An improved the prediction accuracy of the Fractional - order accumulated Grey model by Fourier series and Its application in Container Throughput Forecasting in Danang Port, Vietnam

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Abstract: In order to improve the prediction accuracy of fractional- order accumulated Grey model (GM^r (1,1)), this study using Fourier series to modify their residual error of this model. To verify the effectiveness and the contribution of the proposed approach in modeling the Container Throughput in the future, the volume of Container Throughput in Danang Port from 2011 to 2023 is used for the modeling to forecast the Container Throughput demand from 2025 to 2028. Forecasting results proved that the Fourier- GM^r (1,1) is a better than the GM^r (1,1) model with the lowest MAPE =0% and 5.59% for in sample and out of sample data, respectively. This result is not only show the effectiveness of proposed model but also offers valuable insights for Danang policymakers in orientation and planning management agency so as to boost the development of upcoming port activities.

Keywords: Fractional-order accumulated Grey model; Fourier series; Forecasting, Accuracy; Container; Danang Port

1. Introduction

Grey forecasting is one of main part of Grey system theory, an effective method for modeling and forecasting small sample time series. In the early 1980s, Deng [1, 2] proposed the grey model GM (1,1) based on control theory, which is the core model used in the grey forecasting model. This model utilizes an operator obtained by first -order accumulation to operate on the non-negative original sequence. It demonstrates the approximate exponential growth laws and achieves short-term forecasting accuracy. With Its advantages in dealing with uncertain information and using as few as four data points [3, 5], The GM (1,1) has been validated and widely used in various fields such as tourism [6, 7], transportation [8- 10], financial and economic [11- 13], integrated circuit industry [14-17], energy industry [18-20] etc...

In the recent years, there are many scholars propose new procedures with different ways to improve the precision accuracy of GM (1,1) model. For instant, Lin et al. [21] and Wang et al. [22] used different methods to calculate new background values to improve the background values. Hsu [17] and Wang et al. [23] used different methods to modified internal parameter estimation like development coefficient and grey input coefficient. Some scholars have established GM (1,1) model with residuals modification like Hsu [15] and Wang et al. [24]. Zhou et al. [25] set up a new equation to built the new grey model named as NGBM (1,1). Hsu [16] used the genetic algorithm to optimize parameters of the NGBM (1,1). Chen et al. [26] proposed a Nash NGBM (1,1) based on the Nash equilibrium concept. In addition, many hybrid models based on GM (1,1) were proposed. These include the the grey Markov model [27, 28], and the grey fuzzy model [21], etc. Despite its improvement in prediction accuracy, the prediction accuracy of the GM (1,1) model is always monotonic. As a result, GM (1,1) model may not be always satisfactory.

The recently developed fractional- order accumulated grey model GM^r (1,1) is a new grey forecasting model [29]. It has a fractional order accumulation of r that can effectively manifest the priority of new information of real systems and flexibility determines the level of linear, and an enhanced ability to handle complex, noisy, and uncertain data. It is particularly beneficial for systems exhibiting complex dynamics and long-range dependencies. That reason why the fractional- order accumulated grey model GM^r (1,1) was applied in many cases. Wu and his peers successfully applied fractional accumulation to the fuel production of China [29], tourism demand [30] and electricity consumption [31],high technology equipment [32].

All these improvements focus on the model parameters and the background value. Actually, the initial condition is also an important factor determining the grey modeling accuracy. This is because the initial condition is a part of the predictive function. The current paper aims to develop an approach to increase the predictive precision of the GM^r (1,1) by modifying the residual error obtained from GM^r (1,1) with Fourier series. The practical application in container throughput forecasting in Danang Port shows that the proposed Fourier-GM^r (1,1) model has higher performance on GM^r (1,1) prediction. The remainder of this paper is organized as follows. A brief introduction to the Fractional-order accumulated grey model (Abbreviated *as* GM^r (1,1)) and the residual error modification of GM^r (1,1) model by Fourier series are given in Section 3. Section 4 proves the effectiveness of proposed approach in container throughput forecasting in Danang Port. Finally, the paper concludes with some comments in Section 5.

