overall, the IMC-Tilt Double Derivative Controller is a sophisticated control strategy that offers flexibility and robust performance in various industrial applications.

The below table shows the Disturbance filters applied to different process models.

Process	$F_d$	$\psi$
$\frac{K e^{-\theta s}}{s}$	$\frac{s(1 + \theta s)}{K((\psi s + 1)^2)}$	$\frac{\theta}{2}$
$\frac{K e^{-\theta s}}{s^2}$	$\frac{s^2(1 + \theta s)}{K(\psi s + 1)^3}$	$0.02\theta$
$\frac{K e^{-\theta s}}{s(s + 1)}$	$\frac{s(1 + \theta s)(\tau s + 1)}{K(\psi s + 1)^3}$	$\frac{\theta}{(2 + \tau)}$

Table 1: Disturbance filters applied to different process models.

A disturbance filter, in the context of signal processing or control systems, is a component designed to reduce or eliminate unwanted noise or disturbances from a signal. It's particularly useful in systems where external factors can introduce interference or disturbances that affect the

Process	Fractional filter	$\lambda$ and $\beta$
$\frac{Ke^{-\theta s}}{s}$ $\theta \leq 2$	$F_{f1}$	$\lambda = \frac{0.872\theta^2 + 0.644\theta + 0.101}{\theta^2 - 0.729\theta + 1.549}$ $\beta = \frac{0.014\theta^3 + 0.095\theta^2 - 0.1}{\theta^2 - 1.548 * \theta + 0}$
$\frac{Ke^{-\theta s}}{s}$ $\theta \geq 2$	$F_{f2}$	$\lambda = 0.006\theta^3 - 0.041\theta^2 + 0.118\theta - 0.008$ $\beta = \frac{-0.026\theta^2 + 0.367\theta - 0.266}{\theta - 0.599}$
$\frac{Ke^{-\theta s}}{s^2}$ $\theta \leq 2$	$F_{f1}$	$\lambda = \frac{0.557\theta + 0.056}{\theta^2 - 1.484\theta + 2.709}$ $\beta = 0.159e^{-1.012\theta}$
$\frac{Ke^{-\theta s}}{s^2}$ $\theta \geq 2$	$F_{f2}$	$\lambda = 0.042\theta^{0.345} + 0.046$ $\beta = -0.039\theta^{0.572} + 0.16$
$\frac{Ke^{-\theta s}}{s(s+1)}$ $\theta \leq 2$	$F_{f1}$	$\lambda = 0.288\theta + 0.016$ $\beta = \frac{0.548\theta + 0.565}{\theta^2 - 1.102\theta + 5.83}$
$\frac{Ke^{-\theta s}}{s(s+1)}$ $\theta \geq 2$	$F_{f2}$	$\lambda = 0.005\theta^{1.191} + 0.088$ $\beta = 0.009\theta + 0.236$

Table 2: Fractional IMC filter tuning parameters above Process model.

$$\text{Fractional IMC filter: } F_{f1} = \frac{1+0.5\theta s}{\lambda s^{0.8} + 0.5\theta\lambda s^{1.8} + \theta s^{1-\beta}}$$

$$F_{f2} = \frac{1+0.5\theta s}{\lambda + 0.5\theta\lambda s + \theta s^{1-\beta}} \quad (6)$$

desired output. In this paper we used state feedback control techniques can be employed to actively compensate for disturbances by adjusting control input based on the estimated system state and the detected disturbances.

Subsequently, the fractional IMC filter can be adjusted based on the obtained explicit expressions below mentioned. Table 2 shows the Fractional IMC filter Tuning parameters for above process models. The IMC filter is a type of filter used in control systems for filtering signals. Fractional IMC filters offer advantages such as improved robustness to variations in system dynamics, better noise rejection, and the ability to capture non-linear Behavior more accurately

#### Summary of the tuning parameters:

- Acquire  $\lambda$  and  $\beta$  values from Table 2 corresponding to the process model.
- Implement the fractional IMC filter as recommended in the second column of Table 2.
- Configure the disturbance filter according to Table 1.
- Compute the FOTDD parameters.

### 3. PROPOSED SYSTEM:

In the proposed system, the Firefly Optimization Algorithm is harnessed to devise the Fractional order IMC-TDD Controller for System. This algorithm has found application in a multitude of optimization tasks, spanning function optimization, parameter adjustment, and engineering design optimization.