2. Related work

Wu et al. (2014) in short communication in already using grey model with fractional order accumulation to predict gas emission, this result show that the predicted performance of fractional order accumulation grey model is higher than among other model in three case studies [29]. One year later, Wu also applied fractional GM (1,1) model to predict the life of complex equipment, this result indicated that the fractional GM (1,1) show the effectiveness prediction with the lowest MAPE for in-sample and out-of sample data [32]. With the flexibility of fractional order accumulation r base on the priority of new information, the model has shown promising results.

After few years, many scholars using this model to explore in the real case. Zeng (2018) used the fractional order accumulation grey model to predict the total energy consumption in China and the monthly sales of new products in an enterprise. The result of this paper showed that the proposed model will be enhance the prediction accuracy [31]. Vu and Phan (2023) used the GM^r (1,1) model to forecast the number of tourist visits to Quang Ninh Province, Vietnam. The empirical results show that the proposed model will get a higher accuracy performance with the lowest MAPE =19.722% [30].

All above research focus on the improvement of the accuracy of model by changing the parameters and the background value. In this study provide the different way to enhance the prediction performance.

3. Materials and Methods

3.1. A brief introduction to the fractional- order accumulated Grey model “GM^r (1,1)”

GM^r (1,1) is a fractional-order accumulated and single - variable grey model with an interpolated coefficient in the background value [29]. For predictions involving linear small sample time series, its performance is better than that of the original grey forecasting models. The procedures involved for using the GM^r (1,1) can be summarized as follows:

Step1: Let the non-negative original data sequence X_0

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2),..., x^{(0)}(k),..., x^{(0)}(n)\} \quad n \geq 4 \tag{1}$$

Where n is a total number of modeling data

Step 2: Construct a new series data $X^{(r)}$ by using fractional-order accumulated generating operation (abbreviated as *r-AGO*)

$$X^{(r)} = (x^{(r)}(1), x^{(r)}(2), ...x^{(r)}(n)) = \left[x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(k), ..., x^{(0)}(n)\right] \begin{bmatrix} 1 & C_r^1 & ... & C_{r+n-3}^{n-2} & C_{r+n-2}^{n-1} \\ 0 & 1 & ... & C_{r+n-4}^{n-3} & C_{r+n-3}^{n-2} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & 1 & C_r^1 \\ 0 & 0 & ... & 0 & 1 \end{bmatrix} \quad (2)$$

Step 3: Building the GM^r (1,1) model by establishing the *r*-order differential equation with one variable is expressed as:

$$x^r(k) + az^{(r)}(k) = b\,(\,z^{(r)}(k) = 0.5x^{(r)}((k) + (k-1)), k = 2,3,...,n.) \quad (3)$$

Where $z^{(1)}(k)$ is the average value of consecutive data and is computed by following function:

$$z^{(1)}(k) = 0.5(x_k^r + x_{k-1}^r)$$

So, whiteness equation is the following:

$$\frac{dx^{(r)}(k)}{d(k)} + az^{(r)}(k) = b \quad (4)$$

The value of parameter "a" and "b" can be estimated by using least- square method. That is

$$\begin{bmatrix} a \\ b \end{bmatrix} = (A^T A)^{-1} A^T Y \quad (5)$$

Where

$$Y = \begin{bmatrix} x^{(r)}(2) - x^{(r)}(1) \\ x^{(r)}(3) - x^{(r)}(2) \\ ... \\ x^{(r)}(n) - x^{(r)}(n-1) \end{bmatrix} \quad (6)$$

and

$$A = \begin{bmatrix} -z^{(r)}(2) & 1 \\ -z^{(r)}(3) & 1 \\ ... & 1 \\ -z^{(r)}(n) & 1 \end{bmatrix} \quad (7)$$