In the optimization process facilitated by the Firefly Algorithm, the objective is to fine-tune the gains of the PID controller to ensure optimal control performance under nominal operating conditions. The Firefly Algorithm is employed to optimize the PID parameters  $K_t, K_d, K_{dd}$  using the algorithm and simulation.

The below steps shows the algorithm of Firefly optimization algorithm.

1. Fireflies are randomly initialized in the solution space. Every firefly symbolizes a potential solution to the optimization conundrum.
2. The objective function is evaluated for each firefly, determining its brightness. The brightness is determined based on3. the quality of the solution; higher brightness indicates a better solution.
3. Fireflies are attracted to brighter fireflies in the solution space. This attraction is influenced by two factors: the attractiveness of the brighter firefly and the distance between the fireflies. Fireflies move towards brighter ones with higher attractiveness and shorter distances.
4. Fireflies update their positions based on the attractiveness and distance-dependent movement towards brighter fireflies.
5. Steps 2 and 4 are repeated the process iterates until a termination.

IMC-Tilt controllers is applied to process control applications such as temperature control, pressure control, flow control, and level control in industrial processes. FA can optimize the parameters of IMC-Tilt controllers to achieve desired setpoint tracking performance while maintaining stability and robustness against disturbances and uncertainties in the process. The Figure 2 shows the flowchart of firefly optimization algorithm.

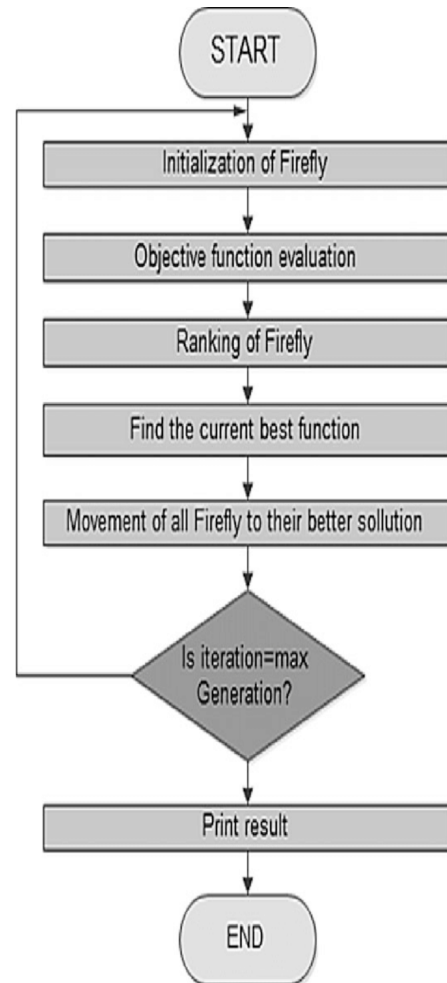


Figure 2: Flowchart of firefly optimization algorithm

#### 4. SIMULATION RESULTS

To evaluate the efficacy for mentioned method, the performance of the closed loop is compared with alternative approaches. Simulation results shows the comparison of proposed and existing method. Here in proposed method we are using the firefly optimization algorithm for existing method. Simulation diagram, graphs and comparison table is shown below which shows the objective is to achieve no oscillations, minimal settling time with reduced overshoot, and minimum Integrated Absolute Error for both setpoint tracking and enhanced disturbance rejection compared to alternative methods. Key indicators such as overshoot ( $OV\%$ ), settling time ( $t_s$  in seconds), and Integrated Absolute Error (IAE) are computed from the proposed technique and other methodologies.

$$IAE = \int_0^{\infty} |y(t) - r(t)| \quad (7)$$

where  $y(t)$  is output and  $r(t)$  is setpoint signals. Overshoot indicates the extent to which surpasses the ultimate value after a step change in the setpoint, and it also reflects

undershoot resulting from disturbance inputs. Settling time, on the other hand, refers to the duration needed for the output to stabilize within a specified tolerance band. The below diagram shows the simulation diagram.