Step 4: The solution of the equation (4) can be expressed as follows

$$\hat{x}^{(r)}(k) = \left[x^{(r)}(1) - \frac{b}{a}\right] e^{-a(k-1)} (1 - e^a) \quad (8)$$

Step 5: The simulations and forecasting value can be obtained by applying the fractional-order inverse accumulated generating operator (abbreviated as *r-IAGO*). Therefore, the fitted and predicted sequence is given $\hat{X}^{(0)}$ as:

$$\hat{X}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), ...\hat{x}^{(0)}(n)) =$$

$$\left[\hat{x}^{(r)}(1), \hat{x}^{(r)}(2), ..., \hat{x}^{(r)}(k), ..., \hat{x}^{(r)}(n)\right] \begin{bmatrix} 1 & -C_r^1 & ... & (-1)^{n-1} C_{r+n-2}^{n-1} \\ 0 & 1 & ... & (-1)^{n-2} C_{r+n-3}^{n-2} \\ ... & ... & ... & ... \\ 0 & 0 & ... & -C_r^1 \\ 0 & 0 & ... & 1 \end{bmatrix} \quad (9)$$

3.2 Modifying the residual error of GM^r (1,1) by Fourier series "Fourier- GM^r (1,1)"

In order to improve the prediction accuracy of fractional- order accumulated Grey model, the Fourier series was used to modify the residual error of the GM^r (1,1). The overall procedure to obtain the modified model is as the followings [24]:

Let x is the original series of m entries and v is the predicted series (obtained from GM (1,1). Based on the predicted series v, a residual series named ε is defined as:

$$\varepsilon = \{\varepsilon(k)\},\ k = 2,3,\ldots m \quad (10)$$

Where $\varepsilon(k) = x(k) - v(k)$, $k = 2,3,\ldots m$ (11)

According to the definition of the Fourier series, the residual sequence of GM^r (1,1) can be approximately expressed as:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_{(0)} + \sum_{i=1}^{Z}\left[a_i \cos\left(\frac{2\pi i}{m-1}(k)\right) + b_i \sin\left(\frac{2\pi i}{m-1}(k)\right)\right],\ k = 1,2,3,,\ m \quad (12)$$

Where $Z = (\frac{m-1}{2}) - 1$ called the minimum deployment frequency of Fourier series [35] and only take integer number, therefore, the residual series is rewritten as:

$$\varepsilon = P.C \quad (13)$$

Where

$$P = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{m-1}\times 2\right) \sin\left(\frac{2\pi\times 1}{m-1}\times 2\right) & \ldots & \cos\left(\frac{2\pi\times Z}{m-1}\times 2\right) \sin\left(\frac{2\pi\times Z}{m-1}\times 2\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{m-1}\times 3\right) \sin\left(\frac{2\pi\times 1}{m-1}\times 3\right) & \ldots & \cos\left(\frac{2\pi\times Z}{m-1}\times 3\right) \sin\left(\frac{2\pi\times Z}{m-1}\times 3\right) \\ \ldots & \ldots & \ldots & \ldots \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{m-1}\times m\right) \sin\left(\frac{2\pi\times 1}{m-1}\times m\right) & \ldots. & \cos\left(\frac{2\pi\times Z}{m-1}\times m\right) \sin\left(\frac{2\pi\times Z}{m-1}\times m\right) \end{bmatrix} \quad (14)$$

And $C = [a_0, a_1, b_1, a_2, b_2, \ldots, a_Z, b_Z]$ (15)

The parameter a0, a1, b1, a2, b2… a_Z, b_Z are obtained by using the ordinary least squares method (OLS) which results in the equation of:

$$C = \left(P^T P\right)^{-1} P^T \varepsilon^T \quad (16)$$

Once the parameters are calculated, the modified residual series is then achieved based on the following expression:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_{(0)} + \sum_{i=1}^{Z}\left[a_i \cos\left(\frac{2\pi i}{m-1}(k)\right) + b_i \sin\left(\frac{2\pi i}{m-1}(k)\right)\right] \quad (17)$$

From the predicted series v and $\hat{\varepsilon}$, the Fourier modified series $\hat{v}$ of series v is determined by:

$$\hat{v} = \{\hat{v}_1, \hat{v}_2, \hat{v}_3, \ldots, \hat{v}_k, \ldots, \hat{v}_m\} \quad (18)$$

Where

$$\hat{v} = \begin{cases} \hat{v}_1 = v_1 \\ \hat{v}_k = v_k + \hat{\varepsilon}_k \quad (k = 2,3,\ldots, m) \end{cases} \quad (19)$$

3.3. Evaluative precision of forecasting models

In order to evaluate the forecast capability of the model, Means Absolute Percentage Error (MAPE) index was used to evaluate the performance and reliability of forecasting technique [33]. It is expressed as follows:

$$MAPE = \frac{1}{m}\sum_{k=2}^{m}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\% \quad (20)$$

Where $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ are actual and forecasting values in time period k, respectively, and n is the total number of predictions. Wang et al. [24] interprets the MAPE results as a method to judge the accuracy of forecasts, where more than 50% is an inaccurate forecast, 20%-50% is a reasonable forecast, 10%-20% is a good forecast, and less than 10% is an excellent forecast.

4. Validation of the Fourier- GM^r (1,1) and It's application in Container Throughput Forecasting in Danang Port

4.1. Collection and setting data

Figure 1: Danang Port planning diagram

Danang Port was established in 1901, located in Danang Bay with an area of 100km^2. The port is protected by a 450m long breakwater, with a convenient transportation system, seamlessly connecting to Danang International Airport, Danang Railway Station industrial parks and the national highway system, convenient for transporting goods to all regions of the country. Danang Port is currently an important link in the Central Logistics service chain. The port was also chosen as the final point of the East-West economic corridor, connecting four ASEAN countries: Myanmar, Thailand, Laos, and Vietnam, and is the main gateway to the East Sea for domestic and foreign regions. The port can receive ships up to 50,000 DWT, container ships of 2,500 TUEs, and passenger ships up to 75,000 GRT [34]. The data of containers throughput in Danang Port was gathered from Danang Port Joint Stock Company during the period from 2011 to 2023 as a whole is visualized by Figure 2 and measured by TEUs [35].

As can we see the Figure 2, the trend of volume of containers throughput Danang Port is a wild fluctuation over the period. There was a steady increase in the volume of containers through Danang Port annually. This data reached 668,446 TEUs, increased more than 200.000 TEUs compared with 2011. However, the volume of containers throughput in the year of 2022 decline because the impact by COVID-19 pandemic. There are totally 13 observations available.

In order to demonstrate the superiority of proposed approach for both of interpolation and extrapolation data, this study sets the samples from 2011 to 2021 (12 data points) for in-sample estimation. And the remainders of the sample are reserved for out-of-sample forecasting purposes (or validation data set).