Example 1: Integrating plus time delay system:

$$G_{p1}(s) = \frac{e^{-0.5s}}{s}$$

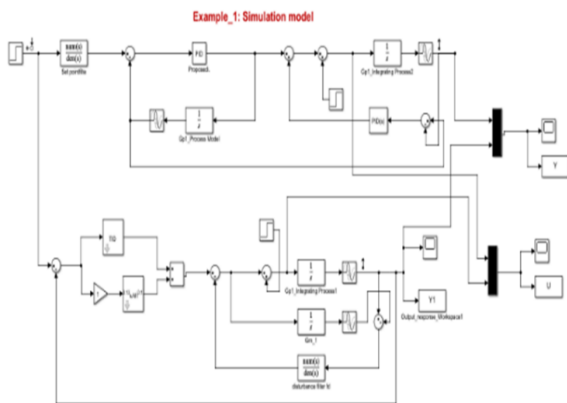


Figure 3: Simulation diagram

The below simulation diagram shows the proposed and existing diagram. We have compared the presented system with FOTDD parameters of equation 1. Subsequently, employing the outlined approach the derived values are  $K_t = 0.01$  and  $K_d = 1.0$ . The tuning parameters  $\lambda$  and  $\beta$  are determined to be 0.45 and 0.17, respectively, aiming for maximum sensitivity.

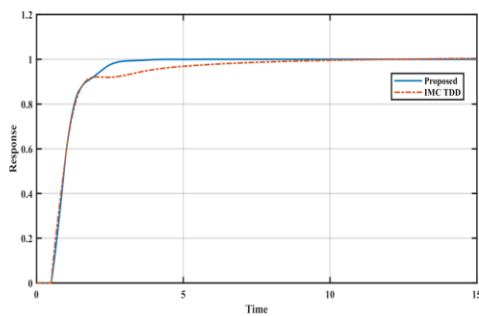


Figure 4: servo response

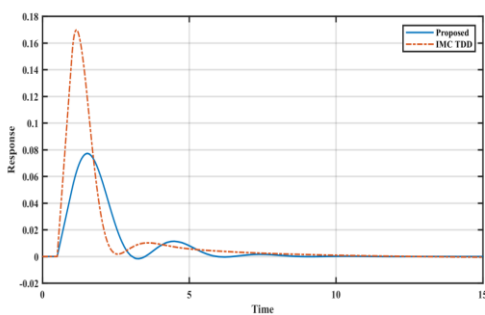


Figure 5: Regulatory response

The servo and regulatory performances are shown in Figs. 4 and 5. Regulatory response shows when the disturbance is applied for system. The findings suggest that it achieves zero oscillations, minimal settling time, reduced overshoot, and minimum IAE for setpoint tracking, along with improved disturbance rejection compared to alternative methods.

Contr ol Meth od	Set Point			Disturbance		
	% Overs hoot	T <sub>s</sub>	IA E	% Overs hoot	T <sub>s</sub>	IA E
Propo sed	0	2.0 42	0.5 36	0	2.0 42	0.0 93
IMC TDD	0.505	5.7 71	0.9 56	0.505	5.7 71	0.1 51

Table 3: Comparison Table

The above table shows the comparison of IMC-TDD and proposed system of example 1. Here we are comparing the overshoot, setting time and IAE. In The above table shows the comparison of IMC-TDD and proposed system of example 1. Here we are comparing the overshoot, setting time and IAE. In proposed system we are using Firefly optimization algorithm for existing system. The table shows that less overshoot, less settling time, less IAE for both setpoint and disturbance methods.

Example 2: Double integrating plus time delay system

$$G_{p2}(s) = \frac{e^{-s}}{s^2}$$

We have compared the presented system with FOTDD parameters of equation 1. The controller settings  $M_s=1.5$  is considered here, the parameters are  $\lambda=1.1$  and  $\beta=0.25$  and controller values are  $K_t=0.0001$ ,  $K_d=0.02$ ,  $K_{ad}=1$ . The parameters were also computed for the case without setpoint filter, resulting  $\lambda=0.9$  and  $\beta=0.05$ . The corresponding servo and regulatory responses for  $M_s=1.5$  both with and without filter is shown in Fig.6.

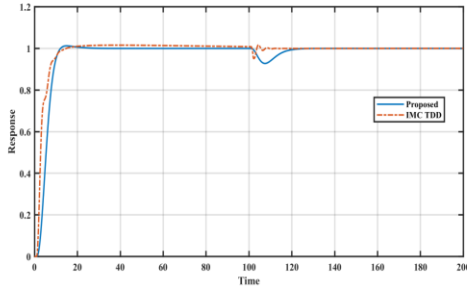


Figure 6: Servo and regulatory response

The servo and regulatory performances are shown in Figure 6 respectively. Regulatory response shows when the disturbance is applied for system. The performance of the system has zero oscillations, minimal settling time, reduced demonstrate overshoot, and minimum Integrated Absolute Error for setpoint tracking, along with superior disturbance rejection.