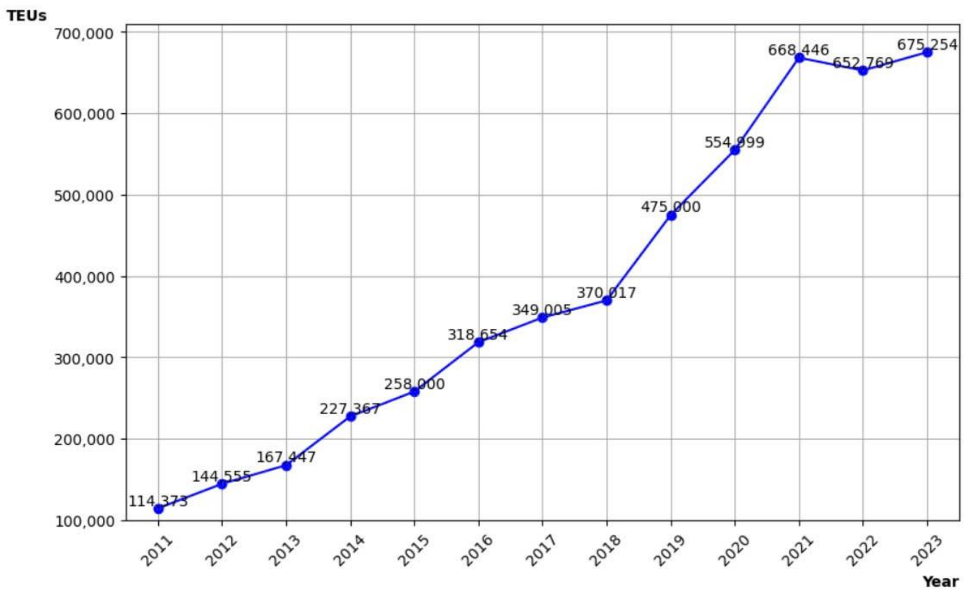

Figure 2: The volume of containers throughput Danang Port (Units: TEUs)

4.2 Tool and functions

For calculation and simulation of GM^r (1,1) and Fourier-GM^r (1,1) model, this research will use Microsoft Excel of Microsoft Corporation. This is a common software among users and multiple functions are integrated for calculation. Beside a basic function in excel, Excel software also offers two useful functions named Mmult (array 1, array 2) to return the matrix product of two relevant arrays and Minverse (array) to return the inverse matrix. They are two fundamental calculations of in model parameter values. After executing these calculations by Microsoft Excel. Forecasting results of two models is shown below.

4.3 The forecasted performance of GM^r (1,1) and Fourier - GM^r (1,1) model

In combination with the data set and algorithm of GM^r (1,1) and Fourier - GM^r (1,1) models which is indicated in section 3.1 and 3.2, respectively, this research found out the parameter value a= -0.048 and b = 33206.714 and r=0.01 of GM^r (1,1) and the parameter of a_0, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, a_5, b_5. The results of both models for forecasting the container volumes throughput in Danang Port as follow:

Table 1: Forecasted and error values of GM^r (1,1) and Fourier - GM^r (1,1) model

Actual value		**$GM^{0.01}$ (1,1)**		**Fourier - $GM^{0.01}$ (1,1)**	
	$x^{(0)}(k)$	$\hat{x}^{(0)}(k)$	Residual error (%)	$\hat{x}^{(0)}(k)$	Residual error **(%)**
2011	114,373	-	0	-	**0**
2012	144,555	152,744.07	5.67	144,555	**0**
2013	167,447	193,202.59	15.38	167,447	**0**
2014	227,367	235,633.91	3.64	227,367	**0**
2015	258,000	280,094.02	8.56	258,000	**0**
2016	318,654	326,670.81	2.52	318,654	**0**
2017	349,005	375,464.19	7.58	349,0055	**0**
2018	370,017	426,581.38	15.29	370,017	**0**
2019	475,000	480,135.67	1.08	475,000	**0**
2020	554,999	536,246.04	3.38	554,999	**0**
2021	668,446	595,037.21	10.98	668,446	**0**
2022	652,769	656,639.86	0.59	652,769	**0**
MAPE (in data samples)			6.79		**0**
Accuracy (%)			93.21 %		**100**
Evaluation			Excellent		Excellent
2023	675,254	721,190.86	6.80	713,001.78	**5.59**
MAPE (in data samples)			6.80		5.59
accuracy			93.2 %		**94.41**
Evaluation			excellent		Excellent

Table 1 show that the MAPE indexes of proposed model for in-sample and out-of sample forecast are 0% and 5.59 %, respectively. These results indicate that the forecasted performance of proposed model is the best fitting performance in this situation. In addition, figure 2 emphasize that the curve of Fourier - $GM^{(0.01)}$ (1, 1) extremely closed with actual data than the curve of $GM^{(0.01)}$ (1,1) model.

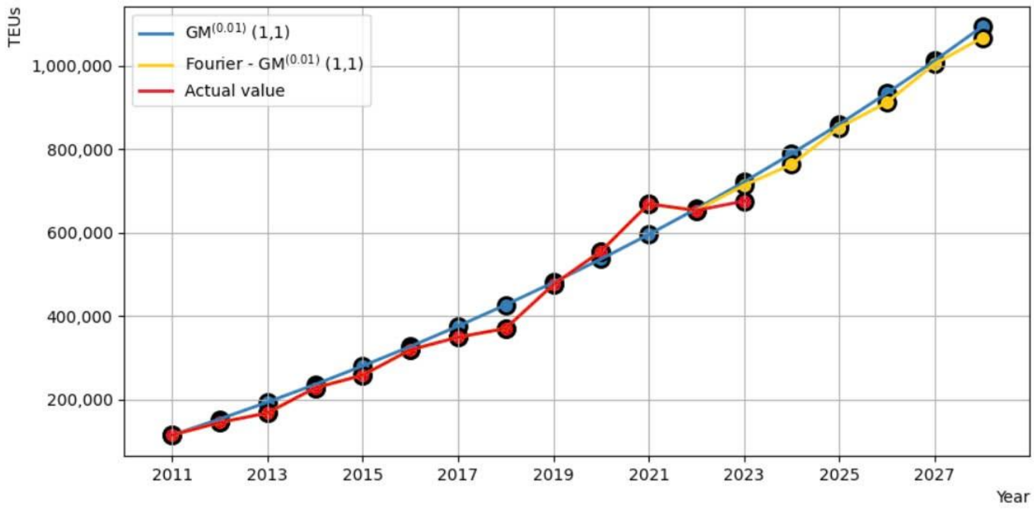

Figure 3: The visualization of $GM^{(0.01)}(1,1)$ and Fourier- $GM^{(0.01)}$ (1,1) model in containers throughput forecasting

4.4. The volume of container throughput in the year of 2025 and 2028

By comparing the accuracy between these two forecasting models aforementioned, this research propose the use of Fourier - $GM^{(0.01)}$ (1,1) model to forecast the containers volume throughput in Danang Port during the period between 2025 and 2028 because the accuracy of Fourier - $GM^{(0.01)}$ (1,1) model is better than $GM^{(0.01)}$ (1,1) forecasting model. The volume of container throughput forecasting in Danang Port during the period from 2025 to 2028 was illustrated in Table 2.

Table 2: The container throughput forecasted in Danang Port

Year	Container volume of Danang Port (Units: TEUs)
2025	851,451.19
2026	911,907.77
2027	1,003,832.61
2028	1,066,974.46

From the results of Figure 4, we can see that container volumes through Danang Port forecast continue to significantly grow during the next years. To be more detailed, containers volume through Danang Port will reach 851,451.19 TEUs in 2025 (26% increase compared with 2023) and this data is projected to hit the milestone of more than 1 million TEUs in the year of 2028. This will be an important data resources for orientation and planning management agency so as to boost the development of upcoming port activities.

5. Conclusions

By using the Fourier series modifies the residual errors of GM^r (1,1), this paper propose an effectiveness GM^r (1,1) model termed as Fourier - GM^r (1,1). Through the simulation in forecast the volume of container through Danang Port, this empirical results displayed the Fourier- GM^r (1,1) is the better model in numerical cases with the value of MAPE is smallest. By the way, the simulation results in forecast the volume of container through Danang Port will be an important data resources for orientation and planning management agency so as to boost the development of upcoming port activities. In the future direction, this study will be modified with other ways like Markov Chain to improve the the accuracy of the GM^r (1,1) model and will be applied this model to expand on these findings and to forecast performance of different industries.

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