Contr ol Meth od	Set Point			Disturbance		
	%Over shoot	T <sub>s</sub>	IA E	%Over shoot	T <sub>s</sub>	IA E
Prop osed	1.865	8.6 24	0.1 81	4.9	9.5 7	0.0 52
IMC TDD	2.648	12. 895	0.2 30	5.7	12. 86	0.0 75

Table 4: Comparison Table

The above table shows the comparison of IMC-TDD and proposed system of example 2. Here we are comparing the overshoot, setting time and IAE. In proposed system we are using Firefly optimization algorithm for existing system. The table shows that less overshoot, less settling time, less IAE for both setpoint and disturbance methods.

Example 3: Integrating plus first order plus time delay system

$$G_{p3(s)} = \frac{Ke^{-\theta s}}{s(s+1)}$$

The tuning parameters  $\lambda$  and  $\beta$  are approximated as 0.115 and 0.20, respectively to provide  $M_s=2.5$ . The controller parameters are  $K_t=0.01$ ,  $K_d=1.01$  and  $K_{da}=1.0$ . Considering a disturbance input occurring at time  $t = 150$  seconds for the simulation study, Figs. 7 and 8 illustrate the corresponding servo and regulatory responses.

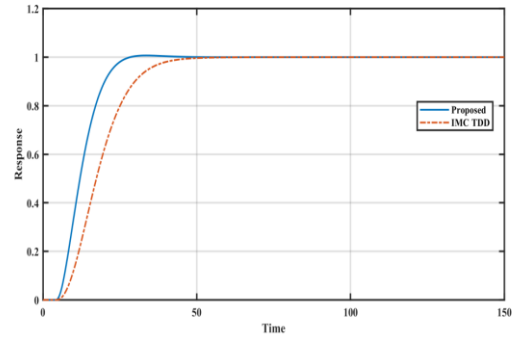


Figure 7: Servo response

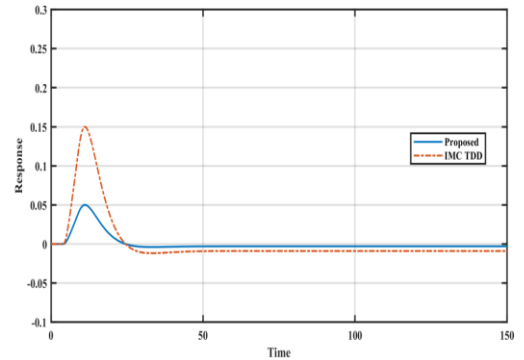


Figure 8: Regulatory response

The servo and regulatory performances are shown in Figs. 7 and 8. Regulatory response shows when the disturbance is applied for system. The performance of the system demonstrates zero oscillations, minimal settling time, reduced overshoot, and minimum Integrated Absolute Error for setpoint tracking, along with superior disturbance rejection.

Contr ol Meth od	Set Point			Disturbance		
	%Over shoot	T <sub>s</sub>	IA E	%Over shoot	T <sub>s</sub>	IA E
Prop osed	0	25. 642	0.2 36	7.246	22. 728	0.0 91
IMC TDD	0	39. 563	0.4 62	19.7	23. 663	0.1 40

Table 4: Comparison Table

The above table shows the comparison of IMC-TDD and proposed system of example 3. Here we are comparing the overshoot, setting time and IAE. In proposed system we are using Firefly optimization algorithm for existing

system. The table shows that less overshoot, less settling time, less IAE for both setpoint and disturbance method.

## 5. CONCLUSION AND FUTURE WORK:

Based on the constraints of the conventional controller applied to system, a new algorithm-based tuning for the controller is observed its servo and regulatory responses and the system output performance for disturbances. it can conclude that the system's performance is superior when utilizing a Fractional Order IMC-TDD controller for integrating process tuned by the Firefly Optimization Algorithm, Exhibits absence of oscillations, minimal settling time with reduced overshoot, achieving minimal Integrated Absolute Error (IAE) for setpoint tracking, along with superior disturbance rejection compared to alternative methods. For further study on this system a non-linear system can be implemented to further enhance its practical applicability and performance.

